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Supra Soft Separation Axioms Based on Supra β-Open Soft Sets

*A. M. Abd El-latif**

Mathematics Department, Faculty of Education, Ain Shams University, Cairo, Egypt.

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Abstract: The notion of supra soft topological spaces was initiated for the first time by El-sheikh et al.[13]. In this paper, we introduce and investigate some weak soft separation axioms by using the notion of supra β-open soft sets, which is a generalization of the supra soft separation axioms mentioned in [2]. We study the relationships between these new soft separation axioms and their relationships with some other properties. As a consequence the relations of some supra soft separation axioms are shown in a diagram. We show that, some classical results in general supra topology are not true if we consider supra soft topological spaces instead. For instance, if (X,μ,E) is supra soft β-T_1-space need not every soft singleton x_E is supra β-closed soft.

Keywords: Soft sets, Soft topological spaces, Supra soft topological spaces Supra β-open soft sets, Supra soft β-T_i spaces $(i = 1,2,3,4)$, Supra soft continuity, Supra β-irresolute open soft function.

1 Introduction

Soft set theory is one of the emerging branches of mathematics that could deal with parameterization inadequacy and vagueness that arises in most of the problem solving methods. It is introduced [36] in 1999 by the Russian mathematician Molodtsov with its rich potential applications in divergent directions such as stability and regularization, game theory, operations research, soft analysis etc [36,37]. Further research works produced so many definitions, results and practical applications. After presentation of the operations of soft sets [32], the properties and applications of soft set theory have been studied increasingly [6,28,37,39]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [4,5,9,16,25,30,31,32,33,37,38,47]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [10].

It got some stability only after the introduction of soft topology [40] in 2011. In [17], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, $\alpha-$open soft and $\beta-$open soft and investigated their properties in detail. Kandil et al. [24] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al. [20]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al.[18,19,21,22,23,26]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [13]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of $b-$open soft sets was initiated for the first time by El-sheikh and Abd El-latif [12], which is generalized to the supra soft topological spaces in [1,14].

The main purpose of this paper, is to generalize the notion of supra soft separation axioms [2] by using the notions of supra β-open soft sets.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the paper.

* Corresponding author e-mail: Alaa_8560@yahoo.com, Alaa8560@hotmail.com

Definition 2.1.[36] Let X be an initial universe and E be a set of parameters. Let $\mathscr{P}(X)$ denote the power set of X and A be a non-empty subset of E. A pair F denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to \mathscr{P}(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of $e-$approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \emptyset$ i.e $F_A = \{F(e) : e \in A \subseteq E,\ F : A \to \mathscr{P}(X)\}$.

Definition 2.2.[11] A soft set F over X is a set valued function from E to $\mathscr{P}(X)$. It can be written a set of ordered pairs $F = \{(e,F(e)) : e \in E\}$. Note that if $F(e) = \emptyset$, then the element $(e,F(e))$ is not appeared in F. The set of all soft sets over X is denoted by $S_E(X)$.

Definition 2.3.[11] Let $F,G \in S_E(X)$. Then,

(1)If $F(e) = \emptyset$ for each $e \in E$, F is said to be a null soft set, denoted by $\tilde{\Phi}$.
(2)If $F(e) = X$ for each $e \in E$, F is said to be absolute soft set, denoted by $\tilde{X}$.
(3)F is soft subset of G, denoted by $F\tilde{\subseteq}G$, if $F(e) \subseteq G(e)$ for each $e \in E$.
(4)$F = G$, if $F\tilde{\subseteq}G$ and $G\tilde{\subseteq}F$.
(5)Soft union of F and G, denoted by $F\tilde{\cup}G$, is a soft set over X and defined by $F\tilde{\cup}G : E \to \mathscr{P}(X)$ such that $(F\tilde{\cup}G)(e) = F(e) \cup G(e)$ for each $e \in E$.
(6)Soft intersection of F and G, denoted by $F\tilde{\cap}G$, is a soft set over X and defined by $F\tilde{\cap}G : E \to \mathscr{P}(X)$ such that $(F\tilde{\cap}G)(e) = F(e) \cap G(e)$ for each $e \in E$.
(7)Soft complement of F is denoted by $F^{\tilde{c}}$ and defined by $F^{\tilde{c}} : E \to \mathscr{P}(X)$ such that $F^{\tilde{c}}(e) = X \setminus F(e)$ for each $e \in E$.

We will consider Definition 2.2 and Definition 2.3. the rest of paper.

Definition 2.4.[48] The soft set $F \in S_E(X)$ is called a soft point if there exist an $e \in E$ such that $F(e) \neq \emptyset$ and $F(e') = \emptyset$ for each $e' \in E \setminus \{e\}$, and the soft point F is denoted by e_F. The soft point e_F is said to be in the soft set G, denoted by $e_F \tilde{\in} G$, if $F(e)\tilde{\subseteq}G(e)$ for the element $e \in E$.

Definition 2.5.[24,40] The soft set (F,E) over X such that $F(e) = \{x\}\ \forall e \in E$ is called singleton soft point and denoted by x_E or (x,E).

Definition 2.6.[3] Let $S_E(X)$ and $S_K(Y)$ be families of soft sets, $u : X \to Y$ and $p : E \to K$ be mappings. Therefore $f_{pu} : S_E(X) \to S_K(Y)$ is called a soft function.

(1)If $F \in S_E(X)$, then the image of F under f_{pu}, written as $f_{pu}(F)$, is a soft set in $S_K(Y)$ such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F(e)), & p^{-1}(k) \neq \emptyset \\ \emptyset, & \text{otherwise.} \end{cases}$$

for each $k \in Y$.

