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Profit Evaluation of a Repairable System with Three-Stage Deterioration

M. Y. Haggag[*]

Department of Mathematics, Faculty of Science, Al-Azhar University, Egypt.

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Abstract: This paper deals with the cost analysis of a repairable system subject to deterioration. The system has three different modes: normal, deterioration and failure. The deterioration can be minor, medium or major. Failure time follows an exponential distribution, while repair time rate and preventive maintenance follow a general distribution. We analyzed the system using supplementary variable technique and developed explicit expressions for availability, busy period and profit function for the system. Laplace transforms of the probabilities of the states of the system have been obtained. Availability, steady-state availability, mean time to system failure (MTSF), and the profit function of the system are studied. Tables and graphs have also been given at the end. Based on assumed numerical values given to the system parameters, some particular cases have also been discussed graphically to see the effect of deterioration, failure and repair rates on profit. The results have indicated that deterioration and failure rates decrease the profit while repair and major maintenance rates increase the profit.

Keywords: Deterioration, repairable system, maintenance, supplementary variable technique.

1 Introduction

Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. Many researchers have studied reliability problem of different systems see Satyavati [6]. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order.

System availability represents the percentage of time the system is available to users. As the age of equipment increases, the equipment slowly deteriorates correspondingly. Deterioration failure is still the inevitable fate of the equipment. In many manufacturing situation, the condition of the system has significant impact on the quantity and quality of the unit produced. Most of these systems are subjected to random deterioration which can results in unexpected failures and disastrous effect on safety and the economy it is therefore important to find a way to slow down the deterioration rate, and to prolong equipment's service life. Maintenance policies are vital in the analysis of deterioration and deteriorating systems as they help in improving reliability and availability of the systems. Maintenance models assume perfect repair, see, among others, [11], minimal repair and imperfect repair which is between perfect and minimal repair.

Large volumes of literature exist on the issue relating to deterioration and prediction of availability of various systems under different maintenance policies. Yusuf and other[10] have studied a stochastic modeling of two unit parallel system under two types of failures, where the system works in normal mode, deterioration (slow, mild, or fast) in model I and normal and failure modes in model II. Yusuf and others [12] dealt with modeling the reliability and availability characteristics of a system with three Stages of deterioration. Marcous and others [4] dealt with the Modeling bridge deterioration, Wirahadikusumah and others [8] have studied Challenging issues in modeling deterioration of combined sewers.

Cost analysis of redundant system working in normal and failure modes are numerous. Various models are developed concerning the cost analysis of a redundant system. Mokaddis and other have studied the cost analysis of two dissimilar

[*]Corresponding author e-mail: mohyas62@yahoo.com

unit cold standby redundant systems subject to inspection and random change of units. Using semi Markov process technique various measures of system effectiveness are obtained. Recently, El-said [1] has studied cost analysis of a system with preventive maintenance. Haggag [2] has studied Cost analysis of a system involving common cause failures and preventive maintenance. Haggag [3] dealt with Cost analysis of K-out-of- n repairable system with dependent failure and standby support. Yusuf1 and others [12] have studied Profit Evaluation of a Repairable System with Three-Stage Deterioration.

The problem considered in this paper is more general than the work of Yusuf and others [12]. This paper is to investigate the system discussed in [12] incorporating the concept that repair and preventive maintenance rates are generally distributed. We analyze the system using supplementary variable technique. Similarly to paper [12], the system has three modes: normal, deterioration and failure. Deterioration stages could be minor, medium or major. Minor, medium or major are control by major maintenance whereas the failure is control by perfect repair.

Our objectives are to develop the explicit expressions for availability, busy period due to failure, busy period due to major maintenance, and profit function. Laplace transforms of the probabilities of the different states and availability have been obtained. Availability, busy period, profit function of the system have also been obtained. MTSF, steady-state availability steady-state busy period and steady-state profit due to failure and repair have been derived. Numerical computations for evaluating availability, steady-state availability, steady-state busy period and steady-state profit of the system are appended and capture the effect of failure rates, deterioration rates, and maintenance rates, perfect repair rates on profit based on assumed numerical values given to the system parameters. Tables of availability vs. time have been illustrated. Graphs of availability vs. time and profit vs. time have also been sketched. A special case for studying the effect of preventive maintenance on the MTSF and steady-state availability and profit are shown theoretically and graphically. The state transition diagram of the system is shown in Figure (1).

2 Notations

$\beta_{12}, \beta_{13}, \beta_{14}$: Minor, medium and major deterioration rates respectively from S_1 to S_2, S_3 and S_4

β_{23}, β_{34}: Deterioration rates from S_2 to S_3 and S_3 to S_4 respectively

$\beta_{15}, \beta_{25}, \beta_{35}, \beta_{45}$: Failure rates

$\alpha_{51}(y)$: General repair rate

$\alpha_{21}(x), \alpha_{31}(x), \alpha_{41}(x)$: General maintenance rates

$A(\infty), B_F(\infty), B_M(\infty), PF$: System availability, busy period due to failure, busy period due to maintenance, profit function.

$P_1(t)$: Probability that the system is in operable state S_1 at time t.

$P_i(t)$: Probability that the system is in state S_i at time t, i = 2, 3, 4. 5.

$P_i(x,t)$: Probability that the system is in state S_i at time t, and under repair, elapsed repair time is x.

$P_i(y,t)$: Probability that the system is in state S_i at time t, and under Preventive Maintenance, elapsed repair time is y.

According to Davis formula, there exists a relation between repair rates $\alpha_{21}(x), \alpha_{31}(x), \alpha_{41}(x), \alpha_{51}(x)$ and their corresponding cumulative functions $F(x), C(x), H(x), V(x)$, i.e.

$$F(x) = \alpha_{21}(x) exp\left(-\int_0^x \alpha_{21}(x)dx\right)dx, \mathrm{G}(x) = \alpha_{31}(x) exp\left(-\int_0^x \alpha_{31}(x)dx\right)dx,$$

$$\mathrm{H}(x) = \alpha_{41}(x) exp\left(-\int_0^x \alpha_{41}(x)dx\right)dx, \mathrm{V}(y) = \alpha_{51}(y) exp\left(-\int_0^y \alpha_{51}(y)dy\right)dy$$

3 Assumptions and Description of the System

1. State of the system can be: Perfect (S_1), minor deterioration (S_2), medium deterioration (S_3), major (S_4) or failed state S_5

