Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/050213

Economic Design of $\overline{X}$ Control Chart for DEWMA under Non-Normal Population

J. R. Singh and Manzoor A. Khanday *

School of Studies in Statistics, Vikram University, Ujjain-456010, M.P. India.

Received: 19 Jan. 2016, Revised: 15 Apr. 2016, Accepted: 1 May 2016
Published online: 1 Jul. 2016

Abstract: In this paper, mathematical investigation has been made to study the effect of double exponentially weighted moving average (DEWMA) model on economic design of $\overline{X}$ control chart for non-normal population. Formulae are derived for calculating the value of n and h when the characteristics of an item posses DEWMA model. The sample mean of DEWMA model has been represented by first four terms of an Edgeworth series. A numerical example is given to verify the performance of DEWMA model in the presence of non-normality. The DEWMA charts working together with non-normality affects the control chart scheme when small to moderate shifts in the mean of the controlled parameter are expected. It is found that when shifts are uncertain the optimal design for DEWMA chart should be more conservative.

Keywords: Economic design Control Chart, DEWMA, Edgeworth Series, Non-Normality.

1 Introduction

Statistical Process Control (SPC) is defined as a set of statistical methods used widely to scrutinize and improve the quality and efficiency of industrial processes and service operations. As quality has become a crucial factor in global market competition, statistical process control (SPC) techniques are becoming significant in both manufacturing and service industries that aim at 6 excellence. With modern measurement and inspection technologies, It is common to collect large volumes of data from individual units usually on very short time intervals. Such nearly continuous measurement unavoidably results in data that tend to be non-normally distributed. However, most existing SPC techniques were not designed for such environments. It is known that conventional SPC techniques are affected by skewed data. Specifically, false alarm rates are so high that true alarms are often ignored. Since the primary purpose of SPC is to detect quickly unusual sources of variability so that their root cause can be properly addressed, data skewness has severe adverse impacts on the economic benefits of implementing SPC. For such purpose control chart is one of the most helpful techniques in SPC. The Shewhart $\overline{X}$ chart has been considered to be the best statistical tool in process surveillance. However, this chart has some limitations towards the detection of small and moderate process mean shifts. Therefore, a popular control chart used to detect and identify small shifts in a process mean is the EWMA given by Roberts [9]. The attempt to increase the sensitivity of EWMA control chart to detect small shifts and drift in a process, a double EWMA (DEWMA) control chart was developed by Shamma and Shamma [11]. Zhang [15] has conducted extensive studies on DEWMA control charts for the mean. Like most commonly used control charts, the traditional EWMA and DEWMA control charts for monitoring process means were developed under the assumption of normality. Further research works have been conducted to suggest more and more effective tools for statistically monitoring the quality of products and processes. Recently, many researchers have contributed to a wide variety of control charts to improve process monitoring, such as Saghaei et al. [10], Amiri et al. [4] and Lee et al. [8]. Simulation studies on the robustness of an EWMA control chart for process mean monitoring have been conducted by Borror et al. [5].

Economic design of control charts is used to determine various design parameters that minimize total economic costs. These charts are the fundamental statistical tool for evaluating process concert, supervision process ability, monitoring processes and getting better processes. The control chart is used to detect the occurrence of assignable cause.

* Corresponding author e-mail: manzoorstat@gmail.com

The design of a control chart, as is well known, has statistical and economic consequences; both are affected by the choice of the control chart or design parameters. These include the sample size, the control limits and the time interval between samples. A variety of costs is associated with control chart usage and is usually broken down into three main categories. These are the costs of allowing non-conforming units, cost of sampling and testing, and correcting the assignable causes. Economic design (ED) of a control chart involves finding the design parameters that minimise the total cost associated with maintaining the control of the process. Unfortunately, economic designs have weaknesses, as Woodall [13] has noted. More recently, Akhavan et al. [[1,2] presented the ESD of the VSI $\overline{X}$ control charts for correlated and non-normal data. They used the Burr distribution to model for the various non-normal situations and implemented Yang and Hancock's [14] model to take into custody the correlation structure. Furthermore, Akhavan et al. [3]studied the ESD of the VSI $\overline{X}$ control charts in the presence of the multiple assignable causes with non-normal population and correlated data. The effect of lot size of production on the quality of the product may also be significant. If it shifts to an out - of - control state at the beginning of the production run, the whole lot will contain more faulty items. Hence it is better to decrease the production cycle to decrease the fraction of defective items and, thus recover output quality. On the other hand, reduction of the production cycle may result in an increase in cost due to frequent setups. A balance must be maintained so that the total cost is minimized. It is assumed that that the cost of maintaining the equipment increases with the age, therefore, an age replacement strategy is needed to minimize the total cost of the system, which will simultaneously improve quality control and maintenance policy. Singh et al. [12] Studies the problem on Variables sampling plan for correlated data, Khanday and Singh [7] study the effect of Markoffs model on Economic design of $\overline{X}$ control charts under independent observations.

