

Estimating Dynamic Panel Data Models with Random Individual Effect: Instrumental Variable and GMM Approach

Johnson T. Olajide^{1,*} and Olusanya E. Olubusoye²

¹ Department of Mathematics & Statistics, The Polytechnic, Ibadan, Ibadan, Nigeria

² Department of Statistics, University of Ibadan, Ibadan, Nigeria

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Abstract: This paper investigated the performance of some Instrumental Variable (IV) estimators and Generalized Method of Moment (GMM) estimators of a dynamic panel data model with random individual effect. The bias and root mean square error criteria were used to assess the sensitivity of the estimators for a serially correlated error term. Monte Carlo study was performed to study the impact of sample size on the performance of different estimators using four different generating schemes for the serial correlation of the error term, namely autoregressive of order one (AR(1)), autoregressive of order two (AR(2)), moving average of order one (MA(1)) and moving average of order two (MA(2)). The results of the simulation showed that Anderson-Hsiao Instrumental Variable Estimator in difference form (AH(d)) performed better when the time dimension is small while the one step Arellano-Bond Generalized Method of Moment (ABGMM(1)) outperformed other estimators when the time dimension is large. The biases of most of the estimators improve as the time dimension increases except in some cases. The effect of serial correlation is minimal using different generating procedures.

Keywords: Autocorrelation, Error term, Estimators, Exogenous variable, Dynamic model

1 Introduction

A repeated measurement on statistical units over a given period of time is called the Panel Data (PD). Among Social and behavioral science researchers PD is increasingly becoming popular and exhibiting phenomenal growth. Panel data models can be specified as a Static or Dynamic panel. The inclusion of a lagged dependent variable on the right-hand side of the equation of a PD models is called a dynamic panel model. It is of interest in a wide range of economic applications, such as Euler equations for household consumption, democracy and education, empirical model of economic growth and so on. Allowing for dynamics in the underlying process may be crucial for recovering consistent estimates of other parameters even when coefficients on the lagged dependent variables are not direct interest.

The most favored form of consistent estimation (of both specifications) is that of instrumental variable (IV). Extending this approach, leads to the more general area of Generalized Method of Moments (GMM). GMM estimation has spawned much interest in attempting to identify the maximum (optimal) number of such conditions [1] and [2]. Prominent among the problems of dynamic panel data model are autocorrelation due to the presence of a lagged dependent variable among the regressors and individual effects characterizing the heterogeneity among individuals.

The dynamic panel data model was first studied by [3]. The estimation of the model with unobserved component using the Generalized Least Squares (GLS) estimator was proposed. After this study, a lot of papers proposed several estimators and discussed their properties. These include [1, 4, 5, 6, 7, 8, 9] to mention a few.

[10] discussed IV estimation in the broader context of GMM and described an extended IV estimation routine that provides GMM estimates as well as additional diagnostic tests. Among empirical and applied researchers GMM has become a very popular tool. Many standard estimators, including IV and OLS can be seen as special cases of GMM estimators.

Several studies have been done on serially correlated error term in panel data but limited work has been done on the dynamic panel data models. Among the notable works on the problem of serial correlation in panel are [11, 12, 13, 14]

* Corresponding author e-mail: taiwoolajide2004@yahoo.co.nz

and so on. [14] carried out a Monte Carlo study to compare the finite sample relative efficiency number of pure and pre-test estimator for an error component with first-order autocorrelation remainder disturbances of a dynamic panel data model using Monte Carlo study. [15] investigates small sample properties of dynamic panel data estimator using different generating mechanism of error component, v_{it} to be serially uncorrelated, moving average of order one (MA(1)) or autocorrelation of order one (AR(1)) process in a fixed effect model. [16] presented an autocorrelation test that is applicable to dynamic panel data models with serial correlated errors AR(1) or MA(1). Several papers considered dynamic panel data model by ignoring the possibility of serial correlation of disturbance term. These include [17, 18, 19, 20, 21, 22] etc.

In this paper we focus on the estimation of random effect dynamic panel data models for a serially correlated disturbance term in a one-way error component model. Specifically we want to consider the AR and MA of order one and two processes. We want to investigate the performance of some IV and GMM estimators (such as Anderson-Hsiao, Arellano-Bond and Blundell-Bond System) and compare them in term of their bias and root mean square error (RMSE). Also, we are examining the effect of sample size on the performance of the estimators under consideration.

The remainder of the paper is organized as follows: section 2 presents the methodology, the model and estimators considered in the study. In Section 3 we present the design of the Monte-Carlo simulation while Section 4 involves results and discussion of the findings. Lastly, Section 5 is the conclusion.

2 Methodology

This work considered a one-way error component model with one exogenous variable. The error component model, u_{it} is assumed to follow AR(1), AR(2), MA(1) or MA(2) process. This work is an extension of the work of [15], by modifying the cross-section, N to be large and time period, T to be fixed. Similarly, the disturbance term is assumed to be serially correlated. Also, the coefficient of the serial correlation is taken to be mild, moderate or high. Dearth work has been done on the presence of serially correlated disturbance term of the dynamic panel model; most of the previous works were focused on the absence or no serial correlation of the disturbance term. However, we considered the first and second order serial correlation of both AR and MA.