2. At any given time *t* the system is either in the operating state, deteriorating states or in the failed state.

3. State S_5 can be access from the previous state

4. The state of the system changes as time progresses

5. System/units work in S_1, S_2, S_3 and S_4

6. The deteriorate stages can be minor, medium or major

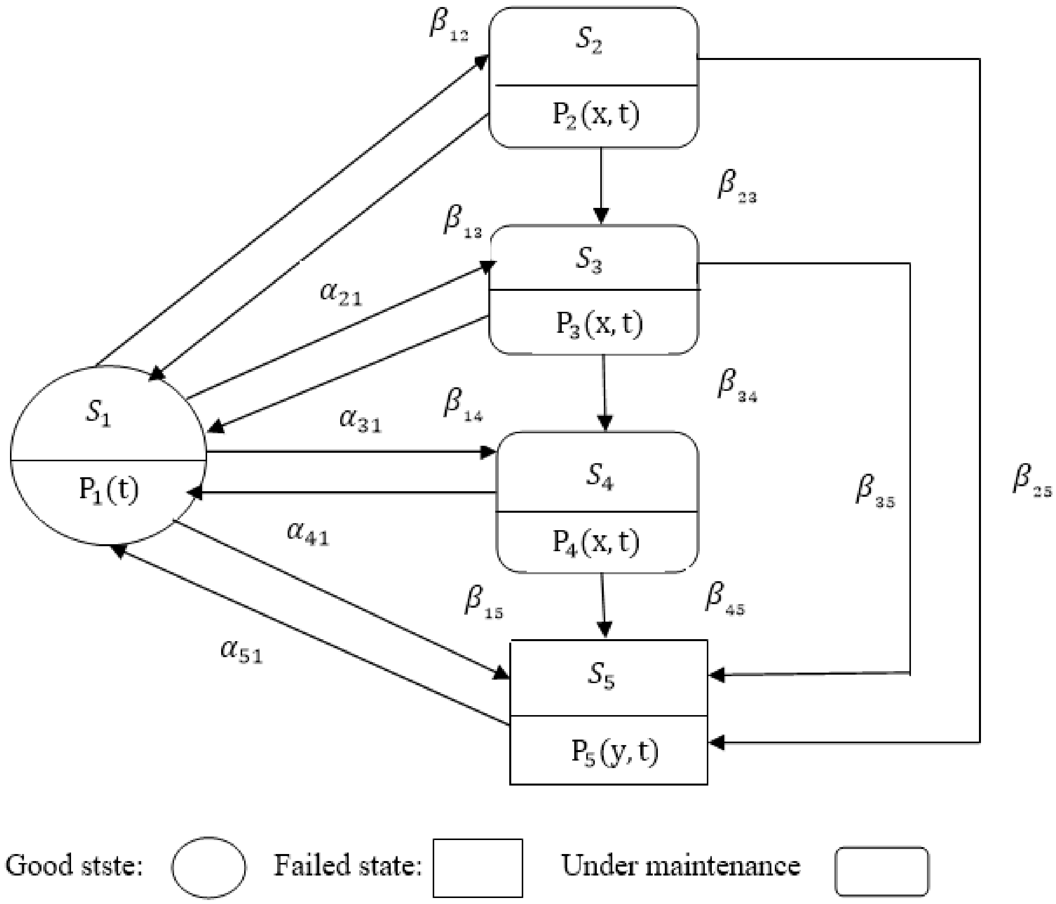

Figure 1: Transition graph of the system.

4 Formulation of Mathematical Model

Let $P_i(t)$ be the probability that the system is in state i at time t. By elementary probability and continuity arguments the difference differential equations for the stochastic process of the system shown in Figure(1), which is continuous in time and discrete in space are shown in Equations (4 – 1) - (4 - 9):

$$\left(\frac{\partial}{\partial t} + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}\right) P_1(t) = \int_0^\infty \alpha_{21}(x)P_2(x,t)\,dx + \int_0^\infty \alpha_{31}(x)P_3(x,t)\,dx + \int_0^\infty \alpha_{41}(x)P_4(x,t)\,dx + \int_0^\infty \alpha_{51}(y)P_5(y,t)dy \quad (4 - 1)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_{23} + \beta_{25} + \alpha_{21}(x)\right) P_2(x,t) = 0 \quad (4 - 2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_{34} + \beta_{35} + \alpha_{31}(x)\right) P_3(x,t) = 0 \quad (4 - 3)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_{45} + \alpha_{41}(x)\right) P_4(x,t) = 0 \quad (4 - 4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \alpha_{51}(y)\right) P_5(y,t) = 0 \quad (4 - 5)$$

Boundary and Initial conditions

$$P_2(0,t) = \beta_{12}P_1(t) \quad (4 - 6)$$

$$P_3(0,t) = \beta_{13}P_1(t) + \beta_{23}P_2(t) \quad (4 - 7)$$

$$P_4(0,t) = \beta_{14}P_1(t) + \beta_{34}P_3(t) \quad (4-8)$$

$$P_5(0,t) = \beta_{15}P_1(t) + \beta_{25}P_2(t) + \beta_{35}P_3(t) + \beta_{45}P_4(t) \quad (4-9)$$

5 Solution of the Model

By taking Laplace transform of equations (4-1)-(4-5), and using initial condition (4-6)-(4-9), we get:

$$(s + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{15})P_1(s) = 1 + \int_0^\infty \alpha_{21}(x)\bar{P}_2(x,s)\,dx + \int_0^\infty \alpha_{31}(x)\bar{P}_3(x,s)\,dx + \int_0^\infty \alpha_{41}(x)\bar{P}_4(x,s)\,dx + \int_0^\infty \alpha_{51}(y)\bar{P}_5(y,s)dy \quad (5\text{-}1)$$

$$\left(\frac{\partial}{\partial x} + s + \beta_{23} + \beta_{25} + \alpha_{21}(x)\right)\bar{P}_2(x,s) = 0 \quad (5-2)$$

$$\left(\frac{\partial}{\partial x} + s + \beta_{34} + \beta_{35} + \alpha_{31}(x)\right)\bar{P}_3(x,s) = 0 \quad (5-3)$$

$$\left(\frac{\partial}{\partial x} + s + \beta_{45} + \alpha_{41}(x)\right)\bar{P}_4(x,s) = 0 \quad (5-4)$$

$$\left(\frac{\partial}{\partial y} + s + \alpha_{51}(y)\right)\bar{P}_5(y,t) = 0 \quad (5-5)$$