2 Duncan's model for the cost function:

Duncan [6] obtained an approximate function for the average net income per hour of using the control chart for mean of normal variables as:

$$I = V_0 - \frac{\eta MB + (\alpha T/h) + \eta W}{1+\eta B} - \frac{b+cn}{h}, \tag{1}$$

Duncans cost model indicates (i) the cost of an out-of control conditions (ii) the cost of false alarms, (iii) the cost of finding an assignable cause and (iv) the cost of sampling inspection, evolution, and plotting. The average cost per hour involved for maintaining the control chart is $\frac{(b+cn)}{h}$. The average net income per hour of the process under the surveillance of the control chart for mean can be rewritten as, $I = V_0 - L$ Where,

$$L = \frac{\eta MB + (\alpha T/h) + \eta W}{1+\eta B} + \frac{b+cn}{h}, \tag{2}$$

L can now be treated as the per hour cost due to the surveillance of the process under the control chart. The probability density function for non normal population is represented by the first four terms of Edgewoth series, $P^{'}$ and $\alpha^{'}$ are determined from the sampling distribution of mean and are written as.

$$P^{'} = 1 - \Phi(\xi) + \frac{\lambda_3}{6\sqrt{n}}\phi^{(2)}(\xi) - \frac{\lambda_4}{24n}\phi^{(3)}(\xi) - \frac{\lambda_3^2}{72n}\phi^{(5)}(\xi), \tag{3}$$

$$\alpha^{'} = \alpha_N - \alpha_C \tag{4}$$

Where $\xi = (k - \delta\sqrt{n})$, $\alpha_c = \frac{3\lambda_4\phi^{(3)}(k)+\lambda_3^2\phi^{(5)}(k)}{36n}$ is the non-normality correction for α

3 Derivation for optimum value of sample size n and sampling interval h:

One can determine the optimum value of sample size n and sampling interval h either by maximizing the gain function I or by minimizing the cost function with respect to n and h, and we get,

$$\frac{\partial L}{\partial n} = \frac{(1+\eta B)(\eta M\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha^{'}}{\partial n}) - (\eta MB + \frac{\alpha^{'}T}{h}) + \eta W)\eta\frac{\partial B}{\partial n}}{(1+\eta B)^2} + \frac{c}{h} \tag{5}$$

$$\frac{\partial L}{\partial h}=\frac{(1+\eta B)(\eta M\frac{\partial B}{\partial h}-\frac{\alpha' T}{h^2})-(\eta MB+\frac{\alpha' T}{h}+\eta W)\eta\frac{\partial B}{\partial n}}{(1+\eta B)^2}-\frac{b+cn}{h^2} \quad (6)$$

Where,

$$\frac{\partial B}{\partial n}=-\frac{h}{P'^2}\frac{\partial P'}{\partial n}+c, \frac{\partial B}{\partial h}=\frac{1}{P'}-\frac{1}{2}+\frac{\eta h}{6} \quad and \quad \frac{\partial \alpha'}{\partial n}=0-\frac{\partial \alpha_c}{\partial n} \quad (7)$$

$\frac{\partial \alpha'}{\partial n}=\frac{3\lambda_4\phi^{(3)}(k)+\lambda_3^2\phi^{(5)}(k)}{36n^2}$

$$\frac{\partial P'}{\partial n}=\frac{\delta}{2\sqrt{n}}\phi(\xi)+\frac{1}{144n^2}(-12\lambda_3[\delta n\phi^3(\xi)-\sqrt{n}\phi^2(\xi)]+3\lambda_4[\delta\sqrt{n}\phi^4(\xi)+2\phi^3(\xi)]+\lambda_3^2[\delta\sqrt{n}\phi^6(\xi))+2\phi^5(\xi)]) \quad (8)$$

