

# Accelerating the Convergence of the Subspace Decomposition Channel Estimation Algorithm in OFDM System

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Received: Nov. 10, 2011; Revised Jan. 23, 2012; Accepted Feb. 13, 2012

**Abstract:** The ESPRIT algorithm is suitable for parametric channel estimation of slow varying channel of OFDM system. In this algorithm, the subspace decomposition technology is usually first used to estimate the multipath delay information of the channel, and then the channel is reconstructed through the estimate of the multipath delay information. This paper focuses on fast subspace tracking technology, and by using the frequency hopping principle, the convergence speed of the subspace tracking is accelerated. Theoretical analysis and simulation results show that, with applying this technology, subspace tracking performance is more stable, and the requirement that channel conditions should maintain unchanged over a longer period of time is not necessary.

**Keywords:** OFDM, channel estimation, ESPRIT, subspace tracking technology

## 1. Introduction

The traditional techniques of channel estimation of the OFDM system, such as linear interpolation based on frequency domain pilot assisted, high order interpolation, interpolation technique based on DFT and so on, have been paid much attention and research[1]. When the multipath time delay positions are not at the sampling points, the performance of the DFT algorithm and the improved DFT-based algorithms will decrease, and suffer from the MSE platform [2].

Parametric channel estimation method can obtain more accurate multipath time delay.

ESPRIT[3]-[5] (Estimating Signal Parameters via Rotational Invariance Techniques) algorithm is a typical kind of parametric channel estimation technique, which is originally used in signal processing field for wave arrival angle estimation[6]and then gradually be applied to frequency estimation [7] and channel estimation.

In the ESPRIT algorithm, by the fast subspace tracking technology is used for the channel subspace and noise subspace separation. The traditional fast subspace tracking techniques generally require that channel remain the

same in a constant period of time, and multiple symbols are needed. In this paper, frequency hopping principle is used to accelerate the convergence speed of the subspace tracking, and the technique is applied in single OFDM symbols.

## 2. Channel estimation algorithm based on accelerating ESPRIT

In sparse multipath environment, the OFDM system parameters of the equivalent baseband channel can be expressed as[13]:

$$h(\tau, t) = \sum_{l=0}^{L-1} h_l(t)\delta(\tau - \tau_l(t)) \quad (1)$$

where  $h_l(t)$  and  $\tau_l(t)$  represent the fading and time delay of the path  $l$  at the time  $t$ , and  $L$  is the number of the multipath. All the paths, which are uncorrelated, are complex Gauss process on the basis of jakes spectrum

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model. Then, in the OFDM system, the frequency domain response of the sparse multipath channels is

$$H(k) = \sum_{l=0}^{L-1} h_l \exp\left(\frac{-j2\pi k\tau_l}{NT}\right), k = 0, 1, \dots, N-1 \quad (2)$$

where  $\tau_l$  represents the multipath time delay (less than the cycle prefix), and the matrix is

$$\mathbf{H} = \mathbf{F}\mathbf{h} \quad (3)$$

where

$$\mathbf{H} = [H(0), H(1), \dots, H(N-1)]^T \quad (4)$$

$$\mathbf{h} = [h_0, h_1, \dots, h_{L-1}]^T \quad (5)$$

$\mathbf{F}$  is a Fourier transform matrix

$$[\mathbf{F}]_{k,l} = \exp(-j2\pi k\tau_l/(NT)) \quad (6)$$

Note that Eq. (3) is slow time-varying channel model. In the slow time-varying system, multipath time delay basically maintain unchanged in a certain period of time, which is far greater than the time needed to estimate the time delay parameter [8]. Therefore, in the following discussion, it's regarded as a constant.

### 2.1. ESPRIT MULTIPATH POSITIONING AND CHANNEL ESTIMATION ALGORITHM

The received OFDM symbols, of which the cyclic prefixes are removed, are FFT transformed. Then, the observed signals at the pilots are

$$\mathbf{Y}_p = \mathbf{X}_p \mathbf{H}_p + \mathbf{W}_p \quad (7)$$

where  $\mathbf{Y}_p$ ,  $\mathbf{H}_p$  and  $\mathbf{W}_p$ , are  $K \times 1$  dimensional column vectors.  $K$  is the total number of pilots.  $\mathbf{H}_p$  is the frequency domain response of the pilots,  $\mathbf{W}_p$  is the frequency response of the white Gaussian noise, with mean zero and variance  $\sigma^2$ , at the pilots. With the application of least square method, the unbiased estimation of  $\mathbf{H}_p$  can be represent as

$$\begin{aligned} \hat{\mathbf{H}}_p &= \mathbf{X}_p^{-1} \mathbf{Y}_p = \mathbf{H}_p + \mathbf{X}_p^{-1} \mathbf{W}_p \\ &= \mathbf{F}_p \mathbf{h} + \mathbf{W}'_p \end{aligned} \quad (8)$$

where  $\mathbf{F}_p$  is consists of all the lines corresponding to the pilots in DFT transformation matrix  $\mathbf{F}$ ,  $\mathbf{W}'_p \sim N(0, \sigma^2)$ .

Assumed that the pilots are distributed on the subcarriers as a uniformly-spaced mode, we can use the ESPRIT algorithm in ref.5, through the decomposition in signal subspace and noise subspace we can estimate the multipath time delay  $\hat{\tau}_l, l = 0, 1, \dots, L-1$ .

The estimating matrix for  $\mathbf{F}$  is

$$\hat{\mathbf{F}} : [\hat{\mathbf{F}}]_{k,l} = \exp(-j2\pi k\hat{\tau}_l/(NT)) \quad (9)$$

For Eq. (8), apply the least square method, we can get the estimating value for the multipath fading coefficient

$$\hat{\mathbf{h}} = (\hat{\mathbf{F}}_p^H \hat{\mathbf{F}}_p)^{-1} \hat{\mathbf{F}}_p^H \hat{\mathbf{H}}_p \quad (10)$$

### 2.2. ACCELERATING THE CONVERGENCE OF THE SUBSPACE TRACKING

The above derivation shows that the basis of the application of ESPRIT algorithm for time delay estimation is the use of the pilot frequency to identify the channel subspace from the noise subspace. Fast subspace tracking technology is one such technology [9].