Boundary and Initial conditions

$$\bar{P}_2(0,s) = \beta_{12}\bar{P}_1(s) \quad (5-6)$$

$$\bar{P}_3(0,t) = \beta_{13}\bar{P}_1(s) + \beta_{23}\bar{P}_2(t) \quad (5-7)$$

$$\bar{P}_4(0,t) = \beta_{14}\bar{P}_1(s) + \beta_{34}\bar{P}_3(t) \quad (5-8)$$

$$\bar{P}_5(0,t) = \beta_{15}\bar{P}_1(s) + \beta_{25}\bar{P}_2(s) + \beta_{35}\bar{P}_3(s) + \beta_{45}\bar{P}_4(s) \quad (5-9)$$

Integrating equations (5 – 2)-(5 – 5)

$$\overline{P_2}(x,s) = \overline{P_2}(0,s)\exp[-(s + \beta_{23} + \beta_{25})x - \int_0^x \alpha_{21}(x)dx \quad (5-10)$$

$$\overline{P_3}(x,s) = \overline{P_3}(0,s)\exp[-(s + \beta_{34} + \beta_{35})x - \int_0^x \alpha_{31}(x)dx] \quad (5-11)$$

$$\bar{P}_4(x,s) = \bar{P}_4(0,s)\exp[-(s + \beta_{45})x - \int_0^x \alpha_{41}(x)dx] \quad (5-12)$$

$$\overline{P_5}(y,s) = \overline{P_5}(0,s)\exp[-sy - \int_0^y \alpha_{51}(y)dy] \quad (5-13)$$

Again integrating by parts equations (5 – 10)-(5 - 13) using equations (5 – 6)-(5 –9)

$$\overline{P_2}(s) = \int_0^\infty \overline{P_2}(x,s)\,dx = \overline{P_2}(0,s)\left\{\int_0^\infty \exp\left[-(s + \beta_{23} + \beta_{25})x - \int_0^x \alpha_{21}(x)dx\right]dx\right\}$$

$$= \overline{P_2}(0,s)\left\{\int_0^\infty \left[exp\left(-\int_0^x \alpha_{21}(x)dx\right)\right] d\left(\frac{\exp[-(s + \beta_{23} + \beta_{25})x}{-(s + \beta_{23} + \beta_{25})}\right)\right\}$$

$$= (s + \beta_{23} + \beta_{25})^{-1}[1 - \bar{F}(s + \beta_{23} + \beta_{25})]\overline{P_2}(0,s)$$

$$\therefore \overline{P_2}(s) = A_2(s)\bar{P}_1(s) \quad (5-14)$$

In the same manner we can get the other integrations:

$$\overline{P_3}(s) = A_3(s)\bar{P}_1(s) \quad (5-15)$$

$$\overline{P_4}(s) = A_4(s)\bar{P}_1(s) \quad (5-16)$$

$$\overline{P_5}(s) = A_5(s)\bar{P}_1(s) \quad (5-17)$$

Where:

$$A_2(s) = \beta_{12}N_2(s),$$

$$A_3(s) = \beta_{13}N_3(s) + \beta_{12}\beta_{23}N_2(s)N_3(s)$$

$$A_4(s) = \beta_{14}N_4(s) + \beta_{34}A_3(s)$$

$$A_5(s) = \beta_{15}N_5(s) + \beta_{12}\beta_{25}N_2(s) + \beta_{35}A_3(s) + \beta_{45}A_4(s)$$

$$N_2(s) = (s + \beta_{23} + \beta_{25})^{-1}[1 - \bar{F}(s + \beta_{23} + \beta_{25})]$$

$$N_3(s) = (s + \beta_{34} + \beta_{35})^{-1}[1 - \bar{G}(s + \beta_{34} + \beta_{35})]$$

$$N_4(s) = (s + \beta_{45})^{-1}[1 - \bar{H}(s + \beta_{45})]$$

$$N_5(s) = s^{-1}[1 - \bar{V}(s)]$$

And:

$$\bar{F}(s) = \int_0^\infty \exp(-sx)\,\alpha_{21}(x)exp\left(-\int_0^x \alpha_{21}(x)dx\right)dx, \bar{G}(s) = \int_0^\infty \exp(-sx)\,\alpha_{31}(x)exp\left(-\int_0^x \alpha_{31}(x)dx\right)dx,$$

$$\bar{H}(s) = \int_0^\infty \exp(-sx)\,\alpha_{41}(x)exp\left(-\int_0^x \alpha_{41}(x)dx\right)dx, \bar{V}(s) = \int_0^\infty \exp(-sy)\,\alpha_{51}(y)exp\left(-\int_0^y \alpha_{51}(y)dy\right)dy$$

Also we have from equations (5 – 10)-(5 – 13) using equations (5 – 6)-(5 –9)

$$\int_0^\infty \overline{P_2}(x,s)\,\alpha_{21}(x)dx = \beta_{12}\bar{\mathrm{F}}(s + \beta_{23} + \beta_{25})\bar{P}_1(s) \qquad (5-18)$$

$$\int_0^\infty \overline{P_3}(x,s)\,\alpha_{31}(x)dx = (\beta_{13} + \beta_{23}A_2(s))\bar{\mathrm{G}}(s + \beta_{34} + \beta_{35})\bar{P}_1(s) \qquad (5-19)$$

$$\int_0^\infty \overline{P_4}(x,s)\,\alpha_{41}(x)dx = \left(\beta_{14} + \beta_{34}A_3(s)\right)\bar{\mathrm{H}}(s + \beta_{45})\bar{P}_1(s) \qquad (5-20)$$

$$\int_0^\infty \overline{P_5}(y,s)\alpha_{51}(y)\,dx = (\beta_{15} + \beta_{25}A_2(s) + \beta_{35}A_3(s) + \beta_{45}A_4(s))\bar{\mathrm{V}}(s)\bar{P}_1(s) \qquad (5-21)$$

Now from equations (5- 18), (5- 19), (5- 20), (5- 21) in equation (5 – 1), we get:

$$\bar{P}_1(s) = \left[(s + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) - \beta_{12}\bar{\mathrm{F}}(s + \beta_{23} + \beta_{25}) - \left(\beta_{13} + \beta_{23}A_2(s)\right)\bar{\mathrm{G}}(s + \beta_{34} + \beta_{35}) - \left(\beta_{14} + \beta_{34}A_3(s)\right)\bar{\mathrm{H}}(s + \beta_{45}) - (\beta_{15} + \beta_{25}A_2(s) + \beta_{35}A_3(s) + \beta_{45}A_4(s))\bar{\mathrm{V}}(s)\right]^{-1} = \frac{1}{A(s)} \qquad (5-22)$$

