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Parameter Estimation of the Marshall-Olkin Exponential Distribution under Type-II Hybrid Censoring Schemes and its Applications

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Abstract: Marshall-Olkin exponential distribution has been studied by Salah, et al. [11,12]. In this paper, we consider the analysis of the hybrid censored data when the life time distribution of the individuals item is a two-parameter Marshall-Olkin exponential distribution. We study the estimations of the shape (α) and scale (λ) parameters of Marshall-Olkin exponential distribution based on Type-II hybrid censoring scheme. Using the EM algorithm to compute the maximum likelihood estimators as it observed that these estimators can not be found explicitly. Finally, we obtain the observed Fisher information matrix that can be used for constructing the asymptomatic confidence intervals. Numerical simulation is performed.

Keywords: order statistics, Marshall-Olkin, exponential distribution, type-I and type-II, hybrid censoring, schemes, half logistic distribution.
AMS *Subject Classification: 62G30,62E99, 60E05.*

1 Introduction

The censored sampling occur in a life-testing experiment when ever the experimenter can't observe the failure times of all units placed on a life-test. For example consider a life-testing experiment where n items are kept under observation, these items could be systems, computers, individuals in a clinical trial, in reliability study experiment, so that the removal of units from the experimentation is pre-planned and intentional, and is done in order to provide saving in terms of time and cost associated with testing. The data obtained from such experiments are called censored data. Here we mention the most common censoring schemes which are the Type-I and Type-II censoring schemes they can be described as follows. Let us consider n unites are placed on a life-test then, type-I (time) censoring: Suppose it is decided to terminate the experiment at a pre-determined time T, so that only failure time of these items that failed prior to this time recorded, the data so obtained from this process constitute a type-I censored sample. Type-II censoring: If the experiment is terminated at the R^{th} failure, that is at time $X_{R:n}$, we obtain type-II censored sample, here R is fixed, while $X_{R:n}$ the duration of the experiment is random. Many articles in this literature have discussed inferential method under type-I and type-II censoring for various parametric families of distributions, for more details, see for example, Balakrishnan and Cohen [13], Pradhan and Kundu [5]. The mixture of type-I and type-II censoring schemes is known as the hybrid censoring scheme and it can be defined as follows. Suppose n identical units are put to test under the same environmental conditions and the lifetime of each unit is independent and identically distributed $(i.i.d.)$ random variables. The test is terminated when a pre-chosen number R, out of n items have failed or a pre-determined time T, on test has been reached. This scheme is known as Type-I hybrid censoring scheme which introduced by Epstin [4]. The Type-I hybrid censoring scheme has been used as a reliability acceptance test in MIL-STD-781 C [16]. For more details about Type-I hybrid censoring schemes one can refer to Chen and Bhattacharya [19], Ebrahimi [14,15], Jeong et al. [9], Gupta and Kundu [17,18], Childs et al. [2], Kundu [6] and Singh, S. K. [20]. Since Type-I hybrid censoring scheme have disadvantages as there may be very few failures occurring up to the pre-fixed time T. Because of this, Childs et al. [2], proposed a new hybrid censoring scheme known as Type-II hybrid censoring scheme which can be defined as follows (see, Banerjee and Kundu [1]). Put n identical items

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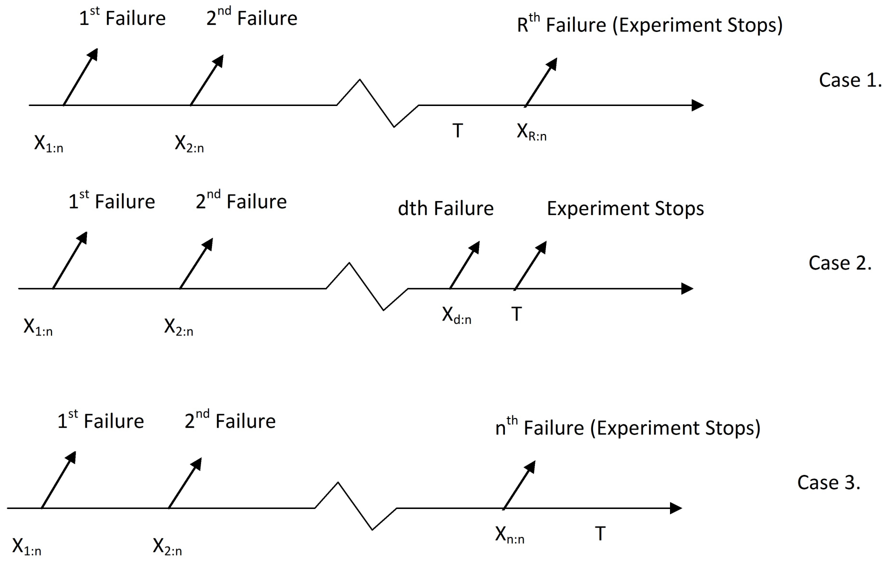

Fig. 1: title

on test, and then stop the experiment at the random time $T^* = max\{x_{R:n}, T\}$, where R, and T are prefixed numbers and $x_{R:n}$ indicates the time of R^{th} failure in a sample of size n.