The solutions of the equations (5) and (6) for n and h yield the required optimum values, and The equations (5) and (6) can be rewritten as follows:

$$\eta h(M-\eta MB-\frac{\alpha' T}{h}-\eta W)\frac{\partial B}{\partial n}+\frac{T}{h}\frac{\partial \alpha'}{\partial n}+\eta B(\eta M\frac{\partial B}{\partial n}+\frac{T}{h}\frac{\partial \alpha'}{\partial n})+c(1+\eta B)^2=0 \quad (9)$$

$$\eta h^2(M-\eta MB-\frac{\alpha' T}{h}-\eta W)\frac{\partial B}{\partial h}-\alpha' T(1+\eta B)+\eta^2 h^2 MB\frac{\partial B}{\partial n}-(b+cn)(1+\eta B)^2=0 \quad (10)$$

By assuming η to be small and noting that the optimum h is roughly of order of $\frac{1}{\sqrt{\eta}}$, we neglect terms containing ηWc , and $\frac{\alpha' T}{h}$ and the terms equating higher powers of η . The equations (9) and (10) are simplified and put in the following form

$$-\frac{\eta h^2 M}{P'^2}\frac{\partial P'}{\partial n}-\eta\alpha' T+\frac{T}{h}.\frac{\alpha_c}{n}+c=0, \quad (11)$$

$$\eta Mh^2(\frac{1}{P'}-\frac{1}{2})-(\alpha' T+b+cn)=0 \quad (12)$$

From the equation (12) we get

$$h=[\frac{\alpha' T+b+cn}{\eta M(\frac{1}{P'}-\frac{1}{2})}]^{\frac{1}{2}} \quad (13)$$

By eliminating h from the equation (11), we get,

$$-\frac{\alpha' T+b+cn}{P'^2(\frac{1}{P'}-\frac{1}{2})}\frac{\partial P'}{\partial n}-\eta\alpha' T+\frac{T\alpha_c}{n}[\frac{\eta M(\frac{1}{P'}-\frac{1}{2})}{\alpha' T+b+cn}]^{\frac{1}{2}}+c=0 \quad (14)$$

The values of n for which the equation (14) satisfy yield us the required optimum value of sample size n. Substituting this value of n in equation (13), we find the optimum value of the sampling interval h.

4 Derivation of the optimum values of sample size n and sampling interval h under DEWMA:

Suppose that a process is on target μ initially and successive measurements $\overline{X}_t$, (t = 1,2,3,) are taken it may be average of several measurements taken at time t to check whether there is a shift from the target. To use a control chart based on the statistic

$$Y_t=\lambda X_t+(1-\lambda)Y_t-1 \quad and \quad Z_t=\lambda Y_t+(1-\lambda)Z_t-1 \quad (15)$$

Such that $0<\lambda<1 \quad and \quad Y_0=Z_0=\mu_0$

$$Z_t=\lambda^2\sum_{j=1}^{t}(t-j+1)(1-\lambda)^{t-j}X_j+t\lambda(1-\lambda)^t Y_0+(1-\lambda)^t Z_0 \quad (16)$$

$$where \quad E(Z_t) = \mu_0 \tag{17}$$

$$\sigma_{Zt}^2 = \lambda^4 \left(\frac{1+(1-\lambda)^2(1-\lambda)^{2t} + (2t^2+2t-1)(1-\lambda)^{2t+2} - t^2(1-\lambda)^{2t+4}}{(1-(1-\lambda)^2)^3} \right) \sigma_0 \tag{18}$$

The control limits for DEWMA control chart are

$$UCL = \mu_0 + L\sigma\sqrt{\sigma_{Zt}^2}, \quad CL = \mu_0, \quad and \quad LCL = \mu_0 - L\sigma\sqrt{\sigma_{Zt}^2} \tag{19}$$

Where L is as defined. For large values of L, the control limits become

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2}{(2-\lambda)^3}}, CL = \mu_0 \quad and \quad LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda(2-2\lambda+\lambda^2}{(2-\lambda)^3}} \tag{20}$$