Where

$$A(s) = \left[(s + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) - \beta_{12}\bar{\mathrm{F}}(s + \beta_{23} + \beta_{25}) - \left(\beta_{13} + \beta_{23}A_2(s)\right)\bar{\mathrm{G}}(s + \beta_{34} + \beta_{35}) - \left(\beta_{14} + \beta_{34}A_3(s)\right)\bar{\mathrm{H}}(s + \beta_{45}) - (\beta_{15} + \beta_{25}A_2(s) + \beta_{35}A_3(s) + \beta_{45}A_4(s))\bar{\mathrm{V}}(s)\right]$$

6 Evaluation of Laplace Transform Of Up And Down State Availability

The Laplace transform of the probability that the system is in up (operable) and down (failed) state at time 't' can be evaluated as follows:

$$\bar{P}_{up}(s) = \overline{P_1}(s) + \overline{P_2}(s) + \overline{P_3}(s) + \bar{P}_4(s)$$

$$= [1 + A_2(s) + A_3(s) + A_4(s)]\bar{P}_1(s) \qquad (6-1)$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \qquad (6-2)$$

Steady state behavior of the system

By using Abel's Lemma, viz

$\lim_{s\to 0}[s.\bar{F}(s)] = \lim_{t\to\infty} F(t) = F\ (say)$, in Equations (5 – 22), (5 – 14) – (5 – 17), we get

$$P_1 = \lim_{s\to 0} s.\bar{P}_1(s) = \left[1 - \beta_{12}\bar{\mathrm{F}}'(\beta_{23} + \beta_{25}) - \left(\beta_{23}A_2{}'(0)\right)\bar{\mathrm{G}}(\beta_{34} + \beta_{35}) - \left(\beta_{13} + \beta_{23}A_2{}'(0)\right)\bar{\mathrm{G}}'(\beta_{34} + \beta_{35}) - \left(\beta_{34}A_3{}'(0)\right)\bar{\mathrm{H}}(\beta_{45}) - \left(\beta_{14} + \beta_{34}A_3{}'(0)\right)\bar{\mathrm{H}}'(\beta_{45}) - (\beta_{25}A_2{}'(0) + \beta_{35}A_3{}'(0) + \beta_{45}A_4{}'(0))\bar{\mathrm{V}}(0) - (\beta_{15} + \beta_{25}A_2(0) + \beta_{35}A_3(0) + \beta_{45}A_4(0))\bar{\mathrm{V}}'(0)\right]^{-1}$$

$$P_2 = A_2(0)P_1,\ P_3 = A_3(0)P_1,\ P_4 = A_4(0)P_1, P_5 = A_5(0)P_1 \qquad (6-3)$$

Steady state availability of the system

By using Abel's Lemma in Equation (6 – 1), we get

$$P_{up}=\lim_{s\to 0}s.\bar{P}_{up}(s)=\lim_{s\to 0}s.[1+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)]\bar{P}_1(s)=[1+A_1(0)+A_2(0)+A_3(0)+A_4(0)]P_1$$
(6 – 4)

Where:

$$A_2(0)=\beta_{12}N_2(0),$$
$$A_2{}'(0)=\beta_{12}N_2{}'(0)$$
$$A_3(0)=\beta_{13}N_3(0)+\beta_{12}\beta_{23}N_2(0)N_3(0)$$
$$A_3{}'(0)=\beta_{13}N_3{}'(0)+\beta_{12}\beta_{13}N_2{}'(0)N_3(0)+\beta_{12}\beta_{13}N_2(0)N_3(0)'$$
$$A_4(0)=\beta_{14}N_4(0)+\beta_{34}A_3(0)$$
$$A'_4(0)=\beta_{14}N_4{}'(0)+\beta_{34}A'_3(0)$$
$$N_2(0)=(\beta_{23}+\beta_{25})^{-1}[1-\bar{F}(\beta_{23}+\beta_{25})]$$
$$N_2{}'(0)=-(\beta_{23}+\beta_{25})^{-2}[1-\bar{F}(\beta_{23}+\beta_{25})]-(\beta_{23}+\beta_{25})^{-1}[\bar{F}'(\beta_{23}+\beta_{25})]$$
$$N_3(0)=(\beta_{34}+\beta_{35})^{-1}[1-\bar{G}(\beta_{34}+\beta_{35})]$$
$$N_3{}'(s)=-(\beta_{34}+\beta_{35})^{-2}[1-\bar{G}(\beta_{34}+\beta_{35})]-(\beta_{34}+\beta_{35})^{-1}[\bar{G}(\beta_{34}+\beta_{35})]$$
$$N_4(0)=(\beta_{45})^{-1}[1-\bar{H}(\beta_{45})]$$
$$N_4(s)=-(\beta_{45})^{-2}[1-\bar{H}(\beta_{45})-\beta_{45}\bar{H}'(\beta_{45})]$$

Mean Time to System Failure

$$MTSF=\lim_{s\to 0}\bar{P}_{up}(s)=[1+A_2(0)+A_3(0)+A_4(0)]\bar{P}_1(s)/A(0) \qquad (6-5)$$

Where

$$A(0)=\big[(\beta_{12}+\beta_{13}+\beta_{14}+\beta_{15})-\beta_{12}\bar{F}(\beta_{23}+\beta_{25})-\big(\beta_{13}+\beta_{23}A_2(0)\big)\bar{G}(\beta_{34}+\beta_{35})-\big(\beta_{14}+\beta_{34}A_3(0)\big)\bar{H}(\beta_{45})-(\beta_{15}+\beta_{25}A_2(0)+\beta_{35}A_3(0)+\beta_{45}A_4(0))\bar{V}(0)\big]$$

7 Particular Case

In this section the Laplace transform of up and down state availability, the mean time to system failure **(**MTSF), the steady state availability of the system and the expected total profit incurred to the system in the steady-state have been evaluated using supplementary variable technique with and without maintenance, when repair times follow exponential distribution.