2.1 The model

The dynamic panel data model with one explanatory variable x_{it} and as well as the lagged endogenous variable $y_{i,t-1}$ is given as:

$$y_{it} = \delta y_{i,t-1} + x_{it}'\beta + \mu_i + u_{it}, \quad i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T \quad (1)$$

where N and T are the cross section and time series dimension respectively, y_{it} is the dependent variable, $y_{i,t-1}$ is the lagged dependent variable, x_{it} is a $(k-1) \times 1$ vector of exogenous regressors, μ_i is an individual specific constant term, δ and β are the unknown parameters of lagged dependent variable and vector of the k explanatory variables respectively, and u_{it} is a disturbance term which varies over the cross section and time. $i = (1, 2, \dots, N)$ is an index over the cross section and $t = (1, 2, \dots, T)$ denotes the time dimension. The disturbance term u_{it} is assumed to follow:

$$AR(1) \text{ process: } u_{it} = \rho u_{i,t-1} + \omega_{it}, \quad (2)$$

$$AR(2) \text{ process: } u_{it} = \rho_1 u_{i,t-1} + \rho_2 u_{i,t-2} + \omega_{it}, \quad (3)$$

$$MA(1) \text{ process: } u_{it} = \theta u_{i,t-1} + \omega_{it}, \quad (4)$$

$$MA(2) \text{ process: } u_{it} = \theta_1 \omega_{i,t-1} + \theta_2 \omega_{i,t-2} + \omega_{it}. \quad (5)$$

where ρ and θ are the autoregressive and moving average parameters, ω_{it} is independently and identically distributed with mean zero and variance σ_{ω}^2 . The explanatory variables are assumed to be weakly exogenous forcing variables with $E(\omega_{it} x_{is}) \neq 0$ for $s \geq t$ and zero otherwise.

2.2 Brief discussion of some estimators of dynamic panel data models

We consider the following estimators in this study: Anderson-Hsiao estimator using lagged levels as instrument (AH(l)), Anderson-Hsiao estimator using lagged differences as instrument (AH(d)), first step Arellano-Bond GMM estimator (ABGMM1), second step Arellano-Bond GMM estimator using estimated covariance matrix (ABGMM2), first step system-estimator using level and differences as instruments proposed by Blundell and Bond (SSY1) and second step system-estimator using estimated covariance matrix proposed by Blundell and Bond (SSY2).

A brief theoretical formulation of the estimators of dynamic panel data models considered in this study is given below:

2.2.1 The Anderson-Hsiao(AH)

In [5] the estimator based on the differenced form of the original equation (i.e. equation (1)) is given as

$$y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1})\beta + u_{it} - u_{i,t-1} \tag{6}$$

which cancelled the individual fixed effects assumed to possibly correlate with the exogenous variables ($E(x'_{it} - \mu_i) \neq 0$). He suggested the use of level instruments y_{t-2} or the lagged difference $y_{i,t-2} - y_{i,t-3}$ as an instrument for the differenced lagged endogenous regressor $y_{i,t-1} - y_{i,t-2}$.

The Anderson-Hsiao estimator is given as

$$\hat{\gamma}^{AH} = (XPX)^{-1} X'Py \tag{7}$$

where $P = Z(Z'Z)^{-1}Z$, the symbol l or d to represent the use of levels or differences as instruments ($\hat{\gamma}^{AH(l)}, \hat{\gamma}^{AH(d)}$) where $\gamma = (\delta, \beta)'$

2.2.2 The Arellano and Bond estimator (ABGMM)

The Arellano and Bond estimator is similar to the one suggested by [5] but exploits additional moment restrictions, which increases the set of instruments. The number of orthogonal conditions (moments), e.g. $T(T-1)(K_1+1/2)$, is much larger than the number of parameters, e.g. K_1+1 . Thus [6] suggested a generalized method of moment (GMM) estimator. The dynamic equation to be estimated in levels is $y_{it} = \delta y_{i,t-1} + X'_{it}\beta + \mu_i + u_{it}$ where the individual effect μ_i is eliminated by differencing:

$$y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1})\beta + u_{it} - u_{i,t-1}. \tag{8}$$

We now look for the instruments available for instrumenting the difference equation for each year. For $t = 3$ the equation to be estimated is

$$y_{i3} - y_{i,2} = \delta(y_{i2} - y_{i1}) + (x'_{i3} - x'_{i,2})\beta + u_{i3} - u_{i2} \tag{9}$$

where the instruments (again assuming x being at least predetermined) $y_{i,1}$, x'_{i2} and x'_{i1} are available.