Assuming that X_t is drawn independently from a normal distribution with variance σ^2 so that t is sufficiently large. One of the disturbing thing here is that λ is quite arbitrary and lies between 0 and 1. Suppose that a machine whose performance can be effectively represented by a single unknown quality μ is inspected regularly to see whether the quality of performance is deteriorated. The successive performance level $\mu_1, \mu_2, \mu_3, ..., \mu_t$ are tracked by the observations x1 , x2, ..., xt The operation continues until a decision is made to overhaul it in which case the level is set to zero instantaneously and the whole sequence begins again. This resetting after overhaul may be subject to error and so it is assumed that μ_0 is $N(0, \frac{\sigma^2}{n})$ and each subsequent state of repair is drawn independently from this distribution. Thus we get

$$E(Z_t) = \mu_0 \quad and \quad V(Z_t) = \frac{\sigma^2}{n} \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3} = \frac{\sigma^2}{n} g^2, \quad where \quad g^2 = \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3} \tag{21}$$

The corresponding measures of skewness and kurtosis for DEWMA are found out to be λ_3g and $\lambda_4 g^2$ respectively. So for the DEWMA model, the probability density function for non-normal population represented by the first four terms of an Edgeworth series is

$$P_e' = 1 - \Phi(\xi_e) + \frac{g\lambda_3}{6\sqrt{n}}\phi^{(2)}(\xi_e) - \frac{g^2\lambda_4}{24n}\phi^{(3)}(\xi_e) - \frac{g^2\lambda_3^2}{72n}\phi^{(5)}(\xi_e), \tag{22}$$

$$\alpha_e' = \alpha_{Ne} - \alpha_{ce}, \xi_e = \frac{k - \delta\sqrt{n}}{g} \tag{23}$$

$\alpha_{Ne} = 2\Phi(\frac{-k}{g})$, $\alpha_{Ce} = \frac{3\lambda_4 g^2 \phi^{(3)}(k) + g^2\lambda_3^2\phi^{(5)}(k)}{36n}$

For DEWMA model, the equation (5) and (6) will reduce in following form

$$\frac{\partial L}{\partial n} = \frac{(1+\eta B)(\eta M\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha_e'}{\partial n}) - (\eta MB + \frac{\alpha_e' T}{h} + \eta W)\eta\frac{\partial B}{\partial n}}{(1+\eta B)^2} \tag{24}$$

$$\frac{\partial L}{\partial h} = \frac{(1+\eta B)(\eta M\frac{\partial B}{\partial h} - \frac{\alpha_e' T}{h^2}) - (\eta MB + \frac{\alpha_e' T}{h} + \eta W)\eta\frac{\partial B}{\partial h}}{(1+\eta B)^2} - \frac{b+cn}{h^2} \tag{25}$$

$where \quad \frac{\partial B}{\partial n} = -\frac{h}{P_e'^2}\frac{\partial P_e'}{\partial n} + c, \frac{\partial B}{\partial h} = \frac{1}{P_e'} - \frac{1}{2} + \frac{\eta h}{6} \quad and \quad \frac{\partial \alpha_e'}{\partial n} = 0 - \frac{\partial \alpha_{ce}}{\partial n},$
$\frac{\partial \alpha'}{\partial n} = \frac{3\lambda_4\phi^{(3)}(k) + \lambda_3^2\phi^{(5)}(k)}{36n^2}$

$$\frac{\partial P_e'}{\partial n} = \frac{\delta}{2\sqrt{ng}}\phi(\xi_e) + \frac{1}{144n^2}(-12g\lambda_3[\frac{\delta}{g}n\phi^3(\xi_e) - \sqrt{2}\phi^2(\xi_e)] + 3g^2\lambda_4[\frac{\delta}{g}\sqrt{n}\phi^4(\xi_e)) + 2\phi^3(\xi_e)] + g^2\lambda_3^2[\frac{\delta}{g^2}\sqrt{n}\phi^6(\xi_e)) + 2\phi^5(\xi_e)]) + c \tag{26}$$

By solving the equation (24) and (25) we get

$$h = [\frac{\alpha_e^{'}T + b + cn}{\eta M(\frac{1}{P_e^{'}} - \frac{1}{2})}]^{\frac{1}{2}} \tag{27}$$

$$-\frac{\alpha_e^{'}T + b + cn}{P_e^{'2}(\frac{1}{P_e^{'}} - \frac{1}{2})}\frac{\partial P_e^{'}}{\partial n} - \eta\alpha_e^{'}T + \frac{T\alpha_c}{n}[\frac{\eta M(\frac{1}{P_e^{'}} - \frac{1}{2})}{\alpha_e^{'}T + b + cn}]^{\frac{1}{2}} + c = 0 \tag{28}$$

The values of n for which the equation (28) satisfy yield us the required optimum value of sample size n. Substituting this value n in equation (27), we find the optimum value of the sampling interval h under non normality for DEWMA model.