(2)If $G \in S_K(Y)$, then the inverse image of G under f_{pu}, written as $f_{pu}^{-1}(G)$, is a soft set in $S_E(X)$ such that

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(e))), & p(e) \in Y \\ \emptyset, & \text{otherwise.} \end{cases}$$

for each $e \in E$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Theorem 2.1.[3] Let $S_E(X)$ and $S_K(Y)$ be families of soft sets. For the soft function $f_{pu} : S_E(X) \to S_K(Y)$, for each $F,F_1,F_2 \in S_E(X)$ and for each $G,G_1,G_2 \in S_K(Y)$ the following statements hold,

(1)$f_{pu}^{-1}(G^{\tilde{c}}) = (f_{pu}^{-1}G)^{\tilde{c}}$.
(2)$f_{pu}(f_{pu}^{-1}(G))\tilde{\subseteq}G$. If f_{pu} is surjective, then the equality holds.
(3)$F\tilde{\subseteq}f_{pu}^{-1}(f_{pu}(F))$. If f_{pu} is injective, then the equality holds.
(4)$f_{pu}(\tilde{X})\tilde{\subseteq}\tilde{Y}$. If f_{pu} is surjective, then the equality holds.
(5)$f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ and $f_{pu}(\tilde{\Phi}) = \tilde{\Phi}$.
(6)If $F_1\tilde{\subseteq}F_2$, then $f_{pu}(F_1)\tilde{\subseteq}f_{pu}(F_2)$.
(7)If $G_1\tilde{\subseteq}G_2$, then $f_{pu}^{-1}(G_1)\tilde{\subseteq}f_{pu}^{-1}(G_2)$.
(8)$f_{pu}^{-1}(G_1\tilde{\cup}G_2) = f_{pu}^{-1}(G_1)\tilde{\cup}f_{pu}^{-1}(G_2)$ and $f_{pu}^{-1}(G_1\tilde{\cap}G_2) = f_{pu}^{-1}(G_1)\tilde{\cap}f_{pu}^{-1}(G_2)$.
(9)$f_{pu}(F_1\tilde{\cup}F_2) = f_{pu}(F_1)\tilde{\cup}f_{pu}(F_2)$ and $f_{pu}(F_1\tilde{\cap}F_2)\tilde{\subseteq}f_{pu}(F_1)\tilde{\cap}f_{pu}(F_2)$. If f_{pu} is injective, then the equality holds.

Definition 2.7.[40] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq S_E(X)$ is called a soft topology on X if

$\tilde{X}, \tilde{\Phi} \in \tau$, where $\tilde{\Phi}(e) = \emptyset$ and $\tilde{X}(e) = X$, for each $e \in E$,
(1)The soft union of any number of soft sets in τ belongs to τ,
(2)The soft intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. A soft set (F,E) over X is said to be open soft set in X if $(F,E) \in \tau$, and it is said to be closed soft set in X, if its relative complement $(F,E)'$ is an open soft set. We denote the set of all open soft sets over X by $OS(X,\tau,E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition 2.8.[40] Let (X,τ,E) be a soft topological space over X and $F \in S_E(X)$. Then, the soft interior and soft closure of F, denoted by $int(F)$ and $cl(F)$, respectively, are defined as,

$$int(F) = \tilde{\bigcup}\{G : G \text{ is open soft set and } G\tilde{\subseteq}F\}$$
$$cl(F) = \tilde{\bigcap}\{H : H \text{ is closed soft set and } F\tilde{\subseteq}H\}.$$

If there exists at least two soft topologies τ_1 and τ_2 over X, then to avoid confusion it can be written $int_{\tau_1}(F)$ and $int_{\tau_2}(F)$ for $F \in S_E(X)$.

Definition 2.9.[48] Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces and $f_{pu} : S_E(X) \to S_K(Y)$ be a function. Then, the function f_{pu} is called,

(1)Continuous soft if $f_{pu}^{-1}(G,K) \in \tau_1$ for each $(G,K) \in \tau_2$.

(2)Open soft if $f_{pu}(F,E) \in \tau_2$ for each $(F,E) \in \tau_1$.

Definition 2.10.[13] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq S_E(X)$ is called supra soft topology on X with a fixed set E if

(1)$\tilde{X}, \tilde{\Phi} \in \mu$,
(2)The soft union of any number of soft sets in μ belongs to μ.

The triplet (X,μ,E) is called supra soft topological space (or supra soft spaces) over X.

Definition 2.11.[13] Let (X,τ,E) be a soft topological space and (X,μ,E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau \subseteq \mu$.

Definition 2.12.[13] Let (X,μ,E) be a supra soft topological space over X, then the members of μ are said to be supra open soft sets in X. We denote the set of all supra open soft sets over X by $supra-OS(X,\mu,E)$, or when there can be no confusion by $supra-OS(X)$ and the set of all supra closed soft sets by $supra-CS(X,\mu,E)$, or $supra-CS(X)$.

Definition 2.13.[13] Let (X,μ,E) be a supra soft topological space over X and $F \in S_E(X)$. Then the supra soft interior of F, denoted by $int^s(F)$ is the soft union of all supra open soft subsets of F. Clearly $int^s(F)$ is the largest supra open soft set over X which contained in F i.e

$$int^s(F) = \tilde{\bigcup}\{G : G \text{ is supra open soft set and } G\tilde{\subseteq}F\}.$$

Definition 2.14.[13] Let (X,μ,E) be a supra soft topological space over X and $F \in S_E(X)$. Then, the supra soft closure of F, denoted by $cl^s(F)$ is the soft intersection of all supra closed super soft sets of F. Clearly $cl^s(F)$ is the smallest supra closed soft set over X which contains F i.e

$$cl^s(F) = \tilde{\bigcap}\{H : H \text{ is supra closed soft set and } F\tilde{\subseteq}H\}.$$

Definition 2.15.[13] Let (X,μ,E) be a supra soft topological space and $(F,E) \in S_E(X)$. Then, (F,E) is called supra β-open soft set if $(F,E)\tilde{\subseteq}cl^s(int^s(cl^s(F,E)))$. We denote the set of all supra β-open soft sets by $S\beta OS(X,\mu,E)$, or $S\beta OS_E(X)$ and the set of all supra β-closed soft sets by $S\beta CS(X,\mu,E)$, or $S\beta CS_E(X)$.