The Arellano-Bond estimator is given by

$$\hat{\gamma}^{ABGMM} = (XW\hat{V}^{-1}W'X)^{-1}X'W\hat{V}^{-1}W'y. \tag{10}$$

Where the first-step estimator makes use of a covariance matrix taking this autocorrelation into account:

$$V = W'GW = \sum_{i=1}^N W'G_TW_i \tag{11}$$

The second-step GMM estimator uses the residuals of the first-step estimation to estimate the covariance matrix as suggested by [23]:

$$\hat{V} = \sum_{i=1}^N W'F_T\hat{v}_i\hat{v}'_iF'_TW_i \tag{12}$$

2.2.3 The Blundell and Bond System GMM Estimator (SSY)

When the instruments are weak the GMM estimator suggested by [6] is known to be rather inefficient because making use of the information contained in differences only. The importance of exploiting the initial condition in generating efficient estimators of the dynamic panel data model when T is small is revisited by [8]. A simple autoregressive panel data model with no exogenous regressors was considered.

$$y_{it} = \delta y_{i,t-1} + \mu_i + u_{it} \quad (13)$$

With $E(\mu_i) = 0$, $E(u_{it}) = 0$, and $E(\mu_i u_{it}) = 0$ for $i = 1, 2, \dots, N$; $t = 1, 2, \dots, T$. [8] focussed on the case where $T = 3$ and therefore there is only one orthogonality condition given by $E(y_{it} \Delta u_{i3}) = 0$, so that δ identified.

The Blundell-Bond estimator is given by

$$\hat{\gamma}^{GMM-SSY} = (XW\hat{V}^{-1}W'X)^{-1}X'W\hat{V}^{-1}W'y \quad (14)$$

The first step estimator makes use of a covariance matrix taking this autocorrelation into account, enlarged for the level equations while the second step GMM estimator uses residuals of the first step estimation to estimate the covariance matrix as pointed out in [23].

3 Monte Carlo study

In this section, we describe the Monte Carlo design used to examine the performances of different techniques in the estimation of the dynamic panel data model. The data generation process is similar to [24]. The model for y_{it} is given in equation (1). $x_{it} = \lambda x_{i,t-1} + \varepsilon_{it}$ where ε_{it} is uniformly distributed on the interval $(-0.5, 0.5)$, $\lambda = 0.5$, $\mu_i \sim IIN(0, 1)$, u_{it} is assumed to follow AR(1), AR(2), MA(1) or MA(2) processes as specified in equations (2), (3), (4) and (5) respectively. Next the values of serial correlation parameters ρ and θ were varied over the following values (0.2, 0.5, 0.8). the values of lagged endogenous variable δ is also varied at (0.1, 0.5, 0.9) and exogenous parameter $\beta = 1$. Two sizes of cross section units (50 and 100) and three time dimensions (5, 10 and 20) for various combinations of N and T were used for simulation and 500 replications are performed since we are considering GMM estimator that is computationally intensive. The bias and root mean square error (RMSE) criteria were used to assess the sensitivity of the estimators. The estimators considered are: AH(l), AH(d), ABGMM1, ABGMM2, SSY1 and SSY2.

4 Results and discussion

The results of the performances of estimators under consideration at various level of serial correlation are discussed in this section. The bias and root mean square errors (RMSE) criteria were used to assess the performance of each estimator. The summary of the results of the performance (on average) of the estimators of δ and β according to RMSE criteria are presented in tables 1 and 2, respectively for possible combinations of N and T . The asterisk sign indicate the estimator that has minimum RMSE while the other one is the one with the maximum. The simulation result for only AR(1) is shown in tables 3-6 to save space. Results for AR(2), MA(1) and MA(2) follow similar pattern and can be released on request from authors.

Estimating δ : RMSE

The Monte-Carlo results show that AH(d) and AH(l) performed better than any other five estimators when $N=50$ and $T=5$ or 10 respectively regardless of the data generating process of the disturbance term, u_{it} and the value of the parameter of lagged dependent variable, δ (i.e. when $\delta = 0.1, 0.5$ or 0.9) with average minimum RMSE of 0.0596 and 0.0421, respectively while ABGMM2 has the worst performance in terms of RMSE. But when the time dimension T increases to 20, ABGMM1 estimator outperforms other estimators with minimum RMSE of 0.0244 on average while SSY1 is the worst in performance (table 3). Also, when $N=100$, AH(l) performs better when $T=5$ or 10, while ABGMM2 performs worst. But here as the time dimension T increases to 20, AH(d) performed better than all other estimators while SSY1 has the worst performance. We also observed that as the serial correlation ρ and θ increase, the performance of nearly all the estimators improve for all the combinations of N and T . In addition, as time dimension, T increases the performance of AH(l), AH(d) and ABGMM1 estimators improves. But for all other estimators the performance improves when T increases from 5 to 10 but deteriorate when it increases to 20. Generally, as sample size increases the performances of the estimators improve.