Fast subspace tracking is a method for space decomposition in the form of iteration, based on sequence orthogonal projection [9]. The early application of this method is in the adaptive filter design. M. R. Raghavendra etc. applied it to the subspace channel estimation. The algorithm can be summarized as references [5]:

Initialization:  $L_m$  is the upper limit of the multipath number

$$\mathbf{Q}_0 = \begin{bmatrix} \mathbf{I}_{L_m} \\ \mathbf{0}_{(K-L_m) \times L_m} \end{bmatrix}, \mathbf{C}_0 = \mathbf{I}_{L_m}, \mathbf{A}_0 = \mathbf{0}_{(K \times L_m)}, \quad 0 \leq \gamma \leq 1 \quad (11)$$

In each iteration, updated:

$$\mathbf{Z}_n = \mathbf{Q}_{n-1}^H \hat{\mathbf{H}}_{ord,n} \quad (12)$$

$$\mathbf{A}_n = \gamma \mathbf{A}_{n-1} \mathbf{C}_{n-1} + (1 - \gamma) \hat{\mathbf{H}}_{ord,n} \mathbf{Z}_n^H \quad (13)$$

$$\mathbf{A}_n = \mathbf{Q}_n \mathbf{R}_n, \text{ (QR Decomposition)} \quad (14)$$

$$\mathbf{C}_n = \mathbf{Q}_{n-1}^H \mathbf{Q}_n, \quad (15)$$

where  $n$  and  $n-1$  represent the iterations,  $\gamma$  represents a constant close to 1 which is a forgetting factor. After iterations,  $\mathbf{Q}_n$  will converge to the eigenvector matrix of  $\hat{\mathbf{H}}_{ord,n}$ 's autocorrelation function, after the convergence of subspace tracking, we need to decide the leading diagonal value of  $\mathbf{R}_n$  to extract the effective path and avoid the interference of the noise path. One of the common methods is MDL (Minimum Description Length [9]):

$$\begin{aligned} MDL(\rho) &= -I(M - \rho) \log \left\{ \frac{(\prod_{k=\rho+1}^M \hat{\lambda}_k)^{1/(M-\rho)}}{\frac{1}{M} \sum_{k=\rho+1}^M \hat{\lambda}_k} \right\} \\ &+ \frac{1}{4} \rho [2M - \rho + 1] \log I \end{aligned} \quad (16)$$

where  $I$  represents the number of the OFDM symbols which make the statistical average.  $M$  represents the exponent number of  $\mathbf{R}_{HH}$ .

While using ESPRIT algorithm based on subspace tracking in OFDM system, it can be seen from the Eq. (3), if the channel changes slowly, then  $\mathbf{h}_n \approx \mathbf{h}_{n-1}$ , the impulse response of the channel on the adjacent two symbol have stronger relevance.

$$\begin{cases} \mathbf{H}_{p,n-1} = \mathbf{F}_{p,n-1} \mathbf{h}_{n-1} \\ \mathbf{H}_{p,n} = \mathbf{F}_{p,n} \mathbf{h}_n \end{cases} \quad (17)$$

It makes the convergence speed of the subspace tracking turns slower even cannot converge. This is also the reason why the traditional fast subspace tracking methods cannot be used in a single OFDM symbol.

Note that, the reason why subspace tracking algorithm converges slow or cannot converge, is the correlation of the channel information is too large in the iterations. Therefore the problem can be solved if the correlation of the pilots in adjacent symbols can be reduced.

In general case, the coherent bandwidth of OFDM system is few subcarriers in multipath fading channel, the authors bring forth new ideas from the frequency hopping system: supposing that the system keeps the pilot structure invariable on the adjacent OFDM symbols, and all the pilots hop the constant frequency point corresponding to the  $N$  subcarriers, the dependence of the adjacent OFDM symbols can be reduced. The idea can be named Tone-Hopping (TH).

When all the pilots in the adjacent OFDM symbols have  $\Delta$  offsets:

$$\mathbf{F}_{p,n} = \mathbf{F}_{p,n-1} \mathbf{\Lambda}, \quad (18)$$

and

$$\mathbf{\Lambda} = \text{diag}(\exp(-j2\pi\Delta\tau_0/N), \dots, \exp(-j2\pi\Delta\tau_{L-1}/N)) \quad (19)$$

Eq. (17) can be signified:

$$\begin{cases} \mathbf{H}_{p,n-1} = \mathbf{F}_{p,n-1} \mathbf{h}_{n-1} \\ \mathbf{H}_{p,n} = \mathbf{F}_{p,n-1} (\mathbf{\Lambda} \mathbf{h}_{n-1}) \end{cases} \quad (20)$$

It's obvious, because of Tone-Hopping, the phase rotation  $h_{n,l} = e^{-j2\pi\Delta\tau_l/N} h_{n-1,l}$  is introduced. (the channel impulse response is time invariant in the adjacent OFDM symbols).  $\mathbf{H}_{p,n-1}$  and  $\mathbf{H}_{p,n}$  have the uncorrelated character, it can accelerate convergence rate of subspace.