5 Numerical illustration and Conclusion:

In order to illustrate the result we take k=2.0, 2.5, 3.0, δ=1.0, 1.5, 2.0, λ_3= -0.5, 0.0, 0.5, λ_4 = -0.5, 0.0, 1.0, 2.0, η =0.01, M=100, W=25, T=50, C=0.05, D=2, b=0.5, c=0.1 and λ =1, 0.8, 0.6 and 0.2 to determine the optimum values of sample size and sampling interval. The values of n and h are presented in Tables 1 to 4., it is clearly seen that for given k and the sample size n and sampling interval h decrease with the increase in the values of , on the other hand the sample size n decreases with the decrease of k, while the sampling interval h increases with decreasing k. When the rate of occurrences of assignable cause is fixed, the value of sample size and sampling interval are different for different values of (shifts). The effect of non-normality is more serious for DEWMA model for different parameters. Results show that in general, the in-control ARL performances of DEWMA control charts were more robust. The degree of robustness of DEWMA control chart to non-normality increases for smaller values of smoothing parameter. Thus the performance of DEWMA is conservative and is recommended as it performs better results than that of EWMA model for smaller shifts. The DEWMA chart working together with non-normality affects the control chart scheme when small shifts to moderate shifts in the mean of the controlled parameter are expected. Thus we conclude that the DEWMA model is more serious under non-normality when the shifts are certain and large. For economic point of view DEWMA chart performs better when there are spoiled data thus we recommend DEWMA model under non-normality.

Table 1: Values of optimal sample size n and sampling interval h under DEWMA for $\lambda = 1$.

λ=	1	k= 3						k= 2.5						k= 2					
	λ_3→	-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
δ	λ_4↓	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h
1.0	-0.5	23	2.3279	23	2.3337	23	2.3423	20	2.4076	20	2.4127	19	2.4157	19	3.0305	19	3.0251	19	3.0166
	0.0	23	2.3307	23	2.3371	23	2.3457	20	2.4089	20	2.4134	19	2.4170	19	3.0315	19	3.0258	19	3.0172
	1.0	23	2.3375	23	2.3439	23	2.3525	20	2.4109	20	2.4154	19	2.4190	19	3.0328	19	3.0266	19	3.0177
	2.0	23	2.3442	23	2.3500	23	2.3587	20	2.4123	20	2.4175	19	2.4205	19	3.0341	19	3.0275	19	3.0182
1.5	-0.5	11	1.7898	11	1.7912	11	1.7980	9	1.9888	9	1.9916	9	1.9894	9	2.7106	9	2.7044	9	2.6935
	0.0	11	1.8015	11	1.8026	11	1.8092	10	1.9936	9	1.9963	9	1.9938	10	2.7127	9	2.7064	9	2.6947
	1.0	11	1.8240	11	1.8249	11	1.8304	10	2.0031	9	2.0052	9	2.0024	10	2.7175	9	2.7104	9	2.6977
	2.0	11	1.8462	11	1.8465	11	1.8515	10	2.0128	10	2.0146	9	2.0109	10	2.7223	9	2.7146	9	2.7008
2.0	-0.5	6	1.5439	6	1.5412	6	1.5477	6	1.8030	6	1.8063	5	1.7996	6	2.5758	6	2.5707	5	2.5582
	0.0	6	1.5677	6	1.5648	6	1.5703	6	1.8130	6	1.8157	5	1.8085	6	2.5796	6	2.5740	5	2.5606
	1.0	7	1.6131	7	1.6091	6	1.6126	6	1.8330	6	1.8346	6	1.8263	6	2.5875	6	2.5812	5	2.5660
	2.0	7	1.6556	7	1.6507	7	1.6526	6	1.8534	6	1.8537	6	1.8440	6	2.5957	6	2.5889	5	2.5724

6 lxicon of parameters and variables:

V_0= the rate per hour at which income accrues from operation of the process is in control and process average is μ

V_1=the rate per hour at which income accrues from operation of the process when process is not in control and process average is $\mu^{'} = \mu + \delta\sigma$,

Table 2: Values of optimal sample size n and sampling interval h under DEWMA for $\lambda = 0.8$.