Setting: $\bar{F}(s)=\frac{\alpha_{21}}{s+\alpha_{21}}, \bar{G}(s)=\frac{\alpha_{31}}{s+\alpha_{31}}, \ , \bar{H}(s)=\frac{\alpha_{41}}{s+\alpha_{41}}$ and $\bar{V}(s)=\frac{\alpha_{51}}{s+\alpha_{51}}$ in Equations (5-14) to (5-17), (5 - 22) one gets:

$$\bar{P}_1(s)=(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})(s+\alpha_{21}+\beta_{23}+\beta_{25})(s+\alpha_{31}+\beta_{34}+\beta_{35})/D(s) \qquad (7-1)$$

$$\bar{P}_2(s)=\beta_{12}(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})(s+\alpha_{31}+\beta_{34}+\beta_{35})/D(s) \qquad (7-2)$$

$$\bar{P}_3(s)=\beta_{12}\beta_{23}(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})+\beta_{13}(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})(s+\alpha_{21}+\beta_{23}+\beta_{25})/D(s) \qquad (7-3)$$

$$\bar{P}_4(s)=\beta_{12}\beta_{23}\beta_{34}(s+\alpha_{51})+\beta_{13}\beta_{34}(s+\alpha_{51})(s+\alpha_{21}+\beta_{23}+\beta_{25})+\beta_{14}(s+\alpha_{51})(s+\alpha_{21}+\beta_{23}+\beta_{25})(s+\alpha_{31}+\beta_{34}+\beta_{35})/D(s) \qquad (7-4)$$

$$\bar{P}_5(s)=\{\beta_{12}\beta_{23}\beta_{35}(s+\alpha_{41}+\beta_{45})-\beta_{12}\beta_{23}\beta_{34}\beta_{45}+\beta_{13}\beta_{34}\beta_{45}(s+\alpha_{21}+\beta_{23}+\beta_{25})+\beta_{13}\beta_{35}(s+\alpha_{41}+\beta_{45})(s+\alpha_{21}+\beta_{23}+\beta_{25})+\beta_{12}\beta_{25}(s+\alpha_{41}+\beta_{45})(s+\alpha_{31}+\beta_{34}+\beta_{35})+\beta_{14}\beta_{45}(s+\alpha_{21}+\beta_{23}+\beta_{25})(s+\alpha_{31}+\beta_{34}+\beta_{35})+\beta_{15}(s+\alpha_{41}+\beta_{45})(s+\alpha_{21}+\beta_{23}+\beta_{25})(s+\alpha_{31}+\beta_{34}+\beta_{35})\}/D(s) \qquad (7-5)$$

From Equations (7 - 1), (7 - 2), (7 - 3), (7 - 4), one may get:

$$\bar{P}_{up}(s)=\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)=(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})(s+\alpha_{21}+\beta_{23}+\beta_{25})(s+\alpha_{31}+\beta_{34}+\beta_{35})+\beta_{12}(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})(s+\alpha_{31}+\beta_{34}+\beta_{35})+\beta_{12}\beta_{23}(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})+\beta_{13}(s+\alpha_{51})(s+\alpha_{41}+\beta_{45})(s+\alpha_{21}+\beta_{23}+\beta_{25})+\beta_{12}\beta_{23}\beta_{34}(s+\alpha_{51})+\beta_{13}\beta_{34}(s+\alpha_{51})(s+\alpha_{21}+\beta_{23}+\beta_{25})+\beta_{14}(s+\alpha_{51})(s+\alpha_{21}+$$

$$\beta_{23} + \beta_{25})(s + \alpha_{31} + \beta_{34} + \beta_{35})/D(s) \qquad (7-6)$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s)$$

Where:

$$D(s) = (s + \alpha_{51})(s + \alpha_{41} + \beta_{45})(s + \alpha_{21} + \beta_{23} + \beta_{25})(s + \alpha_{31} + \beta_{34} + \beta_{35})(s + \beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) - \beta_{12}\alpha_{41}\beta_{23}\beta_{34}(s + \alpha_{51}) - \beta_{12}\alpha_{51}\beta_{23}\beta_{35}(s + \alpha_{41} + \beta_{45}) - \alpha_{31}\beta_{12}\beta_{23}(s + \alpha_{51})(s + \alpha_{41} + \beta_{45}) - \beta_{13}\alpha_{51}\beta_{34}\beta_{45}(s + \alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{41}\beta_{13}\beta_{34}(s + \alpha_{51})(s + \alpha_{21} + \beta_{23} + \beta_{25}) - \beta_{13}\alpha_{51}\beta_{35}(s + \alpha_{41} + \beta_{45})(s + \alpha_{21} + \beta_{23} + \beta_{25}) - \beta_{12}\alpha_{51}\beta_{25}(s + \alpha_{41} + \beta_{45})(s + \alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{31}\beta_{13}(s + \alpha_{51})(s + \alpha_{41} + \beta_{45})(s + \alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{21}\beta_{12}(s + \alpha_{51})(s + \alpha_{41} + \beta_{45})(s + \alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{14}\beta_{45}(s + \alpha_{21} + \beta_{23} + \beta_{25})(s + \alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{41}\beta_{14}(s + \alpha_{51})(s + \alpha_{21} + \beta_{23} + \beta_{25})(s + \alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{15}(s + \alpha_{41} + \beta_{45})(s + \alpha_{21} + \beta_{23} + \beta_{25})(s + \alpha_{31} + \beta_{34} + \beta_{35}) - \beta_{12}\alpha_{51}\beta_{23}\beta_{34}\beta_{45}$$

7.1 Busy Period Analysis

7.1.1 Busy Period of the system due to maintenance

The initial conditions for this problem are the same as for the reliability case. Then the probability that the system is in states $S_2 - S_4$ is given by:

$$\bar{P}_{BM}(s) = \frac{1}{s} - (\bar{P}_1(s) + \bar{P}_5(s)) = \bar{P}_2(s) + \bar{P}_3(s) + \bar{P}_4(s) = \beta_{12}(s + \alpha_{51})(s + \alpha_{41} + \beta_{45})(s + \alpha_{31} + \beta_{34} + \beta_{35}) + (s + \alpha_{51})(s + \alpha_{41} + \beta_{45})(\beta_{13}(s + \alpha_{21} + \beta_{23} + \beta_{25}) + \beta_{12}\beta_{23}) + (s + \alpha_{51})(s + \alpha_{21} + \beta_{23} + \beta_{25})(\beta_{14}(s + \alpha_{31} + \beta_{34} + \beta_{35}) + \beta_{13}\beta_{34}) + \beta_{12}\beta_{23}\beta_{34}(s + \alpha_{51})/D(s) \qquad (7-7)$$