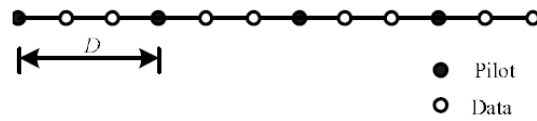


Figure 1 Schematic diagram of interval pilot

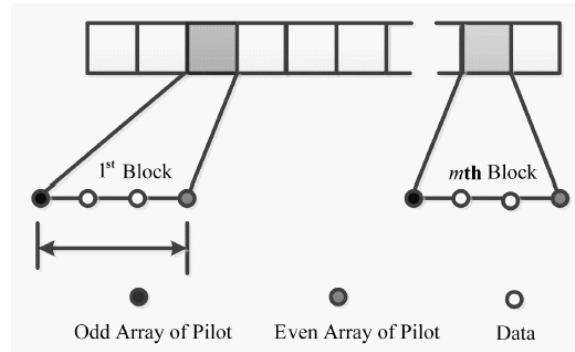


Figure 2 Block pilot structure diagram

### 3. The pilot design optimization and analysis of the ESPRIT

#### 3.1. The number of pilot and pilot interval optimization

Frequency resources are scarce, the number of pilot symbols, directly influences the system transmission efficiency. At the same time, the number of pilot symbols also has some influence on the channel estimation the algorithm complexity. Therefore, to ensure the performance of the system on the basis of, how to balance the frequency utilization rate and the complexity of the algorithm is becoming an important topic.

Because multi-path delay position  $\tau_l$  is calculated by using the phase  $\exp(-j2\pi\tau_l D/(NT))$  obtained, due to  $\exp(-j2\pi\tau_l D/(NT))$  the cycle  $2\pi$ , therefore, to ensure the  $\tau_l$  uniqueness, for all  $0 \leq l \leq L - 1$ , must meet:

$$\frac{\tau_l}{NT/D} < 1 \quad (21)$$

$$NT/D < \tau_{max} \quad (22)$$

Eq. (22) shows that, for a given system (i.e.,  $N$ ,  $T$  and  $\tau_{max}$  identified), ESPRIT algorithm  $D$  pilot interval by the maximum multipath delay decision. For example,  $N = 1024$ ,  $D < 1024/40 = 25.6$ , maximum 25, if the first sub-carrier start placing the pilot, then, a total of  $K = \lfloor 1024/25 \rfloor = 41$  subcarriers used for pilot frequency, which  $\lfloor \cdot \rfloor$  indicates rounding down. What need

points out is, (22) type and no limit to the number of pilot, that is to say, although at this time there were 41 sub-carriers can be used when the pilot, but in fact, in keeping the pilot interval under the premise, the pilot number less than 41, thus, it can reduce the pilot overhead, to improve frequency utilization rate. If so, then the pilot can at least reach a number? Theoretically to ensure that can estimate the full size (i.e.,  $\gamma$  at least one of the  $L$  main eigenvalues), must meet:

$$K - 1 \geq L \tag{23}$$

$$K \geq L + 1 \tag{24}$$

Therefore, theoretically desirable range of pilot symbols:

$$L + 1 \leq K \leq \lfloor N/D \rfloor \tag{25}$$

On the eigenvalue decomposition of ESPRIT algorithm, is also required:

$$K - C + 1 \geq L + 1 \tag{26}$$

That is

$$L + C \leq K \leq \lfloor N/D \rfloor \tag{27}$$

Apparently, when  $C = 1$ , Eq. (27) is (24).

### 3.2. Pilot structure design and optimization

ESPRIT algorithm is the core  $\mathbf{H}_2$  and  $\mathbf{H}_1$  the signal and noise subspace of the translational invariance. It was decided to rearrange the two set of pilot must have a fixed phase shift  $\exp(-j2\pi D/NT)$ . Then, according to the actual demands the pilot structure may have two kinds of choices. The first structure known as the space pilot, two-pilot point spaced the same data point (Figure 1), the upper section of the derivation process is at equal intervals pilot for assumption; second structures called block pilot structure, the whole OFDM frequency point is divided into several blocks the frequency points, each containing not less than two, i.e. at least two frequency for the pilot to use, any selected pieces as a pilot block (Figure 2). This kind of structure, the pilot blocks can be selected at random, increased pilot position randomness.

In fact, the block pilot structure in the pilot into odd and even array vector, wherein the pilot block first pilot, pilot block second pilot component, i.e.

$$\begin{aligned} \hat{\mathbf{H}}_o &= \mathbf{J}_1 \hat{\mathbf{H}}_p \\ \hat{\mathbf{H}}_e &= \mathbf{J}_2 \hat{\mathbf{H}}_p \end{aligned} \tag{28}$$

Among them,  $\mathbf{J}_1$  and  $\mathbf{J}_2$  is an order selection matrix of  $(K/2) \times K$ , i.e.

$$\begin{cases} \mathbf{J}_1 = [\mathbf{e}_1 \mathbf{0} \mathbf{e}_2 \mathbf{0} \dots \mathbf{e}_{K/2} \mathbf{0}] \\ \mathbf{J}_2 = [\mathbf{0} \mathbf{e}_1 \mathbf{0} \mathbf{e}_2 \dots \mathbf{0} \mathbf{e}_{K/2}] \end{cases} \tag{29}$$

Where,  $\mathbf{e}_i$  is unit vectors of the  $i$ th element, whose value is 1.

So, get reset pilot vector:

$$\begin{aligned} \hat{\mathbf{H}}_{ord} &= \begin{bmatrix} \hat{\mathbf{H}}_o \\ \hat{\mathbf{H}}_e \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 \hat{\mathbf{H}}_p \\ \mathbf{J}_2 \hat{\mathbf{H}}_p \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{J}_1 \mathbf{F}_p \mathbf{h} \\ \mathbf{J}_2 \mathbf{F}_p \mathbf{h} \end{bmatrix} + \mathbf{W}'_{ord} \\ &= \begin{bmatrix} \mathbf{F}_o \\ \mathbf{F}_e \end{bmatrix} \mathbf{h} + \mathbf{W}'_{ord} \end{aligned} \tag{30}$$

and

$$\mathbf{F}_e = \mathbf{F}_o \Phi \tag{31}$$

in which,

$$\Phi = \text{diag}[\exp(\frac{-j2\pi \Delta \tau_0}{N}),$$

$$\exp(\frac{-j2\pi \Delta \tau_1}{N}), \dots, \exp(\frac{-j2\pi \Delta \tau_{L-1}}{N})] \tag{32}$$

Where,  $\Delta$  is for the child block within the pilot interval.