Definition 2.16.[2,13] Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu} : S_E(X) \to S_K(Y)$ is called

(1)Supra continuous soft function if $f_{pu}^{-1}(G,K) \in \mu_1$ for each $(F,E) \in \tau_2$.
(2)Supra open soft if $f_{pu}(F,E) \in \mu_1$ for each $(F,E) \in \tau_1$.
(3)Supra irresolute soft if $f_{pu}^{-1}(F,E) \in \mu_1$ for each $(F,E) \in \mu_2$.
(4)Supra irresolute open soft if $f_{pu}(F,E) \in \mu_2$ for each $(F,E) \in \mu_1$.
(5)Supra β-continuous soft if $f_{pu}^{-1}(F,E) \in S\beta OS(X,\mu_1,E)$ for each $(F,E) \in \mu_2$.

3 Supra soft β-separation axioms

Definition 3.1. Let (X,τ,E) be a soft topological space and μ be an associated supra soft topology with τ. Let $x,y \in X$ such that $x \neq y$. Then, (X,μ,E) is called

(1)Supra soft β-T_0-space (S-soft β-T_0 for short) if there exists a μ-supra β-open soft set (F,E) containing one of the points x,y but not the other.
(2)Supra soft β-T_1-space (S-soft β-T_1 for short) if there exist μ-supra β-open soft sets (F,E) and (G,E) such that $x \in (F,E), y \notin (F,E)$ and $y \in (G,E), x \notin (G,E)$.
(3)Supra soft Hausdorff space or supra soft β-T_2-space (S-soft β-T_2 for short) if there exist μ-supra β-open soft sets (F,E) and (G,E) such that $x \in (F,E)$, $y \in (G,E)$ and $(F,E)\tilde{\cap}(G,E) = \tilde{\Phi}$.

Proposition 3.1. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. If there exist μ-supra β-open soft sets (F,E) and (G,E) such that either $x \in (F,E)$ and $y \in (F,E)^{\tilde{c}}$ or $y \in (G,E)$ and $x \in (G,E)^{\tilde{c}}$. Then, (X,μ,E) is supra soft β-T_0-space.

Proof. Let $x,y \in X$ such that $x \neq y$. Let (F,E) and (G,E) be μ-supra β-open soft sets such that either $x \in (F,E)$ and $y \in (F,E)^{\tilde{c}}$ or $y \in (G,E)$ and $x \in (G,E)^{\tilde{c}}$. If $x \in (F,E)$ and $y \in (F,E)^{\tilde{c}}$. Then, $y \in (F(e))^{\tilde{c}}$ for each $e \in E$. This implies that, $y \notin F(e)$ for each $e \in E$. Therefore, $y \notin (F,E)$. Similarly, if $y \in (G,E)$ and $x \in (G,E)^{\tilde{c}}$, then $x \notin (G,E)$. Hence, (X,τ,E) is supra soft β- T_0-space.

Theorem 3.1. A supra soft topological space (X,μ,E) is supra soft β-T_0-space if and only if the supra closures of each distinct points x and y are distinct.

Proof. Necessity: Let (X,μ,E) be a supra soft β-T_0-space and $x,y \in X$ such that $x \neq y$. Then, there exists a μ-supra β-open soft set (F,E) such that $x \in (F,E)$ and $y \notin (F,E)$. Hence, $(F,E)^{\tilde{c}}$ is supra β-closed soft set containing y but not x. It follows that, $cl^s(y_E)\tilde{\subseteq}(F,E)^{\tilde{c}}$. Therefore, $x \notin cl^s(y_E)$. Thus, $cl^s(x_E) \neq cl^s(y_E)$.

Sufficient: Let x,y be two distinct points in X such that $cl^s(x_E) \neq cl^s(y_E)$. Then, there exists a point z belongs to one of the sets $cl^s(x_E), cl^s(y_E)$ but not the other. Say, $z \in cl^s(x_E)$ and $z \notin cl^s(y_E)$. Now, if $x \in cl^s(y_E)$. Then, $cl^s(x_E)\tilde{\subseteq}cl^s(y_E)$, which is a contradiction with $z \notin cl^s(y_E)$. So, $x \notin cl^s(y_E)$. Hence, $[cl^s(y_E)]^{\tilde{c}}$ is supra β-open soft set containing x but not y. Thus, (X,μ,E) is supra soft β-T_0-space.

Proposition 3.2. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. If there exist μ-supra β-open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \in (F,E)^{\tilde{c}}$ and $y \in (G,E)$ and $x \in (G,E)^{\tilde{c}}$. Then (X,τ,E) is supra soft β-T_1-space.

Proof. It is similar to the proof of Proposition 3.1.

Theorem 3.2. Every supra soft β-T_i-space is supra soft β-T_{i-1} for each $i = 1,2$.

Proof. Obvious from Definition 3.1.

Remark 3.1. The converse of Theorem 3.2 is not true in general, as following examples shall show.

Examples 3.1.