7.1.2 Busy Period of the system due to failure

Then the probability that the system is in states S_5 is given by:

$$\bar{P}_{BF}(s) = \bar{P}_5(s) \qquad (7-8)$$

Mean Time to System Failure

$$MTSF = ((\beta_{12}\alpha_{51}\beta_{23}\beta_{34} + \beta_{13}\alpha_{51}\beta_{34}(\alpha_{21} + \beta_{23} + \beta_{25}) + \beta_{13}\alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25}) + \alpha_{51}\beta_{14}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) + \alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) + \beta_{12}{}^2\alpha_{51}{}^2\beta_{23}(\alpha_{41} + \beta_{45})^2(\alpha_{31} + \beta_{34} + \beta_{35}))/(\alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35})(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) - \beta_{12}\alpha_{51}\beta_{23}\beta_{34}\beta_{45} - \alpha_{31}\beta_{12}\alpha_{51}\beta_{23}(\alpha_{41} + \beta_{45}) - \beta_{12}\alpha_{51}\beta_{23}\beta_{35}(\alpha_{41} + \beta_{45}) - \alpha_{41}\beta_{13}\alpha_{51}\beta_{34}(\alpha_{21} + \beta_{23} + \beta_{25}) - \beta_{13}\alpha_{51}\beta_{34}\beta_{45}(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{31}\beta_{13}\alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{21}\beta_{12}\alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{31} + \beta_{34} + \beta_{35}) - \beta_{13}\alpha_{51}\beta_{35}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25}) - \beta_{12}\alpha_{51}\beta_{25}(\alpha_{41} + \beta_{45})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{41}\alpha_{51}\beta_{14}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{14}\beta_{45}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{15}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) - \beta_{12}\alpha_{41}\alpha_{51}\beta_{23}\beta_{34})) \qquad (7-9)$$

The steady state availability of the system is given by:

$$A(\infty) = ((\beta_{12}\alpha_{51}\beta_{23}\beta_{34} + \beta_{13}\alpha_{51}\beta_{34}(\alpha_{21} + \beta_{23} + \beta_{25}) + \beta_{13}\alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25}) + \alpha_{51}\beta_{14}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) + \alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) + \beta_{12}{}^2\alpha_{51}{}^2\beta_{23}(\alpha_{41} + \beta_{45})^2(\alpha_{31} + \beta_{34} + \beta_{35}))/D \qquad (7-10)$$

Where

$$D = \alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) - \beta_{12}\alpha_{41}\beta_{23}\beta_{34} - \alpha_{41}\beta_{13}\alpha_{51}\beta_{34} - \beta_{12}\alpha_{51}\beta_{23}\beta_{35} - \beta_{13}\alpha_{51}\beta_{34}\beta_{45} - \alpha_{21}\beta_{12}\alpha_{51}(\alpha_{41} + \beta_{45}) - \alpha_{31}\beta_{12}\beta_{23}(\alpha_{41} + \beta_{45}) - \alpha_{31}\beta_{13}\alpha_{51}(\alpha_{41} + \beta_{45}) - \beta_{12}\alpha_{51}\beta_{25}(\alpha_{41} + \beta_{45}) - \beta_{13}\alpha_{51}\beta_{35}(\alpha_{41} + \beta_{45}) - \alpha_{31}\beta_{13}\alpha_{51}(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{21}\beta_{12}\alpha_{51}(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{41}\alpha_{51}\beta_{14}(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{41}\beta_{13}\beta_{34}(\alpha_{21} + \beta_{23} + \beta_{25}) - \beta_{13}\alpha_{51}\beta_{35}(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{41}\alpha_{51}\beta_{14}(\alpha_{31} + \beta_{34} + \beta_{35}) - \beta_{12}\alpha_{51}\beta_{25}(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{14}\beta_{45}(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{51}\beta_{14}\beta_{45}(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{31}\beta_{13}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{21}\beta_{12}(\alpha_{41} + \beta_{45})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{15}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25}) - \alpha_{51}\beta_{15}(\alpha_{41} + \beta_{45})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{41}\beta_{14}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{51}\beta_{15}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35}) - \alpha_{31}\beta_{12}\alpha_{51}\beta_{23} + \alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) + \alpha_{51}(\alpha_{41} + \beta_{45})(\alpha_{31} + \beta_{34} + \beta_{35})(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) + \alpha_{51}(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35})(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15}) + (\alpha_{41} + \beta_{45})(\alpha_{21} + \beta_{23} + \beta_{25})(\alpha_{31} + \beta_{34} + \beta_{35})(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15})$$

The steady state busy period due to maintenance of the system is given by:

$$B_M(\infty) = P_2 + P_3 + P_4 = \beta_{12}\alpha_{51}\beta_{23}\beta_{34} + \alpha_{51}(\alpha_{41}+\beta_{45})(\beta_{12}\beta_{23} + \beta_{13}(\alpha_{21}+\beta_{23}+\beta_{25}) + \beta_{12}(\alpha_{31}+\beta_{34}+\beta_{35}) + \beta_{12}(\alpha_{31}+\beta_{34}+\beta_{35})) + \alpha_{51}(\alpha_{21}+\beta_{23}+\beta_{25})(\beta_{13}\beta_{34} + \beta_{14}(\alpha_{31}+\beta_{34}+\beta_{35}))/D \quad (7-11)$$

The steady state busy period due to failure of the system is given by:

$$B_F(\infty) = (P_5) = \beta_{12}\beta_{23}\beta_{34}\beta_{45} + \beta_{12}\beta_{23}\beta_{35}(\alpha_{41}+\beta_{45}) + \beta_{13}\beta_{34}\beta_{45}(\alpha_{21}+\beta_{23}+\beta_{25}) + (\alpha_{21}+\beta_{23}+\beta_{25})(\alpha_{31}+\beta_{34}+\beta_{35})(\beta_{14}\beta_{45}+\beta_{15}(\alpha_{41}+\beta_{45})) + (\alpha_{41}+\beta_{45})(\beta_{13}\beta_{35}(\alpha_{21}+\beta_{23}+\beta_{25}) + \beta_{12}\beta_{25}(\alpha_{31}+\beta_{34}+\beta_{35}))/D$$

(7 – 12)

The expected total profit incurred to the system in the steady-state is given by:

$$\mathrm{PF}(\infty) = R.A(\infty) - \mathrm{C}_1.B_M(\infty) - \mathrm{C}_2.B_F(\infty) \quad (7-13)$$

8 Numerical Simulations

In this section, we numerically obtained the results for variation of availability of the system with respect to time, expected total gain in the interval (0, t] for different values of 't'. Also we computed mean time to system failure (MTSF), steady-state availability of the system, steady-state busy period due to maintenance, steady-state busy period due to failure and the steady-state profit of the system with respect to failure rates to show the effect of failure rates on the profit.

