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Cost Benefit Analysis of a Urea Fertilizer Manufacturing System Model

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Abstract: This paper is concerned with modeling and analysis of a urea fertilizer making system consisting of a number of subsystems of varying nature. Taking constant failure and general repair rates for each subsystem, the system is analysed by using supplementary variable technique. A common cause failure is also considered in system modeling. The expressions for several system characteristics such as reliability, MTSF, steady state availability, busy period and expected profit, which are useful to system managers, engineers, training supervisors and reliability analysts are obtained. The MTSF and profit function have also been studied through graphs in respect of various parameters in a particular case when repair time distributions are taken as exponentials and important conclusions have been drawn from these graphs.

Keywords: Reliability, MTSF, Availability, Supplementary Variable Technique, Busy Period and Profit Function.

1 Introduction

The development of science and technology and the need of modern society are racing against each other. In view of this, industries are trying to introduce more and more automation in their industrial mechanism which results the complexity of industrial systems. The improvement in effectiveness of such complex systems in respect of various reliability and cost affected indices has become important in recent years. An industrial system may consist of a number of subsystems working in varying nature and each subsystem may further be composed of various units connected in different configurations. High system reliability is desirable to reduce the risk of hazards and overall cost of production for the complex and sophisticated plants based on advanced technology such as fertilizer, thermal, paper, sugar, milk powder making plants etc. The reliability and cost-benefit analysis of an industrial plant can help the management in taking timely decision for its smooth functioning as well as to increase the net-expected profit by strengthening the performance of its weak components.

A large number of articles have been already published in the literature of reliability analyzing models based on producing different kind of products. Kumar D. et al.[14] presented the reliability and operational behavior analysis of the pulping system in a paper industry. They further in [15] perform the cost benefit analysis of multi-component screening system in paper industry and in [13] analyzed a stochastic model of the paper production system under different repair policies. More so, in [16] they obtained the availability of crystallization system in sugar industry under common cause failure. Later on, Kaushik and singh [9] performed the reliability analysis of the naphtha fuel oil and water system under priority repair used in thermal power plant. Khan and Kabir [10] described a simulation modeling technique for assessing the availability of ammonia plant. Singh and Goel [24] studied the availability of heating system with warm standby and imperfect switch in sugar industry. Prabhuswami [21] studied the reliability based optimization of manufacturing systems. Kumar, S. et al. [19] presented the study about maintenance management for ammonia synthesis system in a Urea Fertilizer plant. Ma et al. [20] calculated the optimization of a preventive maintenance scheduling for semiconductor manufacturing systems. Ramakrishna and Bawa [22] have discussed optimization of

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machine design criteria for higher reliability and maintainability in food processing industry. Gupta et al. [4,5] discussed the reliability and availability analysis of serial processes of butter oil plant and behavior analysis of the cement industry. Santosh, A. et al. [23] analyzed the reliability of pipelines caring H_2S for risk based inspection of heavy water plant. Kumar and Tewari [17] carried out the analysis for evaluating the performance measures for co-shift conversion system model in a fertilizer plant. Recently, Gupta et al [7] analyzed a stochastic model of milk powder making system in a dairy plant. Thereafter, they [6] analyzed an air condition cooling system model in respect of reliability, MTSF and cost benefit analysis. Kumar and Tewari [18] presented the steady state availability and performance optimization using genetic algorithm technique for CO_2 cooling system of a fertilizer plant. The CO_2 cooling system of a fertilizer plant has five main subsystems of different nature arranged in series network. They have obtained only the steady state results by developing the differential equations through Poisson process. More recently, Khanduja et al. [11] analysed a bleaching system model of a paper plant regarding the steady state behavior and maintenance planning. Some of the other industrial system models producing different products have been already analysed by Damghani, K.K. et al [1]; Kumar, A. et al. [12]; Dev, N. et al [2]; S.M. Famurewa. et al [3] and Vayenas & Peng [25].

In the present paper, keeping in view the purpose of analyzing real existing industrial system model, a urea fertilizer making system is considered. Such type of industrial system model is running by Tata Chemical Ltd. (Fertilizer division, near Dist. Badaun at Indradham, Babrala, India) and the technology of such system is developed by scientist Smemproge Hi. In fact, Urea fertilizer making system is a complex type reparable engineering system involving high risk. The system consists of a number of different subsystems connected in series network. The system may fail if any one of the subsystem fails. The system may also fail completely if all or some units of the system fail due to common cause. Different subsystems are attached with different pressure steam making units whereas wanted. The failure rates of the subsystems are assumed to be exponential while the repair rates are taken to be general. Observing the working network of various subsystems, the possible states of the continuous parametric Markov-Chain have been generated. The following economic related measures of system effectiveness are obtained by using Supplementary Variable Technique:

1. Reliability and mean time to system failure (MTSF).
2. Point-wise and steady-state availabilities.
3. Expected up-time of the system during $(0,t)$.
4. Expected busy period of the repairman during $(0,t)$.
5. Net expected profit earned by the system in $(0,t)$ and in steady-state.

2 Assumptions and System Description

Urea fertilizer making system consists of six subsystems connected in series network. The working of each subsystem is necessary for successful operation of the system. These subsystems are attached with three different pressure steam making units whereas wanted.

The working of different subsystems and pressure making units is shown in **Fig. 1** and explained as follows-

Subsystem A **-**

This subsystem consists of three units in series configuration. First unit is ammonia (NH_3) making unit, second unit is reactor and third unit is striper. First unit makes ammonia and supplies it into reactor where ammonia (NH_3) and carbon dioxide (CO_2, from atmosphere) make urea in liquid form through chemical reaction which is transferred to the stripper in which excess of ammonia is stripped out. In this subsystem, for working of reactor and stripper high pressure steam is needed. Here the resulting urea liquid solution is less concentrated (about 33%). This less concentrated solution of liquid urea is send to subsystem B to make it more concentrate.