(1)Let $X = \{h_1,h_2\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\Phi},(F_1,E)\}$ where (F_1,E) is s soft sets over X defined as follows:

$$F_1(e_1) = \{h_1\},\ F_1(e_2) = X.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X},\tilde{\Phi},(G_1,E),(G_2,E),(G_3,E)\}$, where (G_1,E), (G_2,E) and (G_3,E) are soft sets over X defined as follows:

$$G_1(e_1) = X,\quad G_1(e_2) = \{h_2\},$$
$$G_2(e_1) = \{h_1\},\ G_2(e_2) = X,$$
$$G_3(e_1) = \{h_1\},\ G_3(e_2) = \{h_1\}.$$

Therefore, (X,μ,E) is supra soft β-T_1-space, but it is not supra soft β-T_2-space, for $h_1,h_2 \in X$ and $h_1 \neq h_2$, but there are no μ-supra β-open soft sets (F,E) and (G,E) such that $h_1 \in (F,E)$, $h_2 \in (G,E)$ and $(F,E)\tilde{\cap}(G,E) = \tilde{\Phi}$.

(2)Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\Phi},(F_1,E)\}$ where (F_1,E) is soft set over X defined as follows by

$$F_1(e_1) = \{h_1,h_2\},\ F_1(e_2) = \{h_1,h_2\}.$$

Then, τ defines a soft topology on X. The associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X},\tilde{\Phi},(G_1,E),(G_2,E),(G_3,E)\}$, where $(G_1,E),(G_2,E),\ and\ (G_3,E)$ are soft sets over X defined as follows:

$$G_1(e_1) = \{h_1\},\quad G_1(e_2) = \{h_1\},$$
$$G_2(e_1) = \{h_1,h_2\},\ G_2(e_2) = \{h_1,h_2\},$$
$$G_3(e_1) = \{h_2,h_3\},\ G_3(e_2) = \{h_2,h_3\}.$$

Hence, (X,μ,E) is supra soft β-T_0-space, but it is not supra soft β-T_1-space, since $h_2,h_3 \in X$, and $h_2 \neq h_3$, but every supra β-open soft set which contains h_3 also contains h_2.

Theorem 3.3. Let (X,μ,E) be a supra soft topological space. If x_E is supra β-closed soft set in μ for each $x \in X$, then (X,μ,E) is supra soft β-T_1-space.

Proof. Suppose that $x \in X$ and x_E is supra β-closed soft set in μ. Then, $x_E^{\tilde{c}}$ is supra β-open soft set in μ. Let $x,y \in X$ such that $x \neq y$. For $x \in X$ and $x_E^{\tilde{c}}$ is supra β-open soft set such that $x \notin x_E^{\tilde{c}}$ and $y \in x_E^{\tilde{c}}$. Similarly $y_E^{\tilde{c}}$ is supra β-open soft set in μ such that $y \notin y_E^{\tilde{c}}$ and $x \in y_E^{\tilde{c}}$. Thus, (X,μ,E) is supra soft β-T_1-space over X.

Remark 3.2. The converse of Theorem 3.3 is not true in general, as following examples shall show.

Example 3.1.

Let $X = \{h_1,h_2\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\Phi},(G_1,E)\}$ where (G_1,E) is a soft sets over X defined as follows:

$$G_1(e_1) = X,\ G_1(e_2) = \{h_2\}.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X},\tilde{\Phi},(F_1,E),(F_2,E),(F_3,E)\}$ where (F_1,E), (F_2,E) and (F_3,E) are soft sets over X defined as follows:

$$F_1(e_1) = X,\quad F_1(e_2) = \{h_2\},$$
$$F_2(e_1) = \{h_1\},\ F_2(e_2) = X$$
$$F_3(e_1) = \{h_1\},\ F_3(e_2) = \{h_1\}.$$

Then, μ defines a supra soft topology on X. Therefore, (X,μ,E) is a supra soft β-T_1-space. On the other hand, we note that for the singleton soft points h_{1E} and h_{2E}, where

$$h_1(e_1) = \{h_1\},\ h_1(e_2) = \{h_1\},$$
$$h_2(e_1) = \{h_2\},\ h_2(e_2) = \{h_2\}.$$

The relative complement $h_{1E}^{\tilde{c}}$ and $h_{2E}^{\tilde{c}}$, where

$$h_1^{\tilde{c}}(e_1) = \{h_2\},\ h_1^{\tilde{c}}(e_2) = \{h_2\},$$
$$h_2^{\tilde{c}}(e_1) = \{h_1\},\ h_2^{\tilde{c}}(e_2) = \{h_1\}.$$

Thus, $h_{1E}^{\tilde{c}}$ is not μ-supra β-open soft set. This shows that, the converse of the above theorem does not hold.
Also, we have
$\mu_{e_1} = \{X,\Phi,\{h_1\}\}$, and
$\mu_{e_2} = \{X,\Phi,\{h_1\},\{h_2\}\}$. Therefore,$(X,\mu_{e_1})$ is not a supra β-T_1-space, at the time that (X,μ,E) is a supra soft β-T_1-space.

Definition 3.2. Let (X,τ,E) be a soft topological space and μ be an associated supra soft topology with τ. Let (G,E) be a μ-supra β-closed soft set in X and $x \in X$ such that $x \notin (G,E)$. If there exist μ-supra β-open soft sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\Phi}$, then (X,τ,E) is called supra soft β-regular space. A supra soft β-regular T_1-space is called supra soft β-T_3-space (S-soft β-T_3 for short).

Proposition 3.3. Let (X,τ,E) be a soft topological space and μ be an associated supra soft topology with τ. Let (G,E) be a μ-supra β-closed soft set in X and $x \in X$ such that $x \notin (G,E)$. If (X,τ,E) is supra soft β-regular space, then there exists a μ-supra β-open soft set (F,E) such that $x \in (F,E)$ and $(F,E)\tilde{\cap}(G,E) = \tilde{\Phi}$.

Proof. Obvious from Definition 3.2.

Theorem 3.4. Every supra soft T_i-space is supra soft β-T_i for each $i = 0,1,2,3$.