Table 1: Summary of RMSE performance of δ when $\lambda=0.5$ and ρ or $\theta=0.2, 0.5$ and 0.8

N	T	δ	AR(1)	AR(2)	MA(1)	MA(2)
50	5	0.1	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.5	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.9	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
	10	0.1	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.5	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.9	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
	20	0.1	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1
			SSY1	SSY1	SSY1	SSY1
		0.5	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1
			SSY1	SSY1	SSY1	SSY1
		0.9	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1
			SSY1	SSY1	SSY1	SSY1
100	5	0.1	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.5	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.9	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
	10	0.1	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.5	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.9	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
	20	0.1	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			SSY1	SSY1	SSY1	SSY1
		0.5	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			SSY1	SSY1	SSY1	SSY1
		0.9	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			SSY1	SSY1	SSY1	SSY1

Source: extracted from tables 3-6

Estimating δ : bias

AH(l) tends to perform slightly better than the other estimators especially when T and ρ increase. The bias of AH(l), AH(d), SSY1 and SSY2 are always negative when T =10 or 20 while the bias of ABGMM1 and ABGMM2 are severe and negative when T=5 but it drastically improves in magnitude when T increases.

Estimating β : RMSE

As shown in table 2 and 4, AH(l) outperforms all other estimators when N=50 and T =5 with minimum RMSE of 0.1354 while ABGMM2 is the worst in performance with higher RMSE at different scenario of data generating process but when T=10 or 20 the ABGMM1 performs better while SSY2 performs worst. When N=100, AH(d) is better while ABGMM2 has the worst performance when T=5 but when T=10 ABGMM2 on an average performs better than any other estimators and SSY2 is the worst. Also, when T=20, ABGMM1 outperforms other estimators while SSY2 also performs worst. Hence, as the time period and cross-sections increase most of the estimators improve.

Table 2: Summary of RMSE performance of β when $\lambda=0.5$ and ρ or $\theta = 0.2, 0.5$ and 0.8

N	T	δ	AR(1)	AR(2)	MA(1)	MA(2)
50	5	0.1	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.5	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.9	*AH(l)	*AH(l)	*AH(l)	*AH(l)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
	10	0.1	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1
			SSY2	SSY2	SSY2	SSY2
		0.5	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1
			SSY2	SSY2	SSY2	SSY2
		0.9	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1
			SSY2	SSY2	SSY2	SSY2
20	0.1	ABGMM1	ABGMM1	ABGMM1	ABGMM1	
		SSY2	SSY2	SSY2	SSY2	
	0.5	ABGMM1	ABGMM1	ABGMM1	ABGMM1	
		SSY2	SSY2	SSY2	SSY2	
	0.9	ABGMM1	ABGMM1	ABGMM1	ABGMM1	
		SSY2	SSY2	SSY2	SSY2	
100	5	0.1	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.5	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
		0.9	*AH(d)	*AH(d)	*AH(d)	*AH(d)
			ABGMM2	ABGMM2	ABGMM2	ABGMM2
	10	0.1	*ABGMM2	*ABGMM1	*ABGMM2	*ABGMM2
			SSY2	SSY2	SSY2	SSY2
		0.5	*ABGMM2	*ABGMM1	*ABGMM2	*ABGMM2
			SSY2	SSY2	SSY2	SSY2
		0.9	*ABGMM2	*ABGMM1	*ABGMM2	*ABGMM2
			SSY2	SSY2	SSY2	SSY2
20	0.1	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1	
		SSY2	SSY2	SSY2	SSY2	
	0.5	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1	
		SSY2	SSY2	SSY2	SSY2	
	0.9	*ABGMM1	*ABGMM1	*ABGMM1	*ABGMM1	
		SSY2	SSY2	SSY2	SSY2	

Source: extracted from tables 3-6

Estimating β : bias

For the estimate of the parameter of exogenous variable, AH(l) estimator proved to be better in performance with minimum bias of 0.0064 when N=50 at various values T. Also, when N=100 AH(l) still behave well for T= 5 but when T=10 and 20 ABGMM1 has the overall better bias performance. As the values of ρ and θ increase the bias of most of the estimators improves expect the AH(l) that deteriorates. The magnitude of the bias of most of the estimators is high when the time dimension is low.

5 Conclusion

In this paper, the result of various Monte Carlo study of the instrumental variable and Generalized method of moment estimators of dynamic panel data model for a random individual effects in the presence of serially correlated error term were compared. The results of the Monte Carlo simulation revealed that the estimators performed interchangeably irrespective of the sample size and data generating schemes of disturbance term. The instrumental variable proposed by Anderson-Hsiao (1982) (i.e. AH(l) and AH(d)) performs reasonably well when the time dimension is small(T=5) and in some cases when time dimension is moderate(T=10) while GMM estimator proposed by Arellano-Bond (especially ABGMM1) outperforms other estimators when the time dimension is large (T=20), this may be as a result of finite sample properties. The Blundell-Bond system estimators (SSY1) do not behave well when the panel sample size is large

Table 3: The RMSE and Bias of estimate with respect to δ at $N=50, \lambda=0.5$ True model is AR(1)