Obviously, the pilot structure compared with equal interval pilot structure more flexible, more suitable for OFDMA system, drawback is that the pilot demand increases, i.e.

$$2L \leq K \leq \lfloor N/D \rfloor \tag{33}$$

## 4. SIMULATION RESULTS AND ANALYSIS

OFDM symbol length, cyclic prefix and modulation mode are shown as table 1, channel parameters is in the table 2 and 3(channel 1 and channel 2). The sampling time of channel 1 and 2 respectively is 2.5e-005s and 1.24e-007s, and pilot is uniformly-spaced.

Mean-square error of fading channels is

$$\text{MSE}(\mathbf{H}) = \frac{1}{NI} \sum_{i=1}^I \sum_{k=0}^{N-1} \frac{|\mathbf{H}(i, k) - \hat{\mathbf{H}}(i, k)|^2}{|\mathbf{H}(i, k)|^2} \tag{34}$$

$\mathbf{H}(i, k)$  is the channel frequency response on the  $k$ th frequency point in the  $i$ th OFDM system,  $N$  is the symbol length,  $I$  is the symbol sum during the simulation.

The estimated value of the mean-square error for time delay is:

**Table 1** OFDM system simulation parameters

Parameter Name	Parameter Values
Symbol length N	1024
CP length	120
Modulation mode	QPSK
Pilot number	128

**Table 2** Two path Rayleigh fading channel (channel 1)

Path number	Path location	Fading value(dB)
1	0	0
2	40.5	0

**Table 3** Two path Rayleigh fading channel (channel 2)

Path number	Path location	Fading value(dB)
1	0	-3
2	1.613	0
3	4.839	-2
4	12.903	-6
5	12.903	-8
6	40.323	-10

$$MSE(\tau_N) = \frac{1}{NI} \sum_{i=1}^I \sum_{l=0}^{L-1} |\tau_N(i, l) - \hat{\tau}_N(i, l)|^2 \quad (35)$$

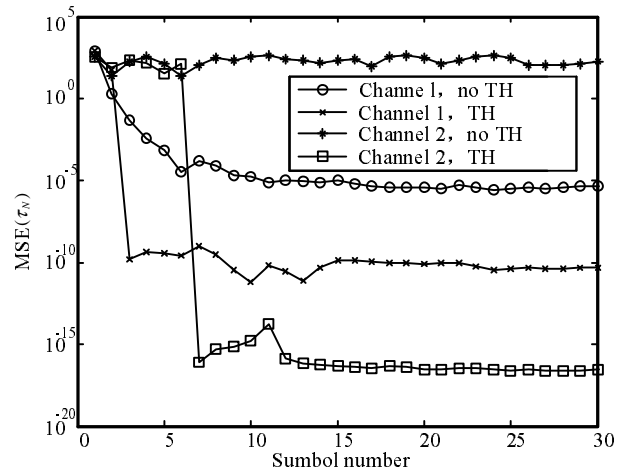
$\tau_N(i, l)$  is the time delay location of the  $l$ th path for the  $i$ th OFDM symbol,  $N$  is the symbol length,  $I$  is the symbol sum during the simulation.

#### 4.1. PERFORMANCE OF THE ALGORITHM

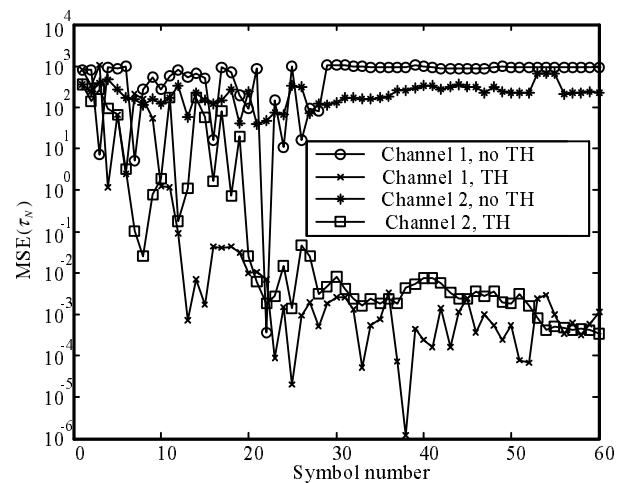
For convenience, pilot interval is 16, pilot number is 32. In Fig. 3, when the noise is negligible, if the TH algorithm is not applied, the subspace convergence in channel 1 during 6 symbols, and can't get the convergence in channel 2.

After the TH algorithm is applied, the subspace gets convergence in channel 1 during 3 symbols, and in channel 2, not only the subspace gets convergence during 7 symbols, but else the results reach the standard.

In fact, when the noise becomes larger (Fig 4), if the TH algorithm is not applied, the subspace can't get the convergence in channel 1, when the TH algorithm is applied, the subspace can gets convergence in channel 1 and 2, Their respective convergence period is 12 and 21 symbols. Fig. 5 is the simulation results from  $MSE(\mathbf{H})$ , the performance gets a improvement.



**Figure 3** TH algorithm convergence rate (the noise is negligible)



**Figure 4** TH algorithm convergence rate (SNR=20dB)

#### 4.2. PERFORMANCE OF SUBSPACE TRACKING IN SINGLE OFDM SYMBOL

Subspace tracking traditionally need track many OFDM symbol, it requires the channel character(path number and path location) keep unchanged during a longer time, these limit the application of ESPRIT algorithm based on subspace tracking.

When the number of subcarriers in OFDM is useful quantity -(such as 1024), and the pilot overhead is too many(such as 64), the subspace tracking can be finished in a single OFDM symbol, using the current symbol, the channel estimation can be accomplished. The results in Fig.6 and 7 prove this idea, in the simulations, the pilot number is 64, in every iteration, 16 pilots is used, other every 16 pilots is successively applied during iteration, the number of iteration is 49.