Proof. It is clear from the fact that, every supra open soft set is supra β-open soft set [13].

Proposition 3.4. Let (X,μ,E) be a supra soft topological space, $(F,E) \in S_E(X)$ and $x \in X$. Then:

(i)$x \in (F,E)$ if and only if $x_E \tilde{\subseteq} (F,E)$.
(ii)If $x_E \tilde{\cap} (F,E) = \tilde{\Phi}$, then $x \notin (F,E)$.

Proof. Obvious.

Theorem 3.5. Let (X,μ,E) be a supra soft topological space and $x \in X$. If (X,μ,E) is supra soft β-regular space, then

(i)$x \notin (F,E)$ if and only if $x_E \tilde{\cap} (F,E) = \tilde{\Phi}$ for every μ-supra β-closed soft set (F,E).
(ii)$x \notin (G,E)$ if and only if $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$ for every μ-supra β-open soft set (G,E).

Proof.

(i)[Let (F,E) be a μ-supra β-closed soft set such that $x \notin (F,E)$. Since (X,τ,E) is supra soft regular space. By Proposition 3.3 there exists a μ-supra β-open soft set (G,E) such that $x \in (G,E)$ and $(F,E) \tilde{\cap} (G,E) = \tilde{\Phi}$. It follows that, $x_E \tilde{\subseteq} (G,E)$ from Proposition 3.4 (1). Hence, $x_E \tilde{\cap} (F,E) = \tilde{\Phi}$. Conversely, if $x_E \tilde{\cap} (F,E) = \tilde{\Phi}$, then $x \notin (F,E)$ from Proposition 3.4 (2).

(ii)Let (G,E) be a μ-supra β-open soft set such that $x \notin (G,E)$. If $x \notin G(e)$ for each $e \in E$, then we get the proof. If $x \notin G(e_1)$ and $x \in G(e_2)$ for some $e_1, e_2 \in E$, then $x \in G^{\tilde{c}}(e_1)$ and $x \notin G^{\tilde{c}}(e_2)$ for some $e_1, e_2 \in E$. This means that, $x_E \tilde{\cap} (G,E) \neq \tilde{\Phi}$. Hence, $(G,E)^{\tilde{c}}$ is μ-supra β-closed soft set such that $x \notin (G,E)^{\tilde{c}}$. It follows by (1) $x_E \tilde{\cap} (G,E)^{\tilde{c}} = \tilde{\Phi}$. This implies that, $x_E \tilde{\subseteq} (G,E)$ and so $x \in (G,E)$, which is contradiction with $x \notin G(e_1)$ for some $e_1 \in E$. Therefore, $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$. Conversely, if $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$, then it obvious that $x \notin (G,E)$. This completes the proof.

Corollary 3.1. Let (X,μ,E) be a supra soft topological space and $x \in X$. If (X,μ,E) is supra soft β-regular space, then the following are equivalent:

(i)(X,τ,E) is supra soft β-T_1-space.
(ii)$\forall\, x,y \in X$ such that $x \neq y$, there exist μ-supra β-open soft sets (F,E) and (G,E) such that $x_E \tilde{\subseteq} (F,E)$ and $y_E \tilde{\cap} (F,E) = \tilde{\Phi}$ and $y_E \tilde{\subseteq} (G,E)$ and $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$.

Proof. Obvious from Theorem 3.5.

Theorem 3.6. Let (X,μ,E) be a supra soft topological space and $x \in X$. Then, the following are equivalent:

(i)(X,μ,E) is supra β-soft regular space.
(ii)For every μ-supra β-closed soft set (G,E) such that $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$, there exist μ-supra β-open soft sets (F_1,E) and (F_2,E) such that $x_E \tilde{\subseteq} (F_1,E)$, $(G,E) \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$.

Proof.

(i) $\Rightarrow$ (ii) Let (G,E) be a μ-supra β-closed soft set such that $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$. Then, $x \notin (G,E)$ from Theorem 3.5 (1). It follows by (1), there exist μ-supra β-open soft sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $(G,E) \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$. This means that, $x_E \tilde{\subseteq} (F_1,E)$, $(G,E) \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$.

(ii) $\Rightarrow$ (i) Let (G,E) be a μ-supra β-closed soft set such that $x \notin (G,E)$. Then, $x_E \tilde{\cap} (G,E) = \tilde{\Phi}$ from Theorem 3.5 (1). It follows by (2), there exist μ-supra β-open soft sets (F_1,E) and (F_2,E) such that $x_E \tilde{\subseteq} (F_1,E)$, $(G,E) \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$. Hence, $x \in (F_1,E)$, $(G,E) \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$. Thus, (X,τ,E) is supra β-soft regular space.

Theorem 3.7. Let (X,μ,E) be a supra soft topological space. If (X,μ,E) is supra soft T_3-space, then $\forall\, x \in X$, x_E is μ-supra β-closed soft set.