Subsystem $B-$

This subsystem consists of two units- separator and exchanger, connected in series network. For the working of this subsystem, medium pressure steam is needed and hence this subsystem is known as medium pressure section. Here the working of separator is to separate out the vapours from the solution of prestage (from subsystem A) and exchanger vapourise the excess of liquid ammonia and water (H_2O) from the solution. Here we get the liquid urea solution more concentrate than previous stage (about 50%). Now this more concentrates liquid urea solution is send to subsystem C to make it more concentrate.

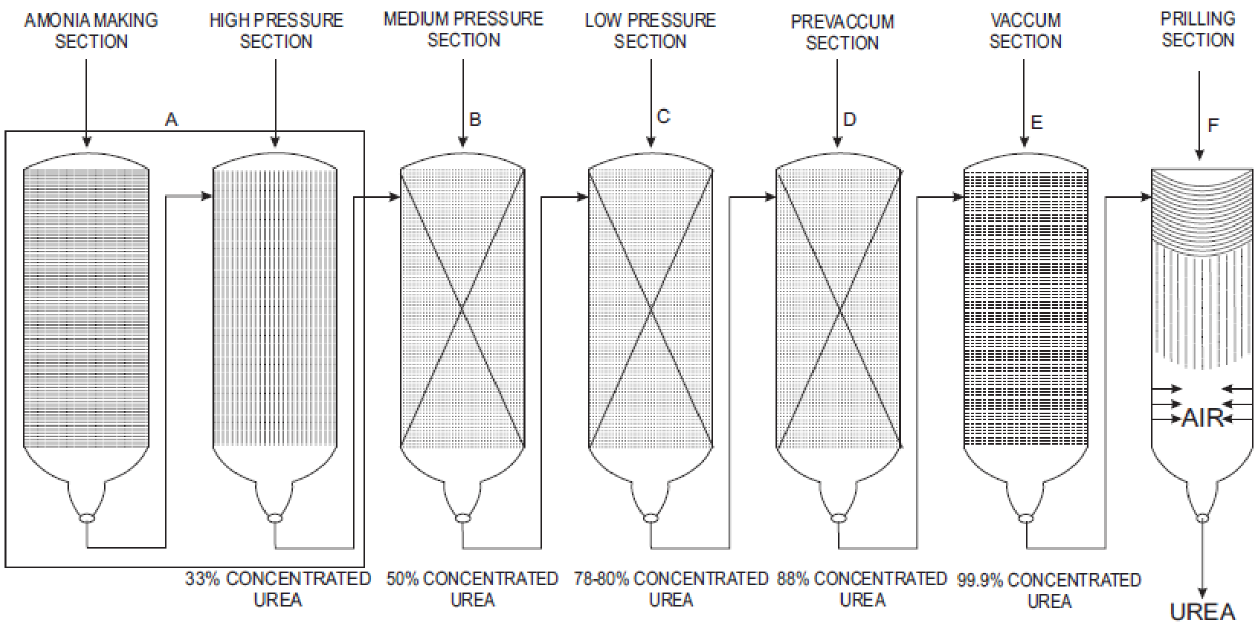

Fig. 1: Urea Fertilizer Making System Network

Subsystem C **-**

This subsystem is similar to the subsystem B and also has separator and exchanger connected in series network. The function of separator and exchanger is same in this state as in subsystem B. For working this subsystem, low pressure steam is needed and hence the subsystem is also known as low pressure unit. The excess of NH_3 and H_2O are vapourised from the liquid urea solution, so that the liquid solution becomes more concentrate than previous stage (78%-80%). Now the resulting solution is transferred into subsystem D.

Subsystem D **-**

This subsystem is used to make more concentrated liquid urea solution (about 88%). The functioning of this subsystem and units are same as that of previous stage subsystems B and C. This subsystem is known as pre-vacuum subsystem because the resulting liquid urea solution from this stage is send to the next subsystem i.e. Vacuum section (Subsystem E).

Subsystem $E-$

This subsystem is also known as vacuum section as vacuum is formed here. This subsystem consists of two units in series network as in subsystem B,C and D but here first unit is exchanger and the second is separator. The liquid urea solution obtained from this section is highly concentrated (99.9%).

Subsystem $F-$

This subsystem is known as prilling section. Here the concentrated liquid urea solution which is obtained from subsystem E is prilled out. This section is of tower form where the concentrated urea is prilled out from up side and simultaneously the atmospheric air is entered from down side resulting that the falling urea solution drops when connected by atmospheric air and its temperature comes down so that it is converted into crystal form i.e. urea.

Pressure units P_1**,** P_2 **and** P_3**-**

For the successful chemical reaction, steams of different pressures are needed. For this there are three pressure units as low pressure, medium pressure and high pressure units (P_1, P_2 and P_3 respectively). Here it should be noted that whenever low and medium pressure units are failed then low and medium pressure steams can be obtained from high pressure units by using some scientific logic and whenever high pressure unit is failed then high pressure steam can't be obtained by any of the rest units. Therefore, process in this condition is stopped and system goes into the failed state. The failed pressure units will be repaired only when the system is failed.