λ=	0.8	k= 3						k= 2.5						k= 2					
	λ_3→	-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
δ	λ_4↓	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h
1.0	-0.5	19	2.1177	19	2.1193	19	2.1222	15	1.9219	15	1.9253	14	1.9277	11	1.8521	11	1.8555	11	1.8572
	0.0	19	2.1201	19	2.1217	19	2.1244	15	1.9231	15	1.9265	14	1.9291	11	1.8520	11	1.8554	11	1.8572
	1.0	19	2.1247	19	2.1260	19	2.1290	15	1.9253	15	1.9289	14	1.9313	11	1.8513	11	1.8548	11	1.8565
	2.0	19	2.1293	19	2.1306	19	2.1332	15	1.9271	15	1.9312	14	1.9335	11	1.8503	11	1.8542	11	1.8559
1.5	-0.5	9	1.6130	9	1.6126	9	1.6145	7	1.5069	7	1.5103	7	1.5093	5	1.5412	5	1.5440	5	1.5415
	0.0	9	1.6206	9	1.6199	9	1.6217	7	1.5113	7	1.5147	7	1.5134	5	1.5420	5	1.5448	5	1.5421
	1.0	9	1.6354	9	1.6345	9	1.6361	7	1.5200	7	1.5231	7	1.5216	5	1.5437	5	1.5463	5	1.5436
	2.0	9	1.6499	9	1.6486	9	1.6499	7	1.5286	7	1.5314	7	1.5299	5	1.5455	5	1.5479	5	1.5449
2.0	-0.5	5	1.3847	5	1.3819	5	1.3841	4	1.3207	4	1.3253	4	1.3206	3	1.4056	3	1.4093	3	1.4033
	0.0	5	1.4001	5	1.3972	5	1.3990	4	1.3300	4	1.3342	4	1.3292	3	1.4080	3	1.4116	3	1.4052
	1.0	5	1.4301	5	1.4267	5	1.4275	4	1.3483	4	1.3518	4	1.3463	3	1.4132	3	1.4162	3	1.4092
	2.0	5	1.4586	5	1.4549	5	1.4549	4	1.3663	4	1.3691	4	1.3630	3	1.4185	3	1.4211	3	1.4133

Table 3: Values of optimal sample size n and sampling interval h under DEWMA for $\lambda = 0.6$.

λ=	0.6	k= 3						k= 2.5						k= 2					
	λ_3→	-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
δ	λ_4↓	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h
1.0	-0.5	16	2.0228	16	2.0225	16	2.0227	12	1.8246	12	1.8256	12	1.8256	9	1.6393	9	1.6410	9	1.6410
	0.0	16	2.0242	16	2.0240	16	2.0243	12	1.8254	12	1.8264	12	1.8264	9	1.6395	9	1.6409	9	1.6411
	1.0	16	2.0273	16	2.0270	16	2.0272	12	1.8270	12	1.8280	12	1.8279	9	1.6395	9	1.6411	9	1.6412
	2.0	16	2.0299	16	2.0297	16	2.0300	12	1.8287	12	1.8297	12	1.8294	9	1.6395	9	1.6411	9	1.6412
1.5	-0.5	7	1.5532	7	1.5520	7	1.5518	6	1.4406	6	1.4416	6	1.4395	4	1.3413	4	1.3430	4	1.3404
	0.0	8	1.5577	7	1.5565	7	1.5562	6	1.4435	6	1.4445	6	1.4422	4	1.3421	4	1.3437	4	1.3411
	1.0	8	1.5668	7	1.5653	7	1.5650	6	1.4491	6	1.4501	6	1.4476	4	1.3437	4	1.3453	4	1.3425
	2.0	8	1.5757	8	1.5742	7	1.5736	6	1.4547	6	1.4556	6	1.4530	4	1.3453	4	1.3468	4	1.3440
2.0	-0.5	4	1.3445	4	1.3422	4	1.3421	3	1.2717	3	1.2737	3	1.2694	3	1.2123	3	1.2151	2	1.2100
	0.0	4	1.3537	4	1.3514	4	1.3511	3	1.2775	3	1.2794	3	1.2749	3	1.2143	3	1.2170	2	1.2117
	1.0	4	1.3719	4	1.3694	4	1.3687	3	1.2891	3	1.2906	3	1.2859	3	1.2183	3	1.2208	2	1.2152
	2.0	4	1.3896	4	1.3869	4	1.3859	3	1.3005	3	1.3018	3	1.2968	3	1.2224	3	1.2246	2	1.2186