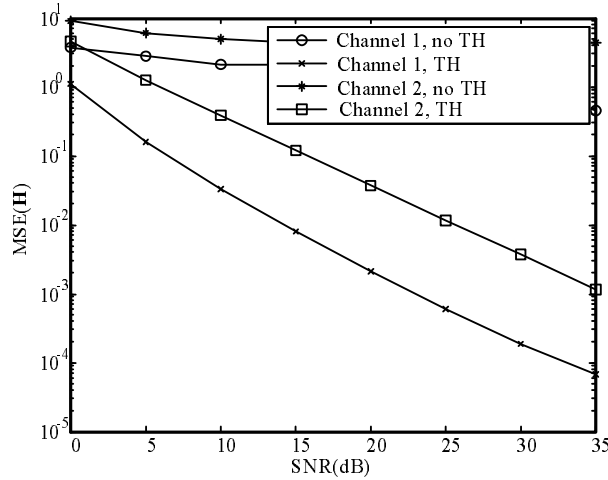


Figure 5 TH algorithm convergence rate (SNR=20dB)

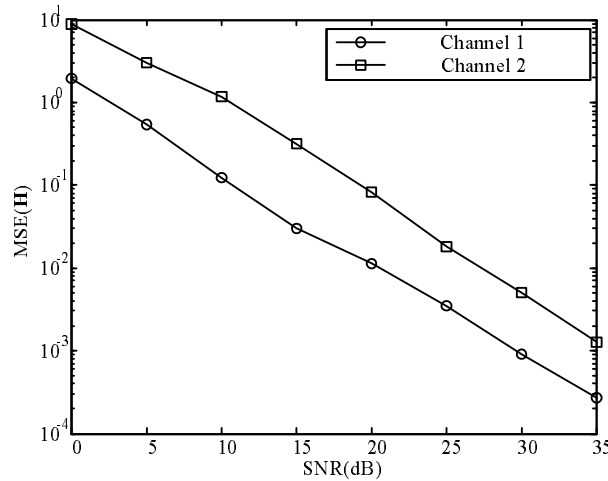


Figure 6 Performance of Subspace tracking applied for channel estimation in single OFDM symbol

### 4.3. Pilot interval and performance

According to the theory (Eq. (22)), the maximum pilot interval of channel one and channel two of is 25, i.e.

$$D_1 = \lfloor 1024/40.5 \rfloor = \lfloor 25.284 \rfloor = 25$$

$$D_2 = \lfloor 1024/40.323 \rfloor = \lfloor 25.3949 \rfloor = 25 \quad (36)$$

Thus, 25 is critical value of the two channel pilot frequency interval. In the following we will analysis in simulation the influence of the pilot interval on the performance of algorithm on ESPRIT when 24, 25, 26, in which the pilot number 32. Fig. 8 to Fig. 10 are respectively the two fractional time delay under Rayleigh fading channel delay variance, channel frequency response MSE and BER

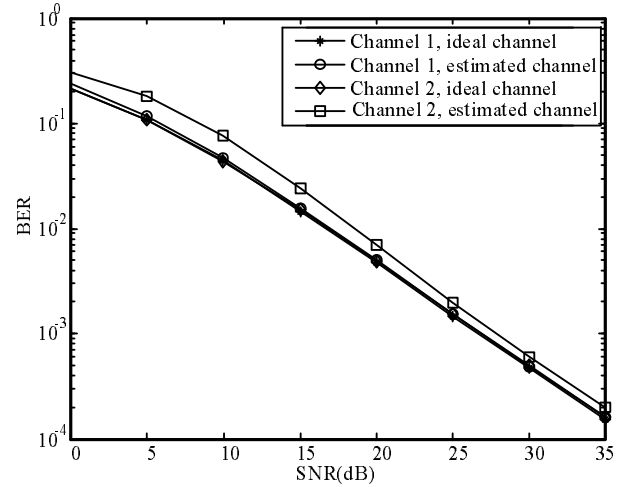


Figure 7 Performance of Subspace tracking applied for channel estimation in single OFDM symbol

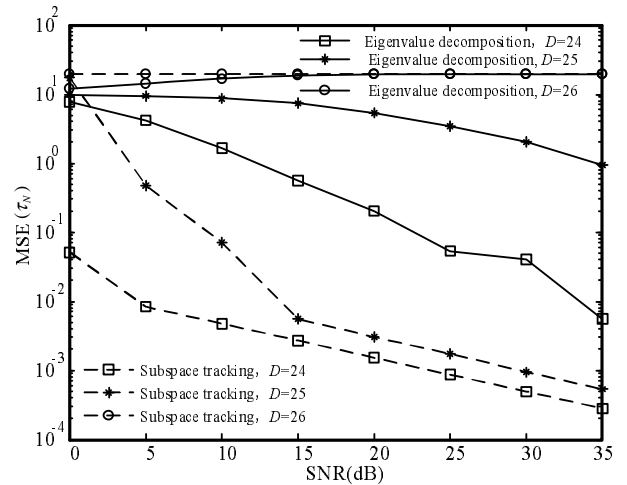
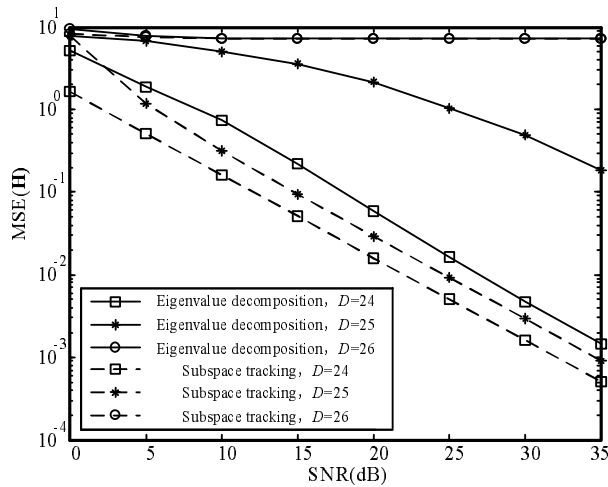
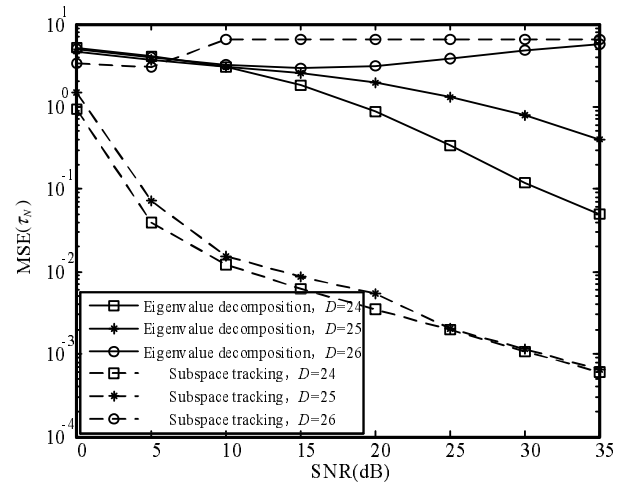