Chemical reactions-

$$\underset{\text{Carbon dioxide}}{CO_2} \quad + \quad \underset{\text{Ammonia}}{2NH_3} \quad \longrightarrow \quad \underset{\text{Carbamate}}{NH_2COOHNH_4}$$

$$\underset{\text{Carbamate}}{NH_2COOHNH_4} \quad \longrightarrow \quad \underset{\text{Urea}}{CO(NH_2)_2} \quad + \quad \underset{\text{Water}}{H_2O}$$

All the failure rates are taken to be exponential while the repair rates are assumed to be general. Since all the subsystems are connected in series network, so it is necessary that they all should be in good condition for successful operation of the system. The system failure may also occur at any stage due to some chance factor (common cause) when some unexpected random happening occurs. The repair is carried out by single repairman only when the system breaks down and after each repair the system becomes as good as new.

3 Notations and States of the System

(a) Notations:

$P_n(t)$	:	Probability that the system is in state S_n at time $t.(n = 0,1,2,\ldots,29)$
$P_u(x,t)dx$	:	Probability that the system is in state S_u at time t and sojourned in this state for the period of time $(x,x+dx)$
α_i	:	Constant failure rates of subsystem A,B,C,D,E and F respectively for $i = 1,2,\ldots,6$.
$\alpha_7,\alpha_8,\alpha_9$	:	Constant failure rate of pressure unit P_1,P_2 and P_3 respectively.
α_c	:	Constant failure rate of entire system from any of its operative state.
$\beta_i(t),g_i(t)$	:	General repair rate of subsystem A,B,C,D,E and F respectively for $i = 1,2,\ldots,6$ and corresponding p.d.f. such that $g_i(t) = \beta_i(t)\exp\left[-\int_0^t \beta_i(u)du\right]$
$h(t),g_h(t)$	:	General repair rate of system failed due to pressure unit P_3 and corresponding p.d.f. such that $g_h(t) = h(t)\exp\left[-\int_0^t h(u)du\right]$
$c(t),g_c(t)$	:	General repair rate of system failed due to common cause failure and corresponding p.d.f. such that $g_c(t) = c(t)\exp\left[-\int_0^t c(u)du\right]$
$\bar{A}$	:	Indicates that subsystem A is failed. Similarly $\bar{B},\bar{C},\bar{D},\bar{E}$ and $\bar{F}$ are defined.
$\bar{P_i}$	:	Indicates that the pressure unit $P_i(i = 1,2,3)$ is failed.
$*,s$	:	Symbols for Laplace transform and dummy variable used in it, viz. $f^*(s) = \int e^{-st}f(t)dt$.

(b) Various states of the system:

S_0	:	Initial operative state of the system where all the subsystems and units are in good condition.
S_7, S_{14}, S_{23}	:	Operative states of the system where the pressure unit P_1, P_2 and (both) P_1, P_2 have failed.
S_{21}	:	Failed state of the system due to the failure of pressure unit P_3.
S_{22}	:	Failed state of the system due to the common cause failure.
S_i, S_j, S_k, S_l	:	Failed states of the system due to the failure of any of the subsystem A, B, C, D, E and F respectively for $i = 1, 2, 3, 4, 5$ and 6 when all the three pressure units are good. $j = 8, 9, 10, 11, 12$ and 13 when pressure unit P_1 is failed. $k = 15, 16, 17, 18, 19$ and 20 when pressure unit P_2 is failed. $l = 24, 25, 26, 27, 28$ and 29 when pressure units P_1 and P_2 are failed.

The transition diagram of the system model is shown in **Fig. 2**.

4 Basic Equations and Their Laplace Transforms

Simple probabilistic considerations give the following set of integro-differential equations.

$$\left[\frac{\partial}{\partial t} + \left(\sum_{i=1}^{6} \alpha_i + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_c\right)\right] P_0(t) = \sum_{i=1}^{6} \int P_i(x,t)\beta_i(x)\, dx + \int P_{21}(x,t)\, h(x)\, dx + \int P_{22}(x,t)\, c(x)\, dx$$

$$\left[\frac{\partial}{\partial t} + \left(\sum_{i=1}^{6} \alpha_i + \alpha_8 + \alpha_9 + \alpha_c\right)\right] P_7(t) = \sum_{j=8}^{13} \int P_j(x,t)\beta_{j-7}(x)\, dx + \alpha_7 P_0(t)$$

$$\left[\frac{\partial}{\partial t} + \left(\sum_{i=1}^{6} \alpha_i + \alpha_7 + \alpha_9 + \alpha_c\right)\right] P_{14}(t) = \sum_{k=15}^{20} \int P_k(x,t)\beta_{k-14}(x)\, dx + \alpha_8 P_0(t)$$

$$\left[\frac{\partial}{\partial t} + \left(\sum_{i=1}^{6} \alpha_i + \alpha_9 + \alpha_c\right)\right] P_{23}(t) = \sum_{l=24}^{29} \int P_l(x,t)\beta_{l-23}(x)\, dx + \alpha_7 P_{14}(t) + \alpha_8 P_7(t)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_i(x)\right] P_i(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_{j-7}(x)\right] P_j(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_{k-14}(x)\right] P_k(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_{l-23}(x)\right] P_l(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + h(x)\right] P_{21}(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + c(x)\right] P_{22}(x,t) = 0 \qquad \text{(1-10)}$$