AR(1)	T	δ	ρ	AH(l)		AH(d)		ABGMM1		ABGMM2		SSY1		SSY2		
				RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	
5	0.1	0.2	0.2	0.06272	0.00742	0.05961	0.00588	1.75235	-0.8214	2.07921	-0.7768	0.3445	0.29646	0.4214	0.3567	
			0.5	0.06272	0.00738	0.05959	0.00586	2.06679	-1.2397	2.06467	-0.1759	0.3359	0.28855	0.4066	0.3387	
			0.8	0.06271	0.00735	0.05957	0.00584	2.313	-1.4754	2.30435	0.6304	0.3084	0.26127	0.3654	0.2945	
	0.5	0.2	0.2	0.06271	0.00743	0.05961	0.00588	1.75235	-0.8214	2.07921	-0.7768	0.3445	0.29646	0.4214	0.3567	
			0.5	0.06271	0.0074	0.05959	0.00586	2.06679	-1.2397	2.06467	-0.1759	0.3359	0.28855	0.4066	0.3387	
			0.8	0.06271	0.00737	0.05957	0.00584	2.313	-1.4754	2.30435	0.6304	0.3084	0.26127	0.3654	0.2945	
	0.9	0.2	0.2	0.0627	0.00743	0.05961	0.00588	1.75235	-0.8214	2.07921	-0.7768	0.3445	0.29646	0.4214	0.3567	
			0.5	0.06271	0.00741	0.05959	0.00586	2.06679	-1.2397	2.06467	-0.1759	0.3359	0.28855	0.4066	0.3387	
			0.8	0.06271	0.00738	0.05957	0.00584	2.313	-1.4754	2.30435	0.6304	0.3084	0.26127	0.3654	0.2945	
	10	0.1	0.2	0.2	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07726	0.00042	0.0453	-0.0108	0.0529	-0.023
				0.5	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07792	0.006	0.0453	-0.0108	0.0525	-0.025
				0.8	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07886	0.00879	0.0453	-0.0108	0.0544	-0.028
0.5		0.2	0.2	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07726	0.00042	0.0453	-0.0108	0.0534	-0.024	
			0.5	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07792	0.006	0.0453	-0.0108	0.0531	-0.025	
			0.8	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07886	0.00879	0.0453	-0.0108	0.0551	-0.029	
0.9		0.2	0.2	0.04211	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07726	0.00042	0.0453	-0.0108	0.0538	-0.024	
			0.5	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07792	0.006	0.0453	-0.0108	0.0537	-0.026	
			0.8	0.04212	-0.0056	0.04752	-0.0049	0.07349	0.00894	0.07886	0.00879	0.0453	-0.0108	0.0558	-0.029	
20		0.1	0.2	0.2	0.03026	-0.0019	0.03362	-0.002	0.02164	0.01889	0.12336	0.11627	0.3496	-0.2692	0.0896	-0.089
				0.5	0.03124	-0.0031	0.03293	-0.0031	0.02864	0.01193	0.18152	0.16754	0.4409	-0.4315	0.0476	-0.001
				0.8	0.03203	-0.0009	0.03106	-0.0037	0.02297	0.01856	0.239	0.23899	0.4092	-0.3738	0.1071	0.106
	0.5	0.2	0.2	0.03101	-0.0033	0.03354	-0.0038	0.01998	0.01992	0.17865	0.07153	0.444	-0.438	0.0956	-0.072	
			0.5	0.02978	-0.0037	0.03501	-0.0033	0.02278	0.01835	0.22723	0.04817	0.3925	-0.3399	0.0631	-0.051	
			0.8	0.03097	-0.0035	0.03375	-0.0032	0.02289	0.01829	0.26581	0.07172	0.419	-0.3889	0.0128	-0.013	
	0.9	0.2	0.2	0.03133	-0.0041	0.03338	-0.0027	0.02693	0.01404	0.33102	0.25793	0.4508	-0.4508	0.005	-0.005	
			0.5	0.03101	-0.0033	0.03354	-0.0038	0.02678	0.01488	0.44101	0.44097	0.4409	-0.4324	0.0433	-0.003	
			0.8	0.03294	0.0002	0.02932	-0.006	0.02289	0.01829	0.439	0.40934	0.4445	-0.4401	0.0264	0.0264	

Source: produced by author

Table 4: The RMSE and Bias of estimate with respect to β at $N=50, \lambda=0.5$ True model is AR(1)