We have one of the following three types of observations under Type-II hybrid censoring scheme :
Case 1:

$$\{x_{1:n} < x_{2:n} < \ldots < x_{R:n}\} \text{ if } x_{R:n} > T.$$

Case 2:

$$\{x_{1:n} < x_{2:n} < \ldots < x_{d:n} < x_{d+1:n}\} \text{ if } R \le d < n \text{ and } x_{d:n} < T < x_{d+1:n}.$$

Case 3:

$$\{x_{1:n} < x_{2:n} < \ldots < x_{n:n} < T\},$$

where $x_{1:n} < x_{2:n} < \ldots < x_{R:n}$ denote the observed ordered failure times of the experimental units. The following digraph describe the schematic representation of the hybrid censoring scheme.

The purpose of this paper is to study the hybrid censored lifetime data when the lifetime of each experimental unit follows a two-parameter Marshall-Olkin exponential distribution (MOE) distribution for the shape and scale parameters. This distribution was studied recently by Salah [11,12]. The cdf of MOE distribution is given as follows:

$$F(x) = 1 - \frac{\alpha}{\left(e^{\lambda x} - (1-\alpha)\right)}, \qquad 0 \le x < \infty, \alpha > 0 \text{ and } \lambda > 0, \tag{1}$$

and its pdf is given by:

$$f(x) = \frac{\alpha \lambda e^{\lambda x}}{\left[e^{\lambda x} - (1-\alpha)\right]^2}, \qquad 0 \le x < \infty, \alpha > 0 \text{ and } \lambda > 0, \tag{2}$$

where α is the shape parameter and λ is the scale parameter.

When $\alpha = 1$, the MOE distribution reduces to the exponential distribution, and when $\alpha = 2$, the MOE distribution reduces to the half logistic distribution. Note that Eqs. (1) and (2) are respectively for the two-parameter MOE distribution.

The rest of this paper is organized as follow: Section 2, presents the maximum likelihood estimators (MLE's) of the shape and scale parameters. In Section 3, we present the EM algorithm to estimate the MLE's $\left(\widehat{\alpha}, \widehat{\lambda}\right)$. In Section 4, we present the Fisher information matrix in order to estimate the 95% asymptotic confidence interval. Finally, simulation study and numerical analysis are presented in Section 5.

2 Maximum Likelihood Estimators

In this section, the MLEs of the model parameters α and λ for the MOE distribution are presented. Suppose $X_{1:n}, X_{2:n}, \ldots, X_{R:n}$ are n independent random variables in presence of Type-II hybrid censored samples from the MOE distribution. The likelihood functions for the three types of observations under Type-II hybrid censoring scheme are given, respectively, by:

Case 1:

$$L(\alpha,\lambda) = \frac{n!}{(n-R)!}\prod_{i=1}^{R} f(x_i)\left[1-F(x_{R:n})\right]^{(n-R)}, \tag{3}$$

for Case 2,

$$L(\alpha,\lambda) = \frac{n!}{(n-d)!}\prod_{i=1}^{d} f(x_i)\left[1-F(T)\right]^{(n-d)}, \tag{4}$$

finally for Case 3 we have

$$L(\alpha,\lambda) = \prod_{i=1}^{n} f(x_i). \tag{5}$$

Therefore, the three cases can be combined together to be

$$L(\alpha,\lambda) = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(x_i)\left[1-F(c)\right]^{(n-r)}, \tag{6}$$

where

$$r = \begin{cases} R & \text{for Case1} \\ d & \text{for Case 2} \\ n & \text{for Case 3} \end{cases},$$

and

$$c = \begin{cases} x_{R:n} & \text{for Case 1} \\ T & \text{for Case 2 and 3.} \end{cases}$$

By substituting Eqs.(1) and (2), respectively, into Eq.(6) and taking the logarithm with ignoring the constant we get

$$\ln L(\alpha,\lambda) = n\ln\alpha + r\ln\lambda - (n-r)\ln\left[e^{\lambda c}-1+\alpha\right] + \lambda\sum_{i=1}^{r} x_i - 2\sum_{i=1}^{r}\ln\left[e^{\lambda x_i}-1+\alpha\right], \tag{7}$$

for simplicity let us denote $\ln L(\alpha,\lambda)$ by $\ln L$. Differentiating Eq.(7) with respect to α and λ , respectively, to get