Boundary conditions:

$$P_i(0,t) = \alpha_i P_0(t)$$

$$P_j(0,t) = \alpha_{j-7} P_7(t)$$

$$P_k(0,t) = \alpha_{k-14} P_{14}(t)$$

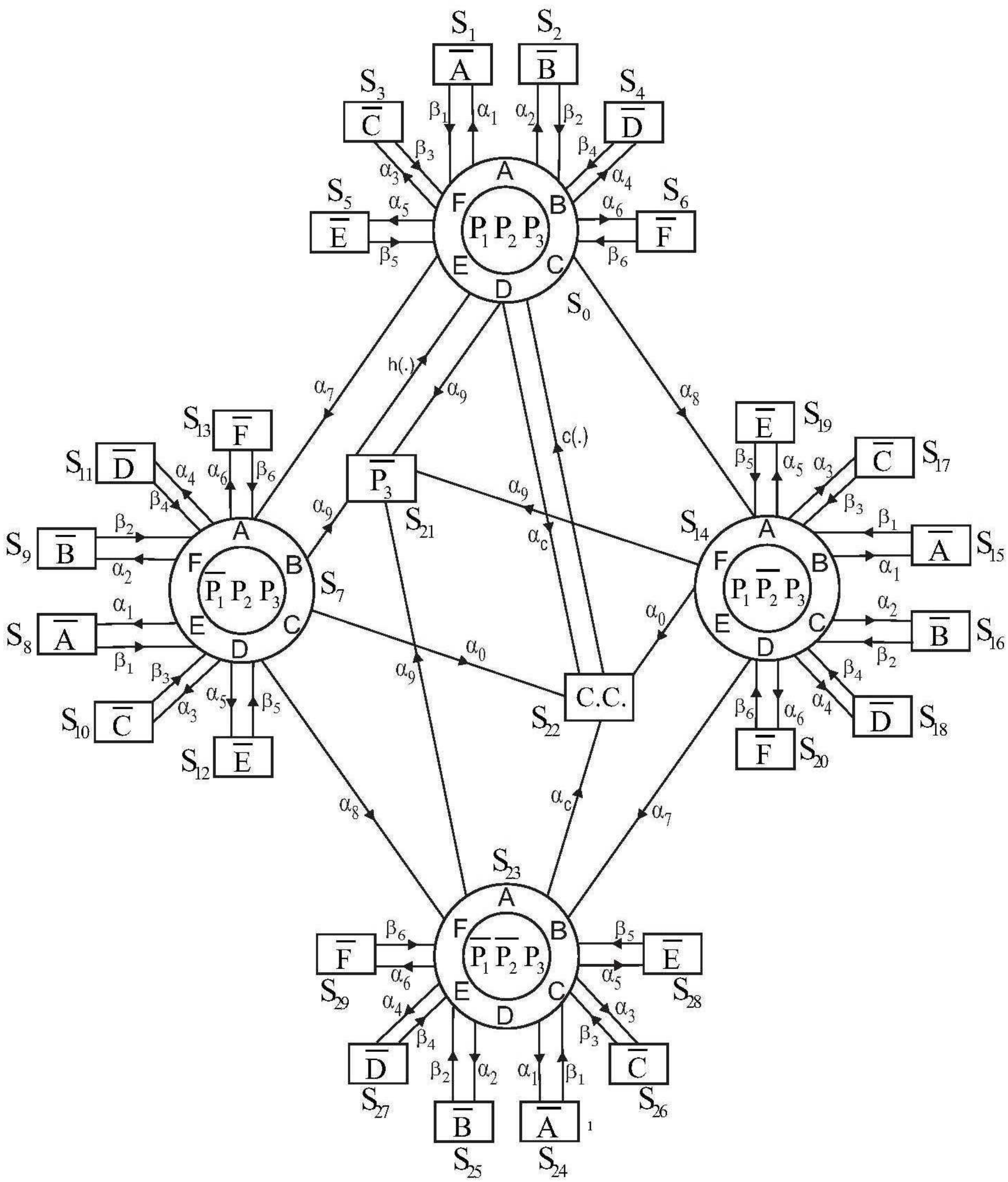

Fig. 2: Transition Diagram

$$P_l(0,t) = \alpha_{l-23} P_{23}(t)$$

$$P_{21}(0,t) = \alpha_9 \left[P_0(t) + P_7(t) + P_{14}(t) + P_{23}(t)\right]$$

$$P_{22}(0,t) = \alpha_c \left[P_0(t) + P_7(t) + P_{14}(t) + P_{23}(t)\right] \tag{11-16}$$

Initial conditions: It is assumed that the system initially starts from normal state S_0 i.e

$$P_0(0) = 1,$$

$$P_n(0) = 0 \qquad \forall \qquad n = 1, 2, \cdots, 29 \tag{17}$$

and

$$P_i(x,0) = P_j(x,0) = P_k(x,0) = P_l(x,0) = 0 \tag{18}$$

5 Solution of $P_n^*(s)$

Taking the Laplace-Transforms of relations (1) to (16) and solving the resulting set of equations in view of the initial conditions (17-18), we get the values of various states probabilities $P_n(t); n = 0, 1, \ldots, 29$ in terms of their Laplace-Transforms as follows-

$$P_i^*(s) = \frac{\alpha_i \left[1 - g_i^*(s)\right] P_0^*(s)}{s}; \quad i = 1 \text{ to } 6$$

$$P_j^*(s) = \frac{\alpha_{j-7} \left[1 - g_{j-7}^*(s)\right] N(s) P_0^*(s)}{s}; \quad j = 8 \text{ to } 13$$

$$P_k^*(s) = \frac{\alpha_{k-14} \left[1 - g_{k-14}^*(s)\right] E(s) P_0^*(s)}{s}; \quad k = 15 \text{ to } 20$$

$$P_l^*(s) = \frac{\alpha_{l-23} \left[1 - g_{l-23}^*(s)\right] H(s) P_0^*(s)}{s}$$