8.1 Availability analysis

For the model analysis the following set of parameters values are fixed throughout the simulations for consistency. If we put: $\beta_{12} = .04, \beta_{13} = .03, \beta_{14} = .02, \beta_{15} = .01, \beta_{23} = .05, \beta_{25} = .025, \beta_{34} = .045, \beta_{35} = .035, \beta_{45} = .055, \alpha_{21} = .4, \alpha_{31} = .3, \alpha_{41} = .2, \alpha_{51} = .1$ in Equation (7 - 6),

We get:

$$\bar{P}_{up}(s) = (s+.8)(s+.2+.055)(s+.4+.05+.025)(s+.3+.045+.035) + .04(s+.8)(s+.2+.055)(s+.3+.045+.035) + (s+.8)(s+.055+.2)((s+.05+.025+.4).03+.04.05) + (s+.8)((s+.05+.025+.4)(s+.045+.035+.3).02 + (s+.05+.025+.4).03.045+.04.05.045)/(s^5+2.01s^4+1.4405\,s^3+0.43788s^2+4.7652\times 10^{-2}s+1.5407\times 10^{-31}) \quad (8-1)$$

By taking inverse Laplace transform of Equation (8 – 2) one may get the probability that the system is in up- state at time’t’.

$$P_{up}(t) = 0.98026exp(-3.2332\times 10^{-30}t) + 4.1845\times 10^{-3}exp(-0.51458t) + 5.4996\times 10^{-3}exp(-0.28024t) + 5.5035\times 10^{-3}exp(-0.80436t) + 4.5490\times 10^{-3}exp(-0.41081t) \quad (8-2)$$

Setting $t = 0, 1, 2, \dots\dots\dots$, in Equation (8 – 2), one can compute Table 1. Variation of availability w.r.t. time is shown in Figure 2.

Table (1): Variation of the availability with respect to time

Time (t)	Availability
0	1.0
1	0.99240
2	0.98800
3	0.98535
4	0.98369
5	0.98262
6	0.98191
7	0.98142
8	0.98109
9	0.98086
10	0.98069
20	0.98028
100	0.98026
1000	0.98026

Figure (2): Variation of the availability with respect to time.

8.2 Cost analysis

The system is subjected to perfect repair, minimal repair, minor and major maintenance respectively. The repairman performed maintenance to the system in states 2, 3, 4 and repair in state 5. Following El-said [1] and Haggag [2], the expected total gain of the system in the interval (0, t) is given by:

Profit = total revenue - total cost

Profit = total revenue generated – cost incurred due to maintenance – cost incurred due to failure

$$PF(t) = R\mu_{up}(t) - C_1.\mu_{BM}(t) - C_2.\mu_{BF}(t) \qquad (8-3)$$

Where:

$PF(t)$: The expected total gain in interval (0, t).

R : The revenue per unit of time of the system.

$C_1,\ C_2$: The service costs per unit of time for maintenance and repair respectively.

$\mu_{up}(t)$: The mean up time in interval (0, t).

$\mu_{BM}(t)$: The expected busy period for maintenance.

$\mu_F(t)$: The expected busy period for repair from failed state.

The mean up time of the system is given by:

From Equation (7 – 6) , one may get:

$$\mu_{up}(t) = \int_0^t \mathrm{P_{up}(t)dt} = -3.0319\times10^{29}(\exp(-3.2332\times10^{-30}\mathrm{t})-1) - 8.1319\times10^{-3}(\exp(-0.51458\mathrm{t})-1) - 1.9625\times10^{-2}(\exp(-0.28024\mathrm{t})-1) - 6.8421\times10^{-3}(\exp(-0.80436\mathrm{t})-1) - 1.1073\times10^{-2}(\exp(-0.41081\mathrm{t})-1) \qquad (8-4)$$

The busy period of the system due to maintenance is given by:

Taking the inverse of Laplace transform of Equation (7 - 7), one may get the probability that the system is in states $(S_2 - S_4)$ at time 't'

$$P_{BM}(t) = 0.20752\exp(-3.2332\times10^{-30}\mathrm{t}) - 0.10465\exp(-0.51458\mathrm{t}) - 5.1215\times10^{-2}\exp(-0.28024\mathrm{t}) - 1.4596\times10^{-3}\exp(-0.80436\mathrm{t}) - 5.0198\times10^{-2}\exp(-0.41081\mathrm{t})$$

And hence,

$$\mu_{BM}(t) = 0.18275(\exp(-0.28024\mathrm{t})-1) + 0.18275(\exp(-0.51458\mathrm{t})-1) + 1.8146\times10^{-3}(\exp(-0.80436\mathrm{t})-1) + 0.12219(\exp(-0.41081\mathrm{t})-1) - 6.4184\times10^{28}(\exp(-3.2332\times10^{-30}\mathrm{t})-1) \qquad (8-5)$$

The busy period of the system due to failure is given by:

Taking the inverse of Laplace transform of Equation (7 – 8), one may get the probability that the system is in state S_5 at time't'

$$P_{BF}(t) = 1.9747\times10^{-2}exp(-3.2332\times10^{-30}t) - 5.9555\times10^{-3}exp(-0.51458t) - 5.8816\times10^{-3}exp(-0.28024t) - 5.1776\times10^{-3}exp(-0.80436t) - 2.7325\times10^{-3}exp(-0.41081t)$$

And hence,

$$\mu_{BF}(t) = \int_0^t P_{BF}(t)dt = -6.1076\times10^{27}(exp(-3.2332\times10^{-30}t)-1) + 1.1574\times10^{-2}(exp(-0.51458t)-1) + 2.0988\times10^{-2}(exp(-0.28024t)-1) + 6.4369\times10^{-3}(exp(-0.80436t)-1) + 6.6515\times10^{-3}(exp(-0.41081t)-1) \quad (8-6)$$

By substituting Equations (8 – 4), (8 – 5), (8 – 6) in Equation (8 – 3), one may get:

$$PF(t) = R\mu_{up}(t) - C_1\mu_{BM}(t) - C_2\mu_{BF}(t) \quad (8-7)$$

Setting values for the cost coefficient C_1, C_2 and revenue R one can get the expected total gain in the interval (0, t] for different values of 't' .Setting t =1, 2, 3,… in Equation (8 – 7) for $R = 1000, C_1 = 100, C_2 = 50$, one compute Table (2). Figure (3) show expected total profit increase in the interval (0, t] for three cases.