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - \frac{n-r}{e^{\lambda c}-1+\alpha} - 2\sum_{i=1}^{r}\frac{1}{e^{\lambda x_i}-1+\alpha}, \tag{8}$$

and

$$\frac{\partial \ln L}{\partial \lambda} = \frac{r}{\lambda} - \frac{(n-r)ce^{\lambda c}}{e^{\lambda c}-1+\alpha} + \sum_{i=1}^{r} x_i\left(1-\frac{2e^{\lambda x_i}}{e^{\lambda x_i}-1+\alpha}\right). \tag{9}$$

From Eq. (8) and Eq. (9) one can find the MLE's of α and λ by solving these equations numerically since it can't be solved analytically.

3 EM algorithm

From the previous section it observed that the MLE's of α and λ can not be solved analytically, so one of the most common method used to find the solution numerically is the Expectation - Maximuzation algorithm (EM algorithm). EM algorithm is a very powerful method tool in handling the incomplete data problem as it presented by Dempster et al. [3]. First let us denote the observed and censored data by $\underset{\sim}{X} = (X_{1:n}, X_{2:n}, \ldots, X_{R:n})$ and $\underset{\sim}{Z} = (Z_{1:n}, Z_{2:n}, \ldots, Z_{R:n})$, respectively. The censored data $\underset{\sim}{Z}$ can be thought of as missing data. the combination of $\underset{\sim}{W} = (\underset{\sim}{X}, \underset{\sim}{Z})$ forms the whole data (complete data). In the next procedure we follow the method of Kundu and Pradhan [7] for missing data introducing. Now the log-likelihood function of uncensored data Eq. (7) is given by

$$L_c(\alpha,\lambda) = n\ln\alpha + n\ln\lambda + \lambda\sum_{i=1}^{r}x_i + \lambda\sum_{i=1}^{n-r}z_i - 2\sum_{i=1}^{r}\ln\left[e^{\lambda x_i}-1+\alpha\right] - 2\sum_{i=1}^{n-r}\ln\left[e^{\lambda z_i}-1+\alpha\right]. \tag{10}$$

For the E-step of the EM algorithm, need to compute the pseudo log-likelihood function as $L_s(\alpha,\lambda) = E\left[L_c(\alpha,\lambda \mid X)\right]$ where

$$L_s(\alpha,\lambda) = n\ln\alpha + n\ln\lambda + \lambda\sum_{i=1}^{r}x_i - 2\sum_{i=1}^{r}\ln\left[e^{\lambda x_i}-1+\alpha\right] + \lambda\sum_{i=1}^{n-r}E\left[Z_i \mid Z_i > c)\right] - 2\sum_{i=1}^{n-r}E\left[\ln\left(e^{\lambda Z_i}-1+\alpha\right) \mid Z_i > c\right]. \tag{11}$$

Applying same procedure as used in Ng, et al. [8]. It can be readily seen that:

$$E\left[Z_i \mid Z_i > c)\right] = c + \frac{e^{\lambda c}-1+\alpha}{\lambda(\alpha-1)}\ln\left[e^{\lambda c}-1+\alpha\right] - \frac{e^{\lambda c}-1+\alpha}{(\alpha-1)}, \tag{12}$$

and

$$E\left[\ln\left(e^{\lambda Z_i}-1+\alpha\right) \mid Z_i > c\right] = \ln\left[e^{\lambda c}-1+\alpha\right]. \tag{13}$$

In M-step if at the k^{th} stage, the estimation of (α,λ) is $\left(\tilde{\alpha}_k, \tilde{\lambda}_k\right)$, then $\left(\tilde{\alpha}_{k+1}, \tilde{\lambda}_{k+1}\right)$ can be obtain by maximizing the pseudo log-likelihood Eq.(10) as

$$L(\alpha,\lambda) = n\ln\alpha + n\ln\lambda + \lambda\sum_{i=1}^{r}x_i - 2\sum_{i=1}^{r}\ln\left[e^{\lambda x_i}-1+\alpha\right] + \lambda(n-r)A(c,\tilde{\alpha}_k,\tilde{\lambda}_k) - 2(n-r)B(c,\tilde{\alpha}_k,\tilde{\lambda}_k), \tag{14}$$

where

$$A(c,\tilde{\alpha}_k,\tilde{\lambda}_k) = E\left[Z_i \mid Z_i > c)\right],$$
$$B(c,\tilde{\alpha}_k,\tilde{\lambda}_k) = E\left[\ln\left(e^{\lambda Z_i}-1+\alpha\right) \mid Z_i > c\right].$$