$$P_{21}^*(s) = \frac{\alpha_9 \left[1 - g_h^*(s)\right] \left[1 + N(s) + E(s) + H(s)\right] P_0^*(s)}{s}$$

$$P_{22}^*(s) = \frac{\alpha_c \left[1 - g_c^*(s)\right] \left[1 + N(s) + E(s) + H(s)\right] P_0^*(s)}{s}$$

$$P_7^*(s) = N(s) P_0^*(s)$$

$$P_{14}^*(s) = E(s) P_0^*(s)$$

$$P_{23}^*(s) = H(s) P_0^*(s)$$

$$P_0^*(s) = \left[J(s) - \{\alpha_9 g_h^* + \alpha_c g_c^*\} \{N(s) + E(s) + H(s)\}\right]^{-1} \tag{19-28}$$

$$N(s) = \frac{\alpha_7}{\left(s + \sum_{i=1}^{6} \alpha_i + \alpha_8 + \alpha_9 + \alpha_c\right) - \sum_{j=8}^{13} \alpha_{j-7} g_{j-7}^*(s)}$$

$$E(s) = \frac{\alpha_8}{\left(s + \sum_{i=1}^{6} \alpha_i + \alpha_7 + \alpha_9 + \alpha_c\right) - \sum_{k=15}^{20} \alpha_{k-14} g_{k-14}^*(s)}$$

$$H(s) = \frac{\alpha_7 E(s) + \alpha_8 N(s)}{\left(s + \sum_{i=1}^{6} \alpha_i + \alpha_9 + \alpha_c\right) - \sum_{l=24}^{29} \alpha_{l-23} g_{l-23}^*(s)}$$

$$J(s) = s + \sum_{i=1}^{6} \alpha_i + \alpha_9 + \alpha_c - \sum_{l=24}^{29} \alpha_{l-23} g_{l-23}^*(s) \tag{29-32}$$

6 Analysis of Characteristics

(a) Reliability

The reliability of the system $R(t)$ in terms of its Laplace Transform is given by

$$R^*(s) = LT\,[R(t)]$$

This can be obtained by assuming the failed states $S_i, S_j, S_k, S_l, S_{21}$ andS_{22} as absorbing i.e. all repair rates are zero. Therefore,

$$R^*(s) = [P_0^*(s) + P_7^*(s) + P_{14}^*(s) + P_{23}^*(s)]_{g_i^*(s)=g_{j-7}^*(s)=g_{k-14}^*(s)=g_{l-14}^*(s)=g_h^*(s)=g_c^*(s)=0}$$

$$= [\{1 + N(s) + E(s) + H(s)\}\, P_0^*(s)]_{g_i^*(s)=g_{j-7}^*(s)=g_{k-14}^*(s)=g_{l-14}^*(s)=g_h^*(s)=g_c^*(s)=0} \tag{33}$$

(b) MTSF

If random variable T is defined as time to reach the system into any of its failed state, then mean time to system failure (MTSF) is given by

$$E(T) = \int R(t)\,dt = \lim_{s\to 0} R^*(s)$$

$$= \frac{[1 + N(0) + E(0) + H(0)]}{J(0)} \tag{34}$$

Where,

$$N(0) = \frac{\alpha_7}{\left(\sum_{i=1}^{6} \alpha_i + \alpha_8 + \alpha_9 + \alpha_c\right)},$$

$$E(0) = \frac{\alpha_8}{\left(\sum_{i=1}^{6} \alpha_i + \alpha_7 + \alpha_9 + \alpha_c\right)}$$

$$H(0) = \frac{\alpha_7 E(0) + \alpha_8 N(0)}{\left(\sum_{i=1}^{6} \alpha_i + \alpha_9 + \alpha_c\right)}$$

$$J(0) = \left(\sum_{i=1}^{6} \alpha_i + \alpha_9 + \alpha_c\right)$$

(c)Steady State Availibility

$$A(\infty) = \lim_{t\to\infty} A(t) = lim_{s\to 0} s\, A^*(s)$$

$$= lim_{s\to 0}\, s\,[1 + N(s) + E(s) + H(s)]\, P_0^*(s)$$

Which is indeterminate form. On applying L. Hospital rule, we get

$$A(\infty) = \frac{1 + N(0) + E(0) + H(0)}{D'(0)} \tag{35}$$

Where,

$$N(0) = \frac{\alpha_7}{\alpha_8 + \alpha_9 + \alpha_c},$$

$$E(0) = \frac{\alpha_8}{\alpha_7 + \alpha_9 + \alpha_c},$$

$$H(0) = \frac{\alpha_7 E(0) + \alpha_8 N(0)}{\alpha_9 + \alpha_c},$$

and in terms of

$$J'(0) = \left(1 + \sum_{i=1}^{6} \alpha_i\, m_i + \alpha_9\, m_h + \alpha_c\, m_c\right)$$

$$N'(0) = \alpha_7 \left(\alpha_8 + \alpha_9 + \alpha_c\right)^{-2} \left(1 + \sum_{j=8}^{13} \alpha_{j-7} m_{j-7}\right)$$

$$E'(0) = \alpha_8 \left(\alpha_7 + \alpha_9 + \alpha_c\right)^{-2} \left(1 + \sum_{k=15}^{20} \alpha_{k-14} m_{k-14}\right)$$

$$\begin{aligned} H'(0) = \alpha_7 & \left[E(0)(\alpha_9 + \alpha_c)^{-2} \left(1 + \sum_{l=24}^{29} \alpha_{l-23} m_{l-23}\right) + E'(0)(\alpha_9 + \alpha_c)^{-1}\right] \\ + \alpha_8 & \left[N(0)(\alpha_9 + \alpha_c)^{-2} \left(1 + \sum_{l=24}^{29} \alpha_{l-23} m_{l-23}\right) + N'(0)(\alpha_9 + \alpha_c)^{-1}\right] \end{aligned}$$

$$m_z = -g_z^*(0) = \int t\, g_z(t)\, dt; \quad z = i, j-7, k-14, l-23, h, c$$

We have

$$D'(0) = \left[1/\frac{d}{ds} P_0^*(s)\right]_{s=0} = \left[J'(0) - \{\alpha_9 + \alpha_c\}\{N'(0) + E'(0) + H'(0)\} + \{\alpha_9 m_h + \alpha_c m_c\}\{N(0) + E(0) + H(0)\}\right]$$