Figure 8 The influence of pilot interval algorithm based on ESPRIT on channel delay estimation performance ( two Rayleigh channel )

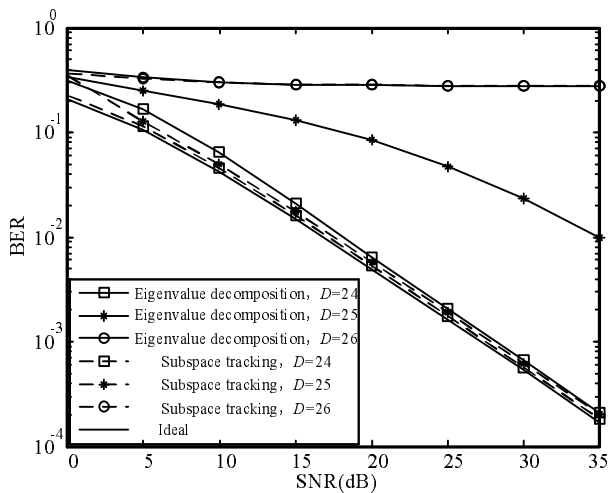
simulation curve, Fig. 11 - 13 COST207TUx6 channel delay under the MSE, channel frequency response MSE and BER simulation curve. From the chart we can see that, when the pilot interval is 24, the algorithm based on the eigenvalue decomposition of the ESPRIT and ESPRIT based on subspace tracking algorithm can obtain better delay and channel estimation performance, when the pilot interval for a critical value of 25, the algorithm based on the subspace tracking algorithm of ESPRIT still can get better performance, but the algorithm based on the eigenvalue decomposition ESPRIT algorithm performance began to deteriorate whose mainly reason is that subspace tracking use before and after the symbol statistical characteris-



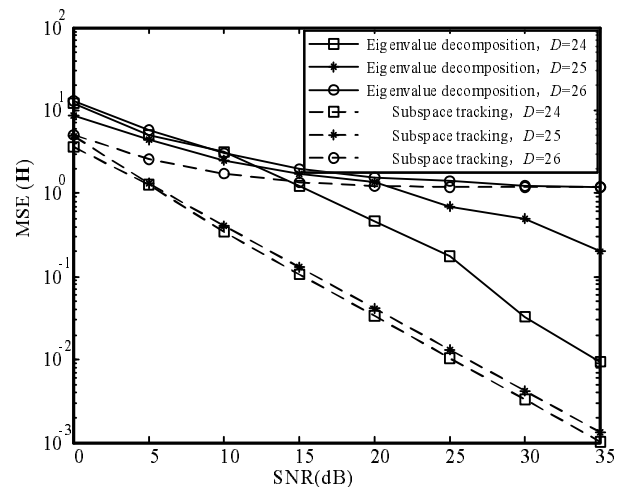
**Figure 9** The influence of pilot interval algorithm based on ESPRIT on channel estimation performance ( two Rayleigh channel )



**Figure 11** The influence of pilot interval algorithm based on ESPRIT on time delay estimation performance ( COST207TUx6)



**Figure 10** The bit error rate performance of different pilot interval ( two Rayleigh channel )



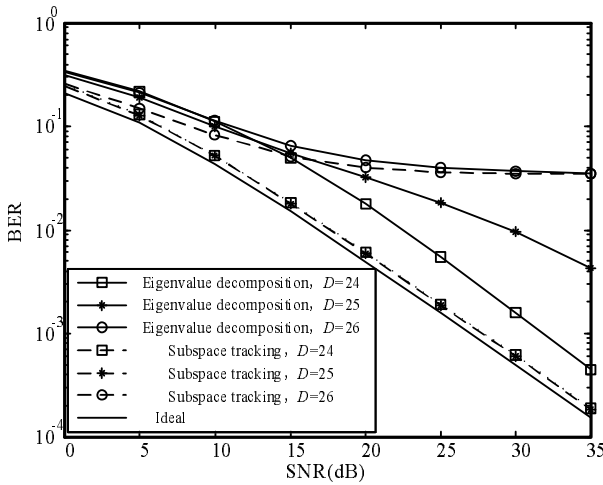
**Figure 12** The influence of pilot interval algorithm based on ESPRIT on channel estimation performance ( COST207TUx6)

tics, and the eigenvalue decomposition method using only the current one of symbols, pilot quantity less, leading to poor statistical properties. It will be further proofed in the next section that increasing the pilot number can improve eigenvalue decomposition based ESPRIT algorithm performance. Whileas two algorithms are both not available when the pilot interval is 26 because when the delay exceeding  $N/(2D)$  a path has been estimated to be not true.