Proof. We want to prove that x_E is μ-supra β-closed soft set, which is sufficient to prove that $x_E^{\tilde{c}}$ is μ-supra β-open soft set for each $y \in \{x\}^{\tilde{c}}$. Since (X,μ,E) is supra soft β-T_3-space. Then, there exist μ-supra β-open soft sets $(F,E)_y$ and (G,E) such that $y_E \tilde{\subseteq} (F,E)_y$ and $x_E \tilde{\cap} (F,E)_y = \tilde{\Phi}$ and $x_E \tilde{\subseteq} (G,E)$ and $y_E \tilde{\cap} (G,E) = \tilde{\Phi}$. It follows that, $\bigcup_{y \in \{x\}^{\tilde{c}}} (F,E)_y \tilde{\subseteq} x_E^{\tilde{c}}$. Now, we want to prove that $x_E^{\tilde{c}} \tilde{\subseteq} \bigcup_{y \in \{x\}^{\tilde{c}}} (F,E)_y$. Let $\bigcup_{y \in \{x\}^{\tilde{c}}} (F,E)_y = (H,E)$, where $H(e) = \bigcup_{y \in \{x\}^{\tilde{c}}} F(e)_y$ for each $e \in E$. Since $x_E^{\tilde{c}}(e) = \{x\}^{\tilde{c}}$ for each $e \in E$ from Definition 2.5. So, for each $y \in \{x\}^{\tilde{c}}$ and $e \in E$, $x_E^{\tilde{c}}(e) = \{x\}^{\tilde{c}} = \bigcup_{y \in \{x\}^{\tilde{c}}} \{y\} = \bigcup_{y \in \{x\}^{\tilde{c}}} y_E(e) \tilde{\subseteq} \bigcup_{y \in \{x\}^{\tilde{c}}} F(e)_y = H(e)$. Thus, $x_E^{\tilde{c}} \tilde{\subseteq} \bigcup_{y \in \{x\}^{\tilde{c}}} (F,E)_y$, and so $x_E^{\tilde{c}} = \bigcup_{y \in \{x\}^{\tilde{c}}} (F,E)_y$. This means that, $x_E^{\tilde{c}}$ is μ-supra β-open soft set for each $y \in \{x\}^{\tilde{c}}$. Therefore, x_E is μ-supra β-closed soft set.

Theorem 3.8. Every supra soft β-T_3-space is supra soft β-T_2-space.

Proof. Let (X,μ,E) be a supra soft T_3-space and $x,y \in X$ such that $x \neq y$. By Theorem 3.7, y_E is μ-supra β- closed soft set and $x \notin y_E$. It follows from the supra soft regularity, there exist μ-supra β-open soft sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $y_E \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$. Thus, $x \in (F_1,E)$, $y \in y_E \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$. Therefore, (X,μ,E) is supra soft β-T_2-space.

Corollary 3.2. The following implications hold from Theorem 3.2, Theorem 3.4 and [[2], Corollary 3.2] for a supra soft topological space (X,μ,E).

$$\begin{array}{ccccccc}
\text{Soft } T_3 & \Rightarrow & \text{Soft } T_2 & \Rightarrow & \text{Soft } T_1 & \Rightarrow & \text{Soft } T_0 \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
\text{S-soft } T_3 & \Rightarrow & \text{S-soft } T_2 & \Rightarrow & \text{S-soft } T_1 & \Rightarrow & \text{S-soft } T_0 \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
\text{S-soft } \beta\text{-}T_3 & \Rightarrow & \text{S-soft } \beta\text{-}T_2 & \Rightarrow & \text{S-soft } \beta\text{-}T_1 & \Rightarrow & \text{S-soft } \beta\text{-}T_0
\end{array}$$

Definition 3.3. Let (X,μ,E) be a supra soft topological space, (F,E) and (G,E) be μ-supra β-closed soft sets in X such that $(F,E) \tilde{\cap} (G,E) = \tilde{\Phi}$. If there exist μ-supra β-open soft sets (F_1,E) and (F_2,E) such that $(F,E) \tilde{\subseteq} (F_1,E)$, $(G,E) \tilde{\subseteq} (F_2,E)$ and $(F_1,E) \tilde{\cap} (F_2,E) = \tilde{\Phi}$, then (X,μ,E) is called supra soft β-normal space. A supra soft β-normal T_1-space is called a supra soft β-T_4-space (S-soft β-T_4 for short).

Theorem 3.9. Let (X,μ,E) be a supra soft topological space and $x\in X$. Then, the following are equivalent:

(i)(X,μ,E) is supra soft β-normal space.
(ii)For every μ-supra β-closed soft set (F,E) and μ-supra β-open soft set (G,E) such that $(F,E)\tilde{\subseteq}(G,E)$, there exists a μ-supra β-open soft set (F_1,E) such that $F,E)\tilde{\subseteq}(F_1,E)$, $cl^s(F_1,E)\tilde{\subseteq}(G,E)$.

Proof.

(i) $\Rightarrow$ (ii) Let (F,E) be a μ-supra β-closed soft set and (G,E) be a μ-supra β- open soft set such that $(F,E)\tilde{\subseteq}(G,E)$. Then, $(F,E),(G,E)^{\tilde{c}}$ are μ-supra β-closed soft sets such that $(F,E)\tilde{\cap}(G,E)^{\tilde{c}}=\tilde{\Phi}$. It follows by (1), there exist μ-supra β- open soft sets (F_1,E) and (F_2,E) such that $(F,E)\tilde{\subseteq}(F_1,E)$, $(G,E)^{\tilde{c}}\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\Phi}$. Now, $(F_1,E)\tilde{\subseteq}(F_2,E)^{\tilde{c}}$, so $cl^s(F_1,E)\tilde{\subseteq}cl^s(F_2,E)^{\tilde{c}}=(F_2,E)^{\tilde{c}}$, where (G,E) is μ-supra β-open soft set. Also, $(F_2,E)^{\tilde{c}}\tilde{\subseteq}(G,E)$. Hence, $cl^s(F_1,E)\tilde{\subseteq}(F_2,E)^{\tilde{c}}\tilde{\subseteq}(G,E)$. Thus, $F,E)\tilde{\subseteq}(F_1,E)$, $cl^s(F_1,E)\tilde{\subseteq}(G,E)$.
(ii) $\Rightarrow$ (i) Let $(G_1,E),(G_2,E)$ be μ-supra β-closed soft sets such that $(G_1,E)\tilde{\cap}(G_2,E)=\tilde{\Phi}$. Then $(G_1,E)\tilde{\subseteq}(G_2,E)^{\tilde{c}}$, then by hypothesis, there exists a μ-supra β-open soft set (F_1,E) such that $G_1,E)\tilde{\subseteq}(F_1,E)$, $cl^s(F_1,E)\tilde{\subseteq}(G_2,E)^{\tilde{c}}$. So, $(G_2,E)\tilde{\subseteq}[cl^s(F_1,E)]^{\tilde{c}}$, $G_1,E)\tilde{\subseteq}(F_1,E)$ and $[cl^s(F_1,E)]^{\tilde{c}}\tilde{\cap}(F_1,E)=\tilde{\Phi}$, where (F_1,E) and $[cl^s(F_1,E)]^{\tilde{c}}$ are μ-supra β-open soft sets. Thus, (X,μ,E) is supra soft β-normal space.