(d) Expected up time of the system during $(0,t)$

$$\mu_{up}(t) = \int_0^t A(u)\, du$$

So that,

$$\begin{aligned} \mu_{up}^*(t) &= A^*(s)/s \\ &= \frac{P_0^*(s) + P_7^*(s) + P_{14}^*(s) + P_{23}^*(s)}{s} \\ &= \frac{[1 + N(s) + E(s) + H(s)]\, P_0^*(s)}{s} \end{aligned} \tag{38}$$

(e) Expected busy period of the repairman during $(0,t)$

The expected time during $(0,t)$ in which the system will be under repair i.e. the repairman will be busy is given by

$$\mu_b(t) = \int_0^t B(u)\, du$$

So that,

$$\mu_{up}^*(s) = B^*(s)/s$$

Where,

$$B^*(s) = \sum_{i=1}^{6} P_i^*(s) + \sum_{j=8}^{13} P_j^*(s) + \sum_{k=15}^{20} P_k^*(s) + \sum_{l=24}^{29} P_l^*(s) + P_{21}^*(s) + P_{22}^*(s) \tag{37}$$

$$= P_0^*(s)[\sum_{i=1}^{6} a_i 1 - g_i^*(s) + \sum_{j=8}^{13} a_{j-7} 1 - g_{j-7}^*(s) N(s)$$
$$+ \sum_{k=15}^{20} a_{k-14} 1 - g_{k-14}^*(s) E(s) + \sum_{l=24}^{29} a_{l-23} 1 - g_{l-23}^*(s) H(s)$$
$$+ \alpha_9 1 - g_h^*(s) 1 + N(s) + E(s) + H(s) + \alpha_c 1 - g_c^*(s) 1 + N(s) + E(s) + H(s)] s^{-1}$$

(f) Steady state probability that the repairman will be busy

The probability that the repairman will be busy in the long run is given by

$$B(\infty) = \lim_{t\to\infty} B(t) = \lim_{s\to 0} sB^*(s)$$

Now this is indeterminate form. On applying L. Hospital rule, we get

$$B(\infty) = \sum_{i=1}^{6} a_i m_i + \sum_{j=8}^{13} a_{j-7} m_{j-7} N(0) + \sum_{k=15}^{20} a_{k-14} m_{k-14} E(0) + \sum_{l=24}^{29} a_{l-23} m_{l-23} H(0)$$
$$+ \alpha_9 m_h \{1 + N(0) + E(0) + H(0)\} + \alpha_c m_c \{1 + N(0) + E(0) + H(0)\} / D'(0) \quad (38)$$

7 The Net Expected Profit Incurred in $(0,t)$

We are now in the position to obtain the net expected profit incurred in time interval $(0,t)$ by considering the characteristics obtained in earlier sections as follows-

$P(t) =$ Total revenue in time interval $(0,t)-$ Expected cost of repair during $(0,t)$.

$$= K_0 \mu_{up}(t) - K_1 \mu_b(t) \quad (39)$$

Where,
$K_0 =$ Revenue per-unit up time; $K_1 =$ Repair cost per-unit of time.

The expected profit per-unit time in steady state is given by-

$$P = \lim_{t\to\infty} \frac{P(t)}{t}$$
$$= K_0 \lim_{t\to\infty} \frac{\mu_{up}(t)}{t} - K_1 \lim_{t\to\infty} \frac{\mu_b(t)}{t}$$
$$= K_0 A(\infty) - K_1 B(\infty) \quad (40)$$

8 Case Study

The system model has wide applicability by considering deferent form of repair time p.d.f.'s of various sub-system, pressure unit and failed system due to common cause. As an illustration, we consider a case when repair times follow exponential distribution with parameters $\mu_i (i = 1, 2, \ldots, 6), \mu_h$ and μ_c i.e

$$g_i(t) = \mu_i e^{-\mu_i t},$$
$$g_{j-7}(t) = \mu_{j-7} e^{-\mu_{j-7} t},$$
$$g_{k-14}(t) = \mu_{k-14} e^{-\mu_{k-14} t},$$
$$g_{l-23}(t) = \mu_{l-23} e^{-\mu_{l-23} t},$$
$$g_h(t) = \mu_h e^{-\mu_h t},$$

$$g_c(t) = \mu_c e^{-\mu_c t} \tag{41-46}$$

In the expressions $N'(0), E'(0), H'(0)$ and $J'(0)$. We have the following changes

$$m_i = 1/\mu_i,$$

$$m_{j-7} = 1/\mu_{j-7},$$

$$m_{k-14} = 1/\mu_{k-14},$$

$$m_{l-23} = 1mu_{l-23},$$

$$m_h = 1mu_h,$$

$$m_c = 1/\mu_c \tag{47-52}$$

9 Graphical Representation

The curves for MTSF have been drawn for different values of parameters α_c, α_8 and α_9. **Fig. 3** depicts the variations in MTSF with respect to common cause failure rate (α_c) for three different values of failure rate $(\alpha_9 = 0.02, 0.03, 0.04)$ of pressure unit P_3 and two different values of the failure rate $(\alpha_8) = 0.03, 0.04$ of pressure unit P_2 when the values of other parameters are kept fix as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0.002$ and $\alpha_7 = 0.004$. It is clearly revealed that MTSF decreases uniformly as the value of α_c increase. It is also observed that the MTSF decreases with the increase in α_9 and increases with the increase in α_8.