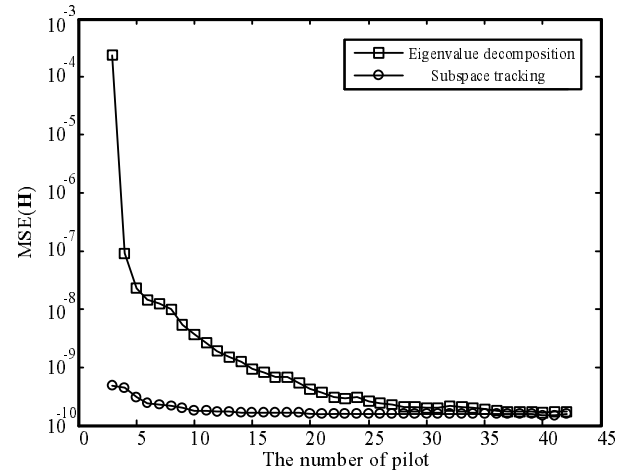
In addition, it is notable that the maximum multipath delay needs  $N/D$  correction when it more than  $N/(2d)$ . When  $d = 25$ , in the ideal environment in noise-free channel, the estimated value for the second size, normalized multipath time delay for 40.5 size is

$$\hat{\tau}_N^{\text{temp}}(2) = 40.5 - 1024/25 = -0.46 \quad (37)$$

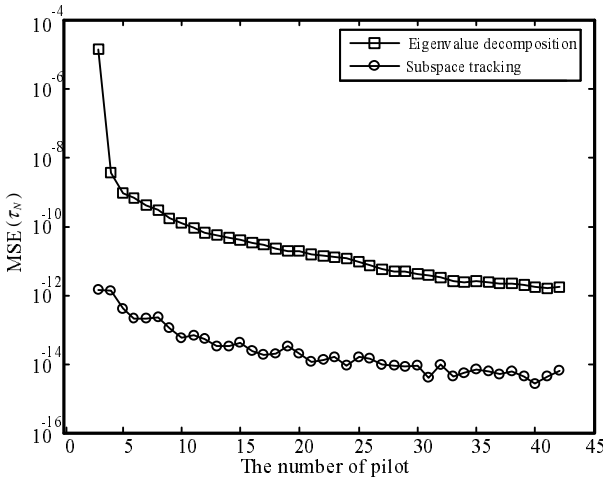
The estimated value and the estimated value is very close to the first radial, and due to the influence of noise, the actual time delay estimation is error and a first radial estimates may also be a less than 0 the number .Thus, it is hard to tell which estimate is a first radial and which estimate is the last one. Therefore, in order to reduce multipath discrimination difficulty and ensure the multipath resolution accuracy (mainly the first radial and finally radial, Pilot interval value should not be too close to the critical value in the actual system.



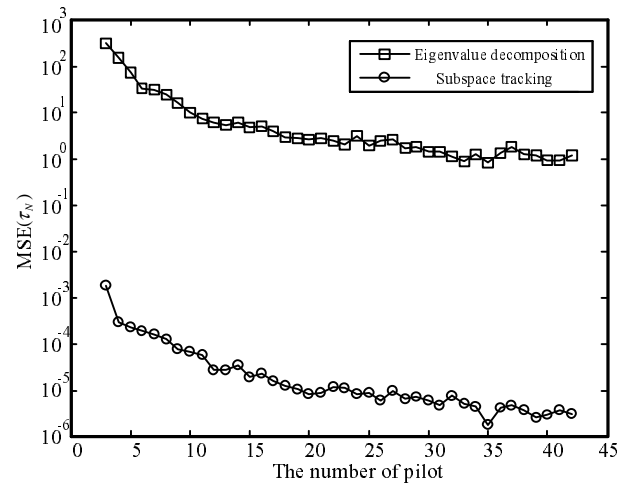
**Figure 13** Different pilot interval of the bit error rate performance ( COST207TUx6)



**Figure 15** The effect of pilot number on channel estimation ( noise is small enough)



**Figure 14** The influence of pilot quantity on time-delay estimation ( noise is small enough)



**Figure 16** The influence of pilot quantity on time-delay estimation ( SNR = 20dB )

#### 4.4. Pilot quantity and performance

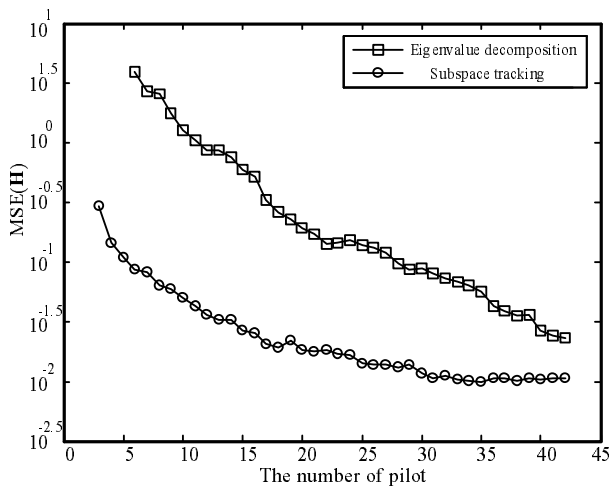
It uses the table 2 to show the two Rayleigh fading channel with pilot interval 24 and pilot frequency maximum 42.

Fig. 14 and Fig.15 show that delay and channel frequency response mean square error performance curve in environment of noise sufficiently small. Although the maximum multi-path delay for 40.5Ts, ESPRIT algorithm requires only 3 pilot point in extreme cases, while the DFT algorithm requires at least 41 pilot. In Figure 14 and Figure 15, eigenvalue decomposition based ESPRIT algorithm can arrive -40dB ( $1e-004$ ) and 30dB ( $1e-003$ ), while, based on the subspace tracking, the  $MSE(\tau_N)$  and  $MSE(\mathbf{H})$  of ESPRIT algorithm can even reach the -120dB and -90dB.