Table 4: Values of optimal sample size n and sampling interval h under DEWMA for $\lambda = 0.2$.

λ=	0.2	k= 3						k= 2.5						k= 2					
	λ_3→	-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
δ	λ_4↓	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h
1.0	-0.5	13	1.8648	13	1.8642	13	1.8633	9	1.6780	9	1.6777	9	1.6767	7	1.5031	7	1.5028	6	1.5016
	0.0	13	1.8653	13	1.8646	13	1.8638	9	1.6784	9	1.6780	9	1.6770	7	1.5032	7	1.5029	6	1.5018
	1.0	13	1.8662	13	1.8655	13	1.8647	9	1.6790	9	1.6786	9	1.6775	7	1.5034	7	1.5031	6	1.5019
	2.0	13	1.8670	13	1.8664	13	1.8655	9	1.6795	9	1.6792	9	1.6781	7	1.5036	7	1.5033	6	1.5021
1.5	-0.5	6	1.4557	6	1.4549	6	1.4540	4	1.3510	4	1.3508	4	1.3492	3	1.2564	3	1.2564	3	1.2545
	0.0	6	1.4570	6	1.4562	6	1.4553	4	1.3519	4	1.3517	4	1.3501	3	1.2568	3	1.2567	3	1.2549
	1.0	6	1.4595	6	1.4587	6	1.4578	4	1.3536	4	1.3534	4	1.3518	3	1.2574	3	1.2574	3	1.2555
	2.0	6	1.4621	6	1.4613	6	1.4603	4	1.3553	4	1.3551	4	1.3534	3	1.2581	3	1.2581	3	1.2561
2.0	-0.5	3	1.2794	3	1.2784	3	1.2776	2	1.2129	2	1.2131	2	1.2109	2	1.1546	2	1.1550	2	1.1525
	0.0	3	1.2820	3	1.2810	3	1.2801	2	1.2146	2	1.2147	2	1.2126	2	1.1553	2	1.1557	2	1.1531
	1.0	3	1.2871	3	1.2861	3	1.2852	2	1.2180	2	1.2181	2	1.2159	2	1.1566	2	1.1570	2	1.1544
	2.0	3	1.2922	3	1.2911	3	1.2902	2	1.2213	2	1.2214	2	1.2192	2	1.1580	2	1.1583	2	1.1556

M= V_0 - V_0

η = the average number of times the assignable cause occur within an interval of time, starting in a state of control at time t=0, the probability that the process will still be in control at time t_1 is $\exp^{\eta t_1}$,

B=ah+Cn+D

$\alpha = (\frac{1}{P} - \frac{1}{2} + \frac{\eta h}{12})$

h= the interval between samples measured in hours,

e = the rate at which the time taken between the sample and plotting of a point on the $\overline{X}$ chart increases with the sample size n.

Cn= the time required to take and inspect a sample of size ,

D= average time taken to find the assignable cause after a point plotted on the chart falls outside the control limits,

P= Probability of detecting an assignable cause when it exists,

$= \int_{\propto}^{\mu+\frac{k\sigma}{\sqrt{n}}} g(\frac{\overline{x}}{\mu'} g(\frac{\overline{x}}{\mu'})d\overline{x} + \int_{\mu-\frac{k\sigma}{\sqrt{n}}}^{\propto} g(\frac{\overline{x}}{\mu'} g(\frac{\overline{x}}{\mu'})d\overline{x}$

=1 - $\Phi(k-\delta\sqrt{n})$

Where $g(\frac{\overline{x}}{\mu'})$ is the density function of $\overline{x}$ when the true mean μ and $\Phi(x)$ is the normal probability.