AR(1)	T	δ	ρ	AH(l)		AH(d)		ABGMM1		ABGMM2		SSY1		SSY2		
				RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	bias	RMSE	bias	RMSE	Bias	
5	0.1	0.2	0.2	0.13455	-0.00014	0.14094	-0.0164	1.12016	-0.1028	1.17103	-0.1179	0.30975	0.2213	0.2194	0.1168	
			0.5	0.13535	0.000654	0.1415	-0.0149	1.21539	-0.3514	1.2267	0.1805	0.28128	0.1828	0.1964	0.0756	
			0.8	0.13556	0.001982	0.14132	-0.0134	1.31747	-0.4986	1.42174	0.6153	0.26063	0.1491	0.1834	0.0324	
	0.5	0.2	0.2	0.13452	-8.64E-05	0.141	-0.0165	1.12016	-0.1028	1.17103	-0.1179	0.30975	0.2213	0.2194	0.1168	
			0.5	0.13533	0.000699	0.14155	-0.0149	1.21539	-0.3514	1.2267	0.1805	0.28128	0.1828	0.1964	0.0756	
			0.8	0.13555	0.002019	0.14137	-0.0133	1.31747	-0.4986	1.42174	0.6153	0.26063	0.1491	0.1834	0.0324	
	0.9	0.2	0.2	0.1345	-3.79E-05	0.14105	-0.0165	1.12016	-0.1028	1.17103	-0.1179	0.30975	0.2213	0.2194	0.1168	
			0.5	0.13531	0.000745	0.1416	-0.0149	1.21539	-0.3514	1.2267	0.1805	0.28128	0.1828	0.1964	0.0756	
			0.8	0.13553	0.002057	0.14141	-0.0133	1.31747	-0.4986	1.42174	0.6153	0.26063	0.1491	0.1834	0.0324	
	10	0.1	0.2	0.2	0.10714	-0.00409	0.11202	-0.0184	0.0665	-0.0059	0.07139	0.0239	0.16396	-0.0049	0.1879	0.0592
				0.5	0.1074	-0.00394	0.11227	-0.0186	0.0665	-0.0059	0.07039	0.0199	0.16396	-0.0049	0.1879	0.0634
				0.8	0.10765	-0.00379	0.11248	-0.0187	0.0665	-0.0059	0.07005	0.0166	0.16396	-0.0049	0.1878	0.0612
0.5		0.2	0.2	0.10714	-0.00408	0.11202	-0.0184	0.0665	-0.0059	0.07139	0.0239	0.16396	-0.0049	0.1888	0.0588	
			0.5	0.10741	-0.00394	0.11227	-0.0186	0.0665	-0.0059	0.07039	0.0199	0.16396	-0.0049	0.1877	0.0628	
			0.8	0.10765	-0.00379	0.11247	-0.0187	0.0665	-0.0059	0.07005	0.0166	0.16396	-0.0049	0.1869	0.0601	
0.9		0.2	0.2	0.10714	-0.00408	0.11201	-0.0184	0.0665	-0.0059	0.07139	0.0239	0.16396	-0.0049	0.189	0.0578	
			0.5	0.10741	-0.00393	0.11227	-0.0186	0.0665	-0.0059	0.07039	0.0199	0.16396	-0.0049	0.1869	0.0608	
			0.8	0.10765	-0.00378	0.11247	-0.0187	0.0665	-0.0059	0.07005	0.0166	0.16396	-0.0049	0.1856	0.0576	
20		0.1	0.2	0.2	0.07162	0.009114	0.07474	-0.0145	0.04122	0.03929	0.10548	0.1025	0.40215	-0.3066	0.5452	-0.54
				0.5	0.07427	0.010997	0.07378	-0.0119	0.03798	0.02546	0.14376	0.1273	0.49877	-0.4892	0.5215	-0.514
				0.8	0.07774	0.011088	0.06998	-0.008	0.03595	0.03299	0.18718	0.1856	0.46405	-0.4243	0.4859	-0.478
	0.5	0.2	0.2	0.07569	0.012339	0.07251	-0.0133	0.04163	0.04028	0.18378	0.1441	0.50343	-0.4976	0.485	-0.423	
			0.5	0.07468	0.010589	0.07286	-0.0114	0.0386	0.03562	0.20737	0.1378	0.44635	-0.3802	0.519	-0.502	
			0.8	0.07671	0.009404	0.07167	-0.0134	0.0361	0.03312	0.23041	0.1414	0.47419	-0.4365	0.5441	-0.544	
	0.9	0.2	0.2	0.07557	0.010676	0.07204	-0.014	0.04034	0.0325	0.28069	0.2264	0.51103	-0.511	0.4495	-0.45	
			0.5	0.07569	0.012322	0.07245	-0.0132	0.03606	0.0286	0.3668	0.3668	0.49965	-0.4885	0.433	-0.377	
			0.8	0.07864	0.008472	0.06798	-0.0032	0.0361	0.03312	0.36072	0.3474	0.50214	-0.4989	0.4675	-0.467	

Source: produced by author

though it has small RMSE and bias. As the time dimension and individual units increase most of the estimators improves drastically. It is therefore established that the characteristics of the data, in particular the size of the panel affect the choice of the best estimator in the dynamic panel data model. Therefore, there is no estimator that is more appropriate in all circumstances. The effect of incorporating different mechanism of serial correlation in the disturbance term of the model is very small for both the bias and RMSE of the parameters of interest.