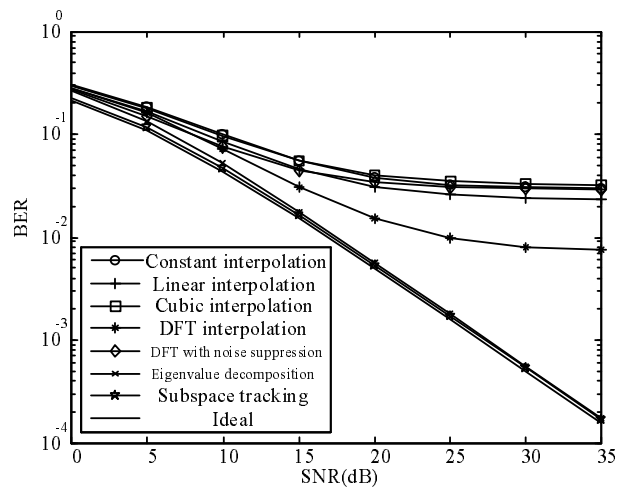
Moreover, when the pilot number is more than 10, the impact of the increasing of the pilot quantity on system performance will stabilized gradually.

However, when the noise cannot be ignored, then the influence of the pilot bechanced deep fading is bigger compared to that not bechanced because the time domain noise still is the Gauss noise, which will be added to each sub-carrier, after transforming into the frequency domain by Gauss DFT linear, if the pilot too little, then it cannot ensure a sufficient effective pilot, which leads to the estimation performance deterioration. It simulates the signal-to-noise ratio of 20dB performance in Figure 16 and Figure 17. Seeing in form of  $MSE(\mathbf{H})$ , eigenvalue decomposition based ESPRIT algorithm requires approximately 30 pi-

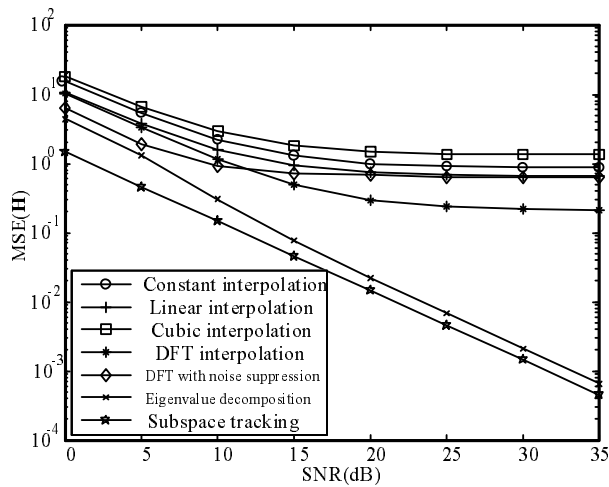




**Figure 17** The effect of pilot number on channel estimation ( SNR =20dB )



**Figure 19** ESPRIT channel estimation algorithm and the conventional channel estimation algorithms in performance comparison: BER



**Figure 18** ESPRIT channel estimation algorithm and the conventional channel estimation algorithms in performance comparison: MSE(H)

lot frequency to achieve the 10dB, the ESPRIT algorithm based on subspace tracking requires about 6-7 pilot. In the actual system pilot volume should not be too little, It can minimize pilot in order to improve frequency utilization rate in guaranteeing the performance of the premise.

#### 4.5. Comparison between ESPRIT algorithm and traditional algorithm performance

Finally, to describe the algorithm of ESPRIT, the paper makes a contraction between the ESPRIT algorithm and the traditional interpolation channel estimation algorithm

(Fig. 18, Fig. 19), it uses the COST207TUx6 channel shows in table 3 with pilot number 64, and pilot interval 16. It is to make a comparison to Fig. 12 and 13 to using the pilot number 64 where MSE(H), which based on the subspace tracking algorithm ESPRIT mean difference of 10dB, reduced to 2dB and BER is almost coincidence, and the two BER curve is almost the ideal channel BER curve coincide. While in Figure 12 and figure 13, based on the eigenvalue decomposition, ESPRIT algorithm performance is poor and BER mean difference 5dB. At the same time, MSE(H) and BER are far superior to the traditional interpolation algorithms.

### 5. CONCLUSIONS

In this paper, based on the subspace tracking algorithm ESPRIT, it puts forward to accelerate tracking speed of algorithm. At the same time, it gives the pilot number and pilot structure optimization design aiming at the character of ESPRIT algorithm. And then the simulation results verify that the algorithm's performance depends mainly on the pilot interval and the pilot quantity can be less than the maximum multi-path delay in the condition of pilot interval meeting the required condition, therefore, in order to improve frequency utilization rate and the data transmission rate, to reduce pilot usage as far as possible, at the same time, it also makes it possible to the application of ESPRIT algorithm in the OFDMA system channel estimation. And then it has been validated through the simulation that it can accelerate the convergence speed of ESPRIT algorithm through the frequency hopping method based on subspace tracking, and combining the thought, it estimates the channel's conclusion of the algorithm based on the eigenvalue decomposition ESPRIT, simulating to the algorithm based on the subspace tracking algorithm of ES-

PRIT, can estimate the channel by symbols, which provides possibility for the algorithm in the center frequency hop variable frequency hopping systems, and improves the algorithm's portability because the algorithm based on the subspace tracking has better performance than ESPRIT eigenvalue decomposition based ESPRIT algorithm performance from the view of simulation. Then, it randomizes pilot structure application of the algorithm based on the subspace tracking algorithm in ESPRIT block by simulation, and come to a conclusion that the pilot structure makes the ESPRIT algorithm can be used for pilot which requires a more flexible scene. Finally, in contrast to two kinds of ESPRIT algorithm and the traditional interpolation algorithms and draw a conclusion, in non integer delay multi-path environment, ESPRIT algorithm for channel estimation mean square error is small and bit error rate is low.

## Acknowledgement

This work was supported by the National Science Foundation of China under Grant number 61032002, 71020107027, 71001075 and 61101090 Chinese Important National Science & Technology Specific Projects under Grant 2011ZX03001-007-01, Program for New Century Excellent Talents in University, NCET-11-0058.

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