Similarly, **Fig. 4** reveals the variations in profit (P) with respect to α_c for varying values of α_9 and μ_h, when the values of other parameters are kept fix as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0.002, \alpha_7 = 0.004, \alpha_8 = 0.03, \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0.3, K_0 = 600$ and $K_1 = 450$ .From the curves we observe that profit decreases uniformly as the values of α_c increase. It also reveals that the profit decreases with the increase in α_9 and increases with the increase in μ_h.

10 Conclusion

From the curves of MTSF, we conclude that to achieve at least a specific value of expected life of the system say 50 units, the common cause failure rate α_c should not exceed 0.075 and 0.01 respectively for $\alpha_8 = 0.03$ and 0.04 when $\alpha_9 = 0.02$. Similarly when $\alpha_9 = 0.03$ and 0.04 one can find the upper bonds of α_c for $\alpha_8 = 0.03$ and 0.04. It is also revealed from the curve that the variations in MTSF for different values of α_8 and α_9 tend to vanish for large values of α_c.

From the curves of Profit function, we conclude that to achieve at least specific value of profit say 60 units, the common cause failure rate α_c should be less than 0.0666, 0.0683 and 0.070 respectively for $\alpha_9 = 0.05, 0.06$ and 0.07 when $\mu_h = 0.7$. Similarly when μ_h =0.2 the upper bonds of α_c may be obtained for different values of α_9.

From smooth curves of **Fig. 4**, it is also concluded that the system is profitable only if failure rate (α_c) is less than 0.0625, 0.0675 and 0.0725 respectively for $\alpha_9 = 0.05, 0.06$ and 0.07 for fixed value of $\mu_h = 0.2$. From dotted curves, we conclude that the system is profitable only if α_c is less than 0.087, 0.088 and 0.089 respectively for $\alpha_9 = 0.05, 0.06$ and 0.07 for fixed value of $\mu_h = 0.7$.

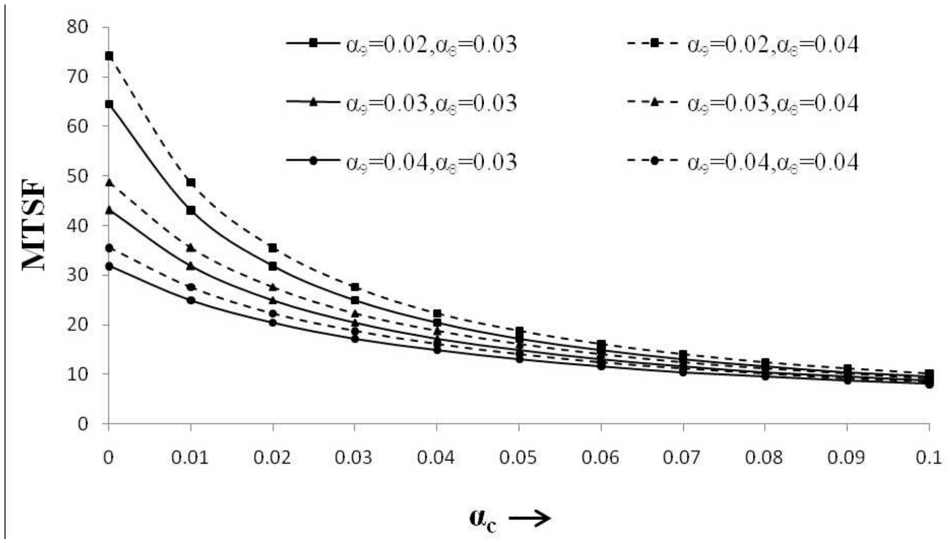

Fig. 3: Behaviour of MTSF with respect to α_c, α_8 and α_9

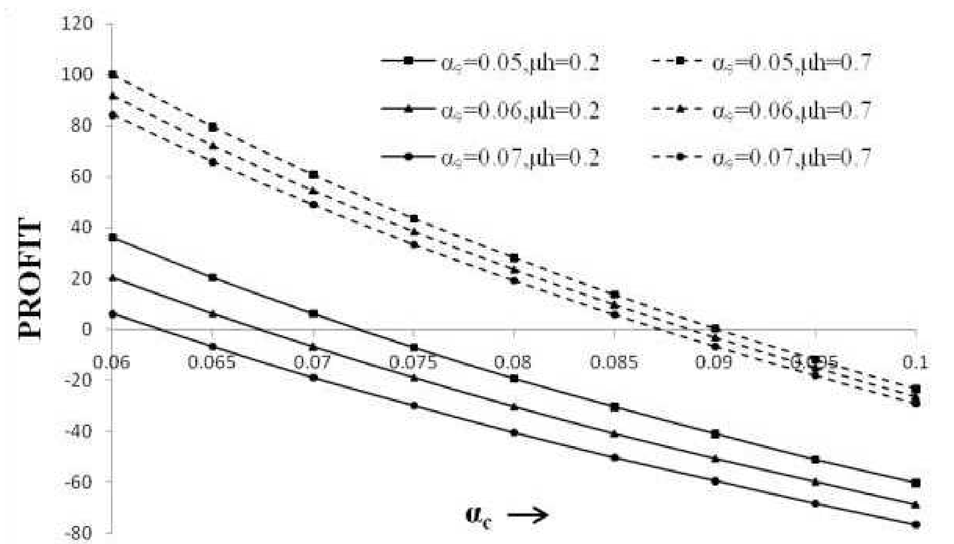

Fig. 4: Behaviour of Profit(P) with respect to α_c, μ_h and α_9

References

[1] K.K Damghani, A.R. Abtahi and M. Tavana, A new multi-objective particle swarm optimization method for solving reliability redundancy allocation problem, Reliability Engineers and system Safety, **111(3)**, 58-75 (2013).