Theorem 3.10. Let (X,μ,E) be a supra soft topological space. If (X,μ,E) is supra soft β-normal space and x_E is μ-supra β-closed soft set for each $x\in X$, then (X,τ,E) is supra soft β-T_3-space.

Proof. Since x_E is μ-supra β-closed soft set for each $x\in X$, then (X,μ,E) is supra soft β-T_1-space from Theorem 3.3. Also, (X,τ,E) is supra β-soft regular space from Theorem 3.6 and Definition 3.3. Hence, (X,μ,E) is supra soft β-T_3-space.

Proposition 3.5. Not every supra soft β-open soft subspace of supra soft β-T_i-space is supra soft β-T_i-space for each $i=0,1,2,3,4$.

Proof. Obvious from the fact that, the soft intersection of two supra β-open soft sets need not to be supra β-open soft.

4 Supra β-irresolute soft functions

Definition 4.1. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu}:S_E(X)\to S_K(Y)$ is called

(1)Supra β-open soft if $f_{pu}(F,E)\in S\beta OS_E(\mu_1)$ for each $(F,E)\in\tau_1$.
(2)Supra β-irresolute soft if $f_{pu}^{-1}(F,E)\in S\beta OS_E(\mu_1)$ for each $(F,E)\in S\beta OS_K(\mu_2)$.
(3)Supra β-irresolute open soft if $f_{pu}(F,E)\in S\beta OS_K(\mu_2)$ for each $(F,E)\in S\beta OS_E(\mu_1)$.

Theorem 4.1. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively and $f_{pu}:S_E(X)\to S_K(Y)$ be a soft function which is bijective and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-T_0-space, then (Y,τ_2,K) is also a supra soft β-T_0-space.

Proof. Let $y_1,y_2\in Y$ such that $y_1\neq y_2$. Since f_{pu} is surjective, then $\exists x_1,x_2\in X$ such that $u(x_1)=y_1$, $u(x_2)=y_2$ and $x_1\neq x_2$. By hypothesis, there exist μ_1-supra β-open soft sets (F,E) and (G,E) in X such that either $x_1\in(F,E)$ and $x_2\notin(F,E)$, or $x_2\in(G,E)$ and $x_1\notin(G,E)$. So, either $x_1\in F_E(e)$ and $x_2\notin F_E(e)$ or $x_2\in G_E(e)$ and $x_1\notin G_E(e)$ for each $e\in E$. This implies that, either $y_1=u(x_1)\in u[F_E(e)]$ and $y_2=u(x_2)\notin u[F_E(e)]$ or $y_2=u(x_2)\in u[G_E(e)]$ and $y_1=u(x_1)\notin u[G_E(e)]$ for each $e\in E$. Hence, either $y_1\in f_{pu}(F,E)$ and $y_2\notin f_{pu}(F,E)$ or $y_2\in f_{pu}(G,E)$ and $y_1\notin f_{pu}(G,E)$. Since f_{pu} is supra β-irresolute open soft function, then $f_{pu}(F,E),f_{pu}(G,E)$ are supra β-open soft sets in Y. Hence, (Y,τ_2,K) is also a supra soft β-T_0-space.

Theorem 4.2. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:S_E(X)\to S_K(Y)$ be a soft function which is bijective and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-T_1-space, then (Y,τ_2,K) is also a supra soft β-T_1-space.

Proof. It is similar to the proof of Theorem 4.1.

Theorem 4.3. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}:S_E(X)\to S_K(Y)$ be a soft function which is bijective and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-T_2-space, then (Y,τ_2,K) is also a supra soft β-T_2-space.

Proof. $y_1,y_2\in Y$ such that $y_1\neq y_2$. Since f_{pu} is surjective, then $\exists x_1,x_2\in X$ such that $u(x_1)=y_1$, $u(x_2)=y_2$ and $x_1\neq x_2$. By hypothesis, there exist μ-supra β-open soft sets (F,E) and (G,E) in X such that $x_1\in(F,E)$, $x_2\in(G,E)$ and $(F,E)\tilde{\cap}(G,E)=\tilde{\Phi}_E$. So, $x_1\in F_E(e)$, $x_2\in G_E(e)$ and $F_E(e)\tilde{\cap}G_E(e)=\Phi$ for each $e\in E$. This implies that, $y_1=u(x_1)\in u[F_E(e)]$, $y_2=u(x_2)\in u[G_E(e)]$ for each $e\in E$. Hence, $y_1\in f_{pu}(F,E)$, $y_2\in f_{pu}(G,E)$ and $f_{pu}(F,E)\tilde{\cap}f_{pu}(G,E)=f_{pu}[(F,E)\tilde{\cap}(G,E)]=f_{pu}[\tilde{\Phi}_E]=\tilde{\Phi}_K$ from Theorem 2.1. Since f_{pu} is supra β-irresolute open soft function, then $f_{pu}(F,E),f_{pu}(G,E)$ are supra β-open soft sets in Y. Thus, (Y,τ_2,K) is also a supra soft β-T_2-space.