Table 5: The RMSE and Bias of estimate with respect to δ at $N=100, \lambda=0.5$ True model is AR(1)

AR(1)	T	δ	ρ	AH(l)		AH(d)		ABGMM1		ABGMM2		SSY1		SSY2	
				RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	bias	RMSE	Bias
5	0.1	0.2	0.0445	-0.0058	0.0484	-0.005	2.078	1.4206	2.5711	-2.1219	0.3003	0.1902	0.3537	0.2119	
			0.0445	-0.0058	0.0484	-0.005	2.0763	1.4222	2.5495	-2.1058	0.2696	0.1707	0.3111	0.1803	
			0.0445	-0.0058	0.0484	-0.005	2.0767	1.427	2.5339	-2.0959	0.2447	0.155	0.2811	0.1631	
	0.5	0.2	0.0445	-0.0058	0.0484	-0.005	2.078	1.4206	2.5711	-2.1219	0.3003	0.1902	0.3537	0.2119	
			0.0445	-0.0058	0.0484	-0.005	2.0763	1.4222	2.5495	-2.1058	0.2696	0.1707	0.3111	0.1803	
			0.0445	-0.0058	0.0484	-0.005	2.0767	1.427	2.5339	-2.0959	0.2447	0.155	0.2811	0.1631	
	0.9	0.2	0.0445	-0.0058	0.0484	-0.005	2.078	1.4206	2.5711	-2.1219	0.3003	0.1902	0.3537	0.2119	
			0.0445	-0.0058	0.0484	-0.005	2.0763	1.4222	2.5495	-2.1058	0.2696	0.1707	0.3111	0.1803	
			0.0445	-0.0058	0.0484	-0.005	2.0767	1.427	2.5339	-2.0959	0.2447	0.155	0.2811	0.1631	
	10	0.1	0.2	0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0609	0.01738	0.0295	0.0009	0.0524	-0.03
				0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0613	0.01717	0.0295	0.0009	0.0517	-0.028
				0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0616	0.01557	0.0295	0.0009	0.0517	-0.028
0.5		0.2	0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0609	0.01738	0.0295	0.0009	0.0524	-0.03	
			0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0613	0.01717	0.0295	0.0009	0.0517	-0.028	
			0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0616	0.01557	0.0295	0.0009	0.0517	-0.028	
0.9		0.2	0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0609	0.01738	0.0295	0.0009	0.0524	-0.03	
			0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0613	0.01717	0.0295	0.0009	0.0517	-0.028	
			0.0282	-0.0013	0.0349	0.00313	0.0529	-0.002	0.0616	0.01557	0.0295	0.0009	0.0517	-0.028	
20		0.1	0.2	0.0219	0.0019	0.0219	0.00082	0.0294	0.0003	0.0779	-0.0484	0.4047	-0.365	0.2375	-0.224
				0.0202	0.00333	0.023	-0.0004	0.0294	0.0003	0.078	-0.0482	0.3865	-0.321	0.233	-0.206
				0.0199	0.0032	0.0233	0.00013	0.0294	0.0003	0.0752	-0.0429	0.2171	-0.104	0.2422	-0.225
	0.5	0.2	0.0247	-0.0011	0.0197	0.00337	0.0294	0.0003	0.0777	-0.0655	0.4387	-0.414	0.2406	-0.228	
			0.0243	-0.0015	0.0201	0.00379	0.0298	-0.014	0.0674	-0.0527	0.2613	-0.144	0.2378	-0.229	
			0.0228	0.00148	0.02	0.00066	0.0294	0.0003	0.078	-0.05	0.4601	-0.449	0.2396	-0.233	
	0.9	0.2	0.0219	0.0019	0.0219	0.00082	0.0299	-0.004	0.0923	-0.0707	0.3974	-0.339	0.2259	-0.211	
			0.021	0.00289	0.0226	-0.0017	0.0294	0.0003	0.0885	-0.0656	0.372	-0.298	0.2049	-0.172	
			0.0235	-0.0002	0.0202	0.00328	0.0296	-0.004	0.0678	-0.0509	0.4275	-0.387	0.222	-0.205	

Source: produced by author

Table 6: The RMSE and Bias of estimate with respect to β at $N=100, \lambda=0.5$ True model is AR(1)