By applying the same procedure in Gupta and Kundu [18] in the M-step, one can get that

$$\tilde{\alpha}(\lambda) = \frac{n}{2\sum_{i=1}^{r}\left[e^{\lambda x_i}-1+\alpha\right]^{-1}}, \tag{15}$$

and

$$\lambda = K(\lambda), \tag{16}$$

where

$$K(\lambda) = \left[\frac{r-n}{n}A(c,\tilde{\alpha}_k,\tilde{\lambda}_k) + \frac{1}{n}\sum_{i=1}^{r}x_i\left(\frac{2e^{\lambda x_i}}{e^{\lambda x_i}-1+\tilde{\alpha}(\lambda)}-1\right)\right]^{-1}.$$

To solve Eq.(16) by iterative technique suppose λ_0 is an initial guess value for λ successive approximation of λ are $\lambda_1 = K(\lambda_0), \lambda_2 = K(\lambda_1), ..., \lambda_{k+1} = K(\lambda_k)$. Stop the iteration at the k^{th} iteration if $|\lambda_{k+1}-\lambda_k| < \varepsilon$, for some pre-specified small value of the error ε. Let $\tilde{\lambda}$ be the estimated value λ obtained by solving Eq.(16), therefore the estimated value of α is obtained by solving Eq.(16) and (15) consequently.

4 Fisher Information Matrix

In this section, we present the 95% asymptotic confidence interval for the parameters α and λ using the observed Fisher information matrix . Using Eq. (7) the Fisher information matrix is given by

$$I(\widetilde{\alpha},\widetilde{\lambda}) = -\begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}, \tag{17}$$

where

$$I_{11} = \frac{\partial^2 \ln L}{\partial \alpha^2} = \frac{-n}{\alpha^2} + \frac{n-r}{\left[e^{\lambda c} - 1 + \alpha\right]^2} + 2\sum_{i=1}^{r} \frac{1}{\left[e^{\lambda x_i} - 1 + \alpha\right]^2},$$

$$I_{12} = \frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} = \frac{(n-r)\, c e^{\lambda c}}{\left[e^{\lambda c} - 1 + \alpha\right]^2} + 2\sum_{i=1}^{r} \frac{x_i e^{\lambda x_i}}{\left[e^{\lambda x_i} - 1 + \alpha\right]^2},$$

$$I_{22} = \frac{\partial^2 \ln L}{\partial \lambda^2} = \frac{-r}{\lambda^2} + \frac{(n-r)(1-\alpha)\, c e^{\lambda c}}{\left[e^{\lambda c} - 1 + \alpha\right]} + 2(1-\alpha)\sum_{i=1}^{r} \frac{x_i^2 e^{\lambda x_i}}{\left[e^{\lambda x_i} - 1 + \alpha\right]^2},$$

and

$$I_{21} = I_{12}.$$

By finding the $I^{-1}(\widetilde{\alpha},\widetilde{\lambda})$ one can get the estimate of the asymptotic variance-covariance matrix of the MLE's where the diagonal elements in $I^{-1}(\widetilde{\alpha},\widetilde{\lambda})$ give asymptotic variance of α and λ respectively. Therefore using large sample theorem, the two sided $100(1-\gamma)\%$ approximate confidence intervals for α and λ can be constructed respectively as

$$\widetilde{\alpha} \pm Z_{1-\frac{\gamma}{2}} \sqrt{var(\widetilde{\alpha})},$$

and

$$\widetilde{\lambda} \pm Z_{1-\frac{\gamma}{2}} \sqrt{var(\widetilde{\lambda})}.$$

5 Numerical Results

In the previous sections, we have found that the MLE's for the unknown parameters α and λ can not be solved analytically. Thus we solved it numerically by generating a random samples of size ($n = 10, 20$ and 30) for fixed values of ($\alpha = 1, 2, 2.5$ and $\lambda = 1, 2$). The simulation study is performed to find the MLE's $\left(\widehat{\alpha}, \widehat{\lambda}\right)$ and the two sided $100(1-\gamma)\%$ approximate confidence intervals for α and λ for different random samples of hybrid censored data and for different choices of n, r, and T values. The MLE's $\left(\widehat{\alpha}, \widehat{\lambda}\right)$ and the two sided $100(1-\gamma)\%$ approximate confidence intervals for α and λ are obtained and tabulated upon using the Mathematica 6.0 package with more than 1000 runs in Tables(1,2,3) below.