[2] N. Dev, Shamsher and S.S. Kachhwahee, System modeling and analysis of a combine cycle power plant, Int. J. of system Assurance Engineering and Management, **4(4)**, 353-364 (2013).

[3] S.M. Famurewa, M. Asplund, D. Galar and U. Kumar, Implementation of performance based maintenance contracting in railway industries, Int. J. of System Assurance Engineering and Management, **4(3)**, 231-240 (2013).

[4] P. Gupta, A.K. Lal, R.K. Sharma and J. Singh, Behavior study of cement manufacturing plant a numerical approch, J. of Mathematics and System Sciences, **1(1)**, 50-70 (2005).

[5] P. Gupta, A.K. Lal, R.K. Sharma and J. Singh, Numerical analysis of reliability and availability of the serial processes in butter oil processing plant, Int. J. of Quality and Reliability Management, **22(3)**, 303-316 (2005).

[6] R. Gupta, P. Patayal and O.K. Belwal, Stochastic analysis of an Air Condition Cooling System Model, J. of Reliability and Statistical Studies, **4(1)**, 95-106 (2011).

[7] R. Gupta, G. Varshney and V. Sharma, Stochastic modeling and analysis of Milk Powder Making system in a dairy plant, Aligarh J. of Statistics, **30**, 1-13 (2010).
[8] S. Kaushik and I.P. Singh, Reliability analysis of feed water system in a thermal power plant, Microelectron. Reliab, **34(4)**, 757-759 (1994).
[9] S. Kaushik and I.P. Singh, Reliability analysis of the naphtha fuel oil in a thermal power plant, Microelectron. Reliab, **34(2)**, 362-372 (1994).
[10] M.R.R. Khan and A.B.M.Z. Kabir, Availability of a simulation modeling of ammonia plant, Reliability Engineering and system safety, **48**, 217-227 (1995).
[11] R. Khanduja, P.C.Tewari and D. Kumar, Steady-state behavior and maintenance planning of bleaching system in paper plant, Int. J. of Industrial Engg., **7(12)**, 39-44 (2012).
[12] A. Kumar, S.P.Sharma and D. Kumar, Performance analysis of reparable system using GA and fuzzy lambda- tau methodology, Int. J. of Quality and Reliability Management, **30(9)**, 1017-1032 (2013).
[13] D. Kumar, J. Singh and P.C. Pandey, Behavior analysis of paper production system with different repair policies, Microelectron. Reliab., **3(1)**, 47-51 (1991).
[14] D. Kumar, J. Singh and P.C. Pandey, Maintenance planning for the pulping system in paper industry, Reliability Engineering and System Safety, **25(4)**, 293-303 (1989).
[15] D. Kumar, J. Singh and P.C. Pandey, Cost analysis of a multi-component screening system in paper industry, Microelectron. Reliab., **30(3)**, 457-461 (1990).
[16] D. Kumar, J. Singh and P.C. Pandey, Availability of crystallization system in the sugar industry under common cause failur, IEEE Trans. Reliab., **41(1)**, 85-91 (1992).
[17] S. Kumar and P.C. Tiwari, Development of performance evaluating model for co-shift conversion system in the fertilizer plant, International J. of Engineering Research and Industrial Applications, **1(6)**, 369-382 (2008).
[18] S. Kumar and P.C. Tiwari, Mathematical modeling and performance optimization of CO_2 cooling system of a fertilizer plant, Int. J. of Industrial Engineering Computations, **2(3)**, 689-698 (2011).
[19] S. Kumar, D. Kumar and N.P. Mehta, Maintenance management for ammonia synthesis system in a Urea Fertilizer Plant, International Journal of Management and System, **15(3)**, 211-214 (1999).
[20] Y. Ma, J. Han and K.S. Trivendi, Composite performance and availability analysis of wireless communication networks, IEEE trans. On Vehicular Technology, **5(50)**, 1216-1223 (2001).
[21] M.S. Prabhuswamy, Studies in reliability base optimization of manufacturing systems, Ph.D. Thesis, I.I.T. Delhi (1997).
[22] A. Ramakrishna, and A.S. Bawa, Optimization of machine design criteria for higher reliability and maintainability in food processing industry, Proceeding of International conference on Reliability and Safety Engineering, I. I. T. Kharagpur, 151-157 (2005).
[23] A. Santosh, G. Vinod, O.P. Shrivastva, and R.K. Saraf, Reliability analysis of pipelines carrying H_2S for risk based inspection of heavy water plants, Reliability Engineering and System Safety, **91(2)**, 163-170 (2006).
[24] J. Singh and P. Goel, Availability analysis of heating system with warm standby and imperfect switch in sugar industry, Proceedings of National Conference on Operations Research in Modern Technology, 39-45 (1996).
[25] N. Vayenas and S. Peng, Reliability analysis of underground mining equipment using genetic algorithms: A case study of two mine hoists, J. of Quality in Maintenance Engineering, **31(1)**, 32-50 (2014).

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