AR(1)	T	δ	ρ	AH(l)		AH(d)		ABGMM1		ABGMM2		SSY1		SSY2	
				RMSE	bias	RMSE	Bias	RMSE	Bias	RMSE	bias	RMSE	bias	RMSE	Bias
5	0.1	0.2	0.1113	0.0074	0.1079	-0.0178	1.224	0.7719	1.3874	-0.9843	0.2659	-0.2025	0.267	-0.202	
			0.1126	0.0064	0.1094	-0.0175	1.2251	0.7727	1.3714	-0.9658	0.2633	-0.2	0.2599	-0.195	
			0.1129	0.0054	0.1104	-0.0173	1.2273	0.7753	1.3596	-0.9526	0.2612	-0.1981	0.256	-0.191	
	0.5	0.2	0.1113	0.0073	0.1079	-0.0178	1.224	0.7719	1.3874	-0.9843	0.2659	-0.2025	0.267	-0.202	
			0.1126	0.0063	0.1094	-0.0174	1.2251	0.7727	1.3714	-0.9658	0.2633	-0.2	0.2599	-0.195	
			0.1128	0.0054	0.1104	-0.0173	1.2273	0.7753	1.3596	-0.9526	0.2612	-0.1981	0.256	-0.191	
	0.9	0.2	0.1113	0.0073	0.1079	-0.0178	1.224	0.7719	1.3874	-0.9843	0.2659	-0.2025	0.267	-0.202	
			0.1126	0.0063	0.1094	-0.0174	1.2251	0.7727	1.3714	-0.9658	0.2633	-0.2	0.2599	-0.195	
			0.1128	0.0053	0.1104	-0.0173	1.2273	0.7753	1.3596	-0.9526	0.2612	-0.1981	0.256	-0.191	
	10	0.1	0.2	0.0677	-0.003	0.0721	-0.006	0.0537	-0.002	0.0535	-0.0136	0.0934	0.0068	0.1171	0.0069
				0.0676	-0.003	0.0721	-0.0061	0.0537	-0.002	0.0538	-0.0118	0.0934	0.0068	0.1183	0.0145
				0.0674	-0.003	0.0721	-0.0063	0.0537	-0.002	0.0546	-0.0124	0.0934	0.0068	0.1199	0.0215
0.5		0.2	0.0677	-0.003	0.0721	-0.006	0.0537	-0.002	0.0535	-0.0136	0.0934	0.0068	0.1171	0.0069	
			0.0676	-0.003	0.0721	-0.0061	0.0537	-0.002	0.0538	-0.0118	0.0934	0.0068	0.1183	0.0145	
			0.0674	-0.003	0.0721	-0.0063	0.0537	-0.002	0.0546	-0.0124	0.0934	0.0068	0.1199	0.0215	
0.9		0.2	0.0677	-0.003	0.0721	-0.006	0.0537	-0.002	0.0535	-0.0136	0.0934	0.0068	0.1171	0.0069	
			0.0676	-0.003	0.0721	-0.0061	0.0537	-0.002	0.0538	-0.0118	0.0934	0.0068	0.1183	0.0145	
			0.0674	-0.003	0.0721	-0.0063	0.0537	-0.002	0.0546	-0.0124	0.0934	0.0068	0.1199	0.0215	
20		0.1	0.2	0.0528	-0.002	0.0533	-0.0022	0.029	-0.004	0.0514	-0.0248	0.6624	-0.6433	1.2823	-1.277
				0.0509	-0.006	0.0546	0.00101	0.029	-0.004	0.0482	-0.0141	0.3603	-0.302	0.4715	-0.428
				0.0509	0.0005	0.0579	0.00529	0.029	-0.004	0.0473	-0.0041	0.2072	-0.0968	0.4824	-0.462
	0.5	0.2	0.0521	-0.001	0.0487	-0.0034	0.029	-0.004	0.0435	-0.0136	0.4104	-0.3851	0.4834	-0.464	
			0.0506	0.0016	0.0486	-0.0049	0.0247	0.0028	0.0328	0.00461	0.2472	-0.1263	0.487	-0.473	
			0.0523	-0.005	0.0497	0.00442	0.029	-0.004	0.0489	-0.011	0.4237	-0.4128	0.4923	-0.488	
	0.9	0.2	0.0528	-0.002	0.0533	-0.0022	0.0282	-0.002	0.0561	-0.0333	0.3728	-0.3203	0.4686	-0.449	
			0.049	-0.004	0.054	0.00107	0.029	-0.004	0.0529	-0.0252	0.3472	-0.2796	0.4386	-0.389	
			0.0509	-0.002	0.0508	-0.0026	0.0281	-0.002	0.0425	0.0015	0.3948	-0.3563	0.4827	-0.466	

Source: produced by author

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Johnson Taiwo OLAJIDE is a Lecturer, in the Department of Mathematics and Statistics, The Polytechnic Ibadan, Nigeria. He received his M.Sc. degree in Statistics and currently undergoing his PhD degree in Statistics (Econometrics) at the University of Ibadan (Nigeria). He had published research articles in reputable local and international journals. He is a member of American Statistical Association (ASA) and International Biometric Society (IBS) Group Nigeria.



Olusanya Elisa OLUBUSOYE is a Senior Lecturer and formerly the Acting Head (May 2009 - April 2011), Department of Statistics, University of Ibadan. He has taught in other universities including Addis Ababa University, Ethiopia, and Pan African University (Institute for Basic Sciences, Technology and Innovation), Kenya. His research interests include macroeconomic modelling, Applied Bayesian Analysis and Statistical Model Simulation. He has served as a consultant and research fellow to several organizations both locally and abroad including MacArthur Foundation, African Union (AU) on Education Management Information System (EMIS), African Economic Research Consortium (AERC) and Lagos State Government on Lagos State Baseline Data and Service Delivery Projects funded by World Bank. He was a visiting scholar to the International Monetary Fund (IMF)

in 2006. He is a member of many Professional and Academic associations including Royal Statistical Society (RSS), London, African Econometric Society (AES), International Biometric Society (IBS) Group Nigeria, and The Nigerian Statistical Association (NSA). He holds the highest professional qualification of Chartered Statistician (CStat) of the RSS. He is a Senior Research Fellow with the Centre for Econometric and Allied Research (CEAR), University of Ibadan.