*Example 1.*Real Data Set: (see, Bain [10]) Suppose 20 items from an exponential population are put on life-test and observed for 150 hours. During that period 13 items fail with the following lifetime, measured in hours: 3, 19, 23, 26, 37, 38, 41, 45, 58,84,90, 109 and 138. Now suppose we have the following setup of the same experiment, n = 20, R = 10 and T = 150. Then we have the following data;, 3, 19, 23, 26, 37, 38, 41, 45, 58 and 84. One can easily conclude that the MOE distribution gives better fits for the given data. We find the MLE's $\left(\widehat{\alpha}, \widehat{\lambda}\right)$ are as follows $\widehat{\alpha} = 7.608$ with MSE equal to 0.0002 and $\widehat{\lambda} = 0.0202$ with MSE equal to 0.2203. For the 95% asymptotic intervals we have $\widehat{\alpha} \in [-13.7287, 28.9462]$ and $\widehat{\lambda} \in [-0.0024, 0.0429]$.

Table 1: The maximum likelihood estimators, the 95% asymptotic intervals and MSE for the parameters α and λ when ($\alpha = 2$ and $\lambda = 1$)

			$\widehat{\alpha}$				$\widehat{\lambda}$			
n	r	T	MLE	α_L	α_U	MSE	MLE	λ_L	λ_U	MSE
10	5	8	4.1496	−3.4031	11.7023	0.0289	0.3220	0.1461	0.4979	1.2408
10	6	8	4.2067	−2.0585	10.4720	0.0751	0.5963	0.3495	0.8431	1.9062
20	10	5	2.9435	−0.7317	6.6187	0.0448	0.4801	0.2972	0.6630	0.9003
20	15	5	2.6415	−0.2655	5.5485	0.0730	0.5809	0.3343	0.8274	0.8616
30	20	10	1.2994	−0.0102	2.6091	0.0150	0.2441	0.1236	0.3645	0.1631
30	25	10	0.5300	−0.0370	1.0971	0.0240	0.2593	0.0774	0.4412	0.0750

Table 2: The maximum likelihood estimators, the asymptotic intervals and MSE for the parameters α and λ when ($\alpha = 2.5$ and $\lambda = 2$)

			$\widehat{\alpha}$				$\widehat{\lambda}$			
n	r	T	MLE	α_L	α_U	MSE	MLE	λ_L	λ_U	MSE
10	5	8	4.7459	−3.6337	13.1256	0.0286	0.3337	0.1656	0.5018	1.4269
10	6	8	4.8406	−2.7215	12.4029	0.0349	0.3851	0.2073	0.5629	1.4859
20	10	5	12.0169	−0.8542	24.8881	0.03255	0.6147	0.5109	0.7185	4.0372
20	15	5	5.0674	0.0137	10.1212	0.0666	0.6881	0.4982	0.8780	1.7743
30	20	10	4.8805	0.3993	9.3618	0.0386	0.5268	0.3832	0.6704	1.2045
30	25	10	0.6056	−0.0330	1.2442	0.0168	0.2273	0.0820	0.3726	0.0740

Table 3: The maximum likelihood estimators, the asymptotic intervals and MSE for the parameters α and λ when ($\alpha = 1$ and $\lambda = 1$)

			$\widehat{\alpha}$				$\widehat{\lambda}$			
n	r	T	MLE	α_L	α_U	MSE	MLE	λ_L	λ_U	MSE
10	5	8	1.9353	−1.9254	5.7960	0.0271	0.27184	0.0762	0.4674	0.5354
10	6	8	1.1033	−0.9884	3.1950	0.02910	0.2638	0.0476	0.4800	0.2815
20	10	5	2.5655	−0.7978	5.9289	0.04488	0.4610	0.2702	0.6518	0.7911
20	15	5	1.0497	−0.2518	2.3513	0.0865	0.5058	0.1704	0.8413	0.3359
30	20	10	0.4576	−0.0073	0.9225	0.0098	0.1944	0.0950	0.2938	0.0461
30	25	10	0.3316	−0.0175	0.6808	0.0186	0.2272	0.0661	0.3883	0.0404

6 Conclusion

This paper, considered the analysis of the hybrid censored data when the life time distribution of the individuals item is a two-parameter Marshall-Olkin exponential distribution. It also studied the maximum likelihood estimations of the unknown shape (α) and scale (λ) parameters of Marshall-Olkin exponential distribution based on Type-II hybrid censoring scheme. The EM algorithm, Fisher information matrix with numerical results were presented.

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