α= probability of wrongly indicating the presence of assignable cause.

$=\int_{\mu-\frac{k\sigma}{\sqrt{n}}}^{\mu+\frac{k\sigma}{\sqrt{n}}} g(\frac{\overline{x}}{\mu'})d\overline{x}$ $=2\Phi(-k)$

T=The cost per occasions of looking for an assignable cause when no assignable cause exists,

W= the average cost per occasion of finding the assignable cause when it exist,

b=per sample cost of sampling and plotting, that is independent of sample size,

and c = the cost per unit of measuring an item in a sample.

Acknowledgements

Authors are very much thankful to the reviewer and the editor of the journal for their timely suggestions in preparations of this paper.

References

[1] Akhavan Niaki, S. T., F. Gazaneh, and M. Toosheghanian (2013a). Economic Design of Variable Sampling Interval X-Bar Control Charts for Monitoring Correlated Non Normal Samples. Communications in Statistics Theory and Methods 42 (18): 3339 to 3358.

[2] Akhavan Niaki, S. T., F. Gazaneh, and M. Toosheghanian. (2013b). A Parameter-tuned Genetic Algorithm for Economic-statistical Design of Variable Sampling Interval X-Bar Control Charts for Non-normal Correlated Samples. Communications in Statistics Simulation and Computation 43 (5): 1212 to1240.

[3] Akhavan Niaki, S. T., F. Gazaneh, and M. Toosheghanian. (2013c). Economic Design of VSI X Control Chart with Correlated Nonnormal Data under Multiple Assignable Causes. Journal of Statistical Computation and Simulation 83 (7): 1279 to 1300.

[4] Amiri, A., Moslemi, A., and Doroudyan, M. H. (2015). Robust economic and economic statistical design of EWMA control chart. International Journal of Advanced Manufacturing Technology, 78(1 to 4)

[5] Borror, C. M., Montgomery, D. C, and Runger, G. C. (1999). Robustness of the EWMA Control Chart to Non-Normality, Journal of Quality Technology, 31, 309 to 316.

[6] Duncan, A. J. (1956): The economic design of X chart used to maintain current control of a process. Journal of the American Statistical Association, 51, 228 to 242.

[7] Khanday, M. A. and Singh, J. R. (2015). Effect of Markoffs model on Economic design of X- bar control chart for independent observations. International Journal of Scientific Research in Mathematical and Statistical Sciences Vol. 2 (2), 1 to 6.

[8] Lee, S. H., Park, J. H., and Jun, C. H. (2014). An exponentially weighted moving average chart controlling false discovery rate. Journal of Statistical Computation and Simulation, 84(8), 1830 to 1840.

[9] Roberts, S. W. (1959). Control charts tests based on geometric moving average. Technimetrics, 1(3), 239 to 250.

[10] Saghaei, A., Ghomi, S. M. T. F., and Jaberi, S. (2014). Economic design of exponentially weighted moving average control chart based on measurement error using genetic algorithm. Quality and Reliability Engineering International, 30(8), 1153 to1163.

[11] Shamma, S. E., and Shamma, A. K. (1992). Development and evaluation of control charts using double exponentially weighted moving averages. International journal of Quality and Reliability Management, 9(6), 18 to 25.

[12] Singh J. R., Sankle R. and M. Ahmad Khanday (2013). Variables Sampling Plan For Correlated Data. Vol. 12 (2), 184 to190.

[13] Woodall, W. H. (1986). Weaknesses of the Economic Design of Control Charts. Technometrics 28: 408 to 409.

[14] Yang, K. A. I., and W. M. Hancock (1990). Statistical Quality Control for Correlated Samples. International Journal of Production Research 28 (3): 595 to 608.

[15] Zhang, L. Y. (2002). EWMA control charts and extended EWMA control charts. Unpublished doctoral dissertation. University of Regina, Saskatchewan, Canada.

Jais Raj Singh Professor and Head, School of Studies in Statistics, Vikram University Ujjain, (M. P.) India - 456010

Manzoor Ahmad Khanday Ph. D. Research Scholar at School of Studies in Statistics, Vikram University Ujjain, (M. P.) India 456010.