Theorem 4.4. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra β-irresolute soft and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-regular space, then (Y,τ_2,K) is also a supra soft β-regular space.

Proof. Let (G,K) be a supra β-closed soft set in Y and $y \in Y$ such that $y \notin (G,K)$. Since f_{pu} is surjective and supra β-irresolute soft, then $\exists x \in X$ such that $u(x) = y$ and $f_{pu}^{-1}(G,K)$ is supra β-closed soft set in X such that $x \notin f_{pu}^{-1}(G,K)$. By hypothesis, there exist supra open soft sets (F,E) and (H,E) in X such that $x \in (F,E)$, $f_{pu}^{-1}(G,K)\tilde{\subseteq}(H,E)$ and $(F,E)\tilde{\cap}(H,E) = \tilde{\Phi}_E$. It follows that, $x \in F_E(e)$ for each $e \in E$ and $(G,K) = f_{pu}[f_{pu}^{-1}(G,K)]\tilde{\subseteq} f_{pu}(H,E)$ from Theorem 2.1. So, $y = u(x) \in u[F_E(e)]$ for each $e \in E$ and $(G,K)\tilde{\subseteq} f_{pu}(H,E)$. Hence, $y \in f_{pu}(F,E)$ and $(G,K)\tilde{\subseteq} f_{pu}(H,E)$ and $f_{pu}(F,E)\tilde{\cap} f_{pu}(H,E) = f_{pu}[(F,E)\tilde{\cap}(H,E)] = f_{pu}[\tilde{\Phi}_E] = \tilde{\Phi}_K$ from Theorem 2.1. Since f_{pu} is supra β-irresolute open soft function. Then, $f_{pu}(F,E), f_{pu}(H,E)$ are supra β-open soft sets in Y. Thus, (Y,τ_2,K) is also a supra soft β-regular space.

Theorem 4.5. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra β-irresolute soft and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-T_3-space, then (Y,τ_2,K) is also a supra soft β-T_3-space.

Proof. Since (X,τ_1,E) is supra soft β-T_3-space, then (X,τ_1,E) is supra soft β-regular T_1-space. It follows that, (Y,τ_2,K) is also a supra soft β-T_1-space from Theorem 4.2 and supra soft β-regular space from Theorem 3.8. Hence, (Y,τ_2,K) is also a supra soft β-T_3-space.

Theorem 4.6. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra β-irresolute soft and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-normal space, then (Y,τ_2,K) is also a supra soft β-normal space.

Proof. Let $(F,K),(G,K)$ be supra β-closed soft sets in Y such that $(F,K)\tilde{\cap}(G,K) = \tilde{\Phi}_K$. Since f_{pu} is supra β-irresolute soft, then $f_{pu}^{-1}(F,K)$ and $f_{pu}^{-1}(G,K)$ are supra β-closed soft set in X such that $f_{pu}^{-1}(F,K)\tilde{\cap} f_{pu}^{-1}(G,K) = f_{pu}^{-1}[(F,K)\tilde{\cap}(G,K)] = f_{pu}^{-1}[\tilde{\Phi}_K] = \tilde{\Phi}_E$ from Theorem 2.1. By hypothesis, there exist supra β-open soft sets (K,E) and (H,E) in X such that $f_{pu}^{-1}(F,K)\tilde{\subseteq}(K,E)$, $f_{pu}^{-1}(G,K)\tilde{\subseteq}(H,E)$ and $(F,E)\tilde{\cap}(H,E) = \tilde{\Phi}_E$. It follows that, $(F,K) = f_{pu}[f_{pu}^{-1}(F,K)]\tilde{\subseteq} f_{pu}(K,E)$,$(G,K) = f_{pu}[f_{pu}^{-1}(G,K)]\tilde{\subseteq} f_{pu}(H,E)$ from Theorem 2.1 and $f_{pu}(K,E)\tilde{\cap} f_{pu}(H,E) = f_{pu}[(K,E)\tilde{\cap}(H,E)] = f_{pu}[\tilde{\Phi}_E] = \tilde{\Phi}_K$ from Theorem 2.1. Since f_{pu} is supra β-irresolute open soft function. Then, $f_{pu}(K,E), f_{pu}(H,E)$ are supra β-open soft sets in Y. Thus, (Y,τ_2,K) is also a supra soft β-normal space.

Corollary 4.1. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra β-irresolute soft and supra β-irresolute open soft. If (X,τ_1,E) is supra soft β-T_4-space, then (Y,τ_2,K) is also a supra soft β-T_4-space.

Proof. It is obvious from Theorem 4.2 and Theorem 4.6.

5 Conclusion

The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [13]. In this paper, we introduce and investigate some weak soft separation axioms by using the notion of supra β-open soft sets, which is a generalization of the supra soft separation axioms mentioned in [2]. We study the relationships between these new soft separation axioms and their relationships with some other properties. As a consequence the relations of some supra soft separation axioms are shown in a diagram. We show that, some classical results in general supra topology are not true if we consider supra soft topological spaces instead. For instance, if (X,μ,E) is supra soft β-T_1-space need not every soft singleton x_E is supra β-closed soft. Our next work, is to generalize this paper by using the notion of b-open soft sets. We hope that, the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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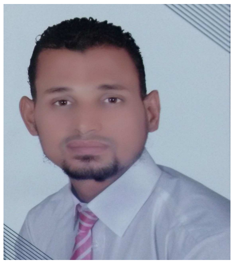

Alaa Mohamed Abd El-Latif received the Ph. D degree in pure Mathematics (Topology) from Ain Shams University, Faculty of Education, Mathematic Department, Cairo, Egypt. His primary research areas are set theory, general topology, fuzzy topology, bitopology, ditopology and soft topology. He is referee of several international journals in the pure mathematics. Dr. Alaa has published many papers in refereed journals.