Table (2): Expected total profit in the interval (0, t] for different values of 't'.

t	PF(t)
0	0.0000
1	32. 392
2	53. 167
3	66. 792
4	75. 901
5	82. 092
6	86. 358
7	89. 334
8	7575. 3
9	7576. 8
10	7577. 9
20	15064.0
100	89903.0
1000	9.5803×10^5

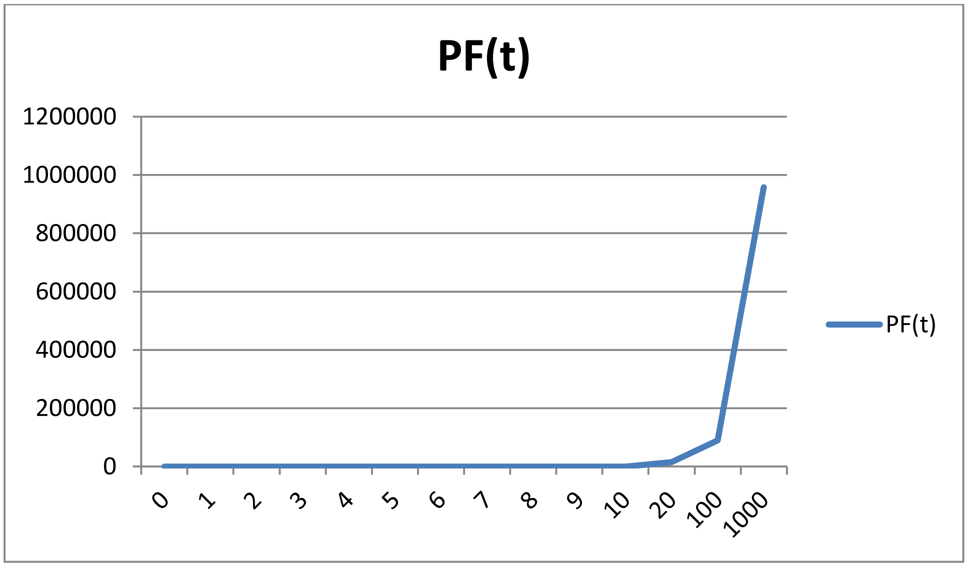

Figure (3): Expected total gain increase in the interval (0, t) for first case.

8.3 MTSF Analysis

If we put: $\beta_{12} = .04, \beta_{13} = .03, \beta_{14} = .02, \beta_{15} = .01, \beta_{23} = .05, \beta_{25} = .025, \beta_{34} = .045, \beta_{35} = .035, \beta_{45} = .055, \alpha_{21} = .4, \alpha_{31} = .3, \alpha_{41} = .2, \alpha_{51} = .1$ in Equation (7 – 9), and so on for $\beta_{13}, \beta_{14}, \beta_{15}$, one can compute Table 3. Variation of MTSF of the system with different values of failure rates is shown in Figure 4.

Table (3): Variation of MTSF with different values of failure rates.

Failure rates	MTSF β_{12}	MTSF β_{13}	MTSF β_{14}	MTSF β_{15}
.0	73.181	74.589	223.29	86.13
.01	72.227	72.815	106.01	69.508
.02	71.297	71.123	69.508	58.264
.03	70.391	69.508	51.704	50.151
.04	69.508	67.965	41.161	44.021
.05	68.647	66.489	34.189	39.227
.06	67.806	65.076	29.237	35.374
.07	66.987	63.721	25.538	32.210
.08	66.186	62.422	22.670	29.566
.09	65.405	61.174	20.381	27.323
.10	64.642	59. 976	18.512	25.397

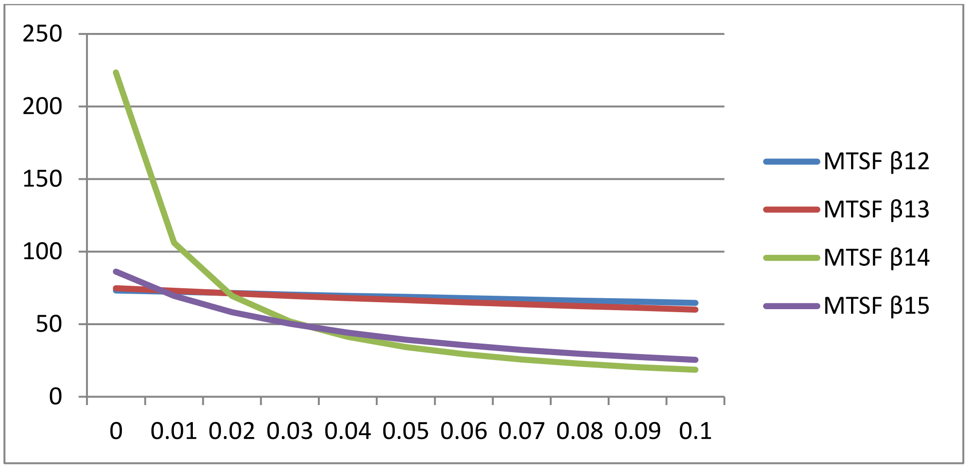

Figure (4): Effect of failure rates on MTSF.

8.4 Steady-state availability

If we put: $\beta_{12} = .0, .01, .02, \ldots$at constant$\beta_{13} = .03, \beta_{14} = .02, \beta_{15} = .01, \beta_{23} = .05, \beta_{25} = .025, \beta_{34} = .045, \beta_{35} = .035, \beta_{45} = .055, \alpha_{21} = .4, \alpha_{31} = .3, \alpha_{41} = .2, \alpha_{51} = .1$, in Equations (7 – 10), and so on for $\beta_{13}, \beta_{14}, \beta_{15}$we get Table 4. Variation of steady state availability with respect to failure rates is shown in Figure 5.

Table (4): Variation of steady-state availability with respect to failure rate

failure rates	availability β_{12}	availability β_{13}	availability β_{14}	availability β_{15}
0	0.92922	0.93289	0.93653	0.96046
.01	0.92821	0.93026	0.93077	0.92543
.02	0.92725	0.92778	0.92543	0.89286
.03	0.92632	0.92543	0.92046	0.86251
.04	0.92543	0.92320	0.91584	0.83415
.05	0.92458	0.92109	0.91151	0.80760
.06	0.92375	0.91908	0.90746	0.78269
.07	0.92296	0.91717	0.90366	0.75926
.08	0.92220	0.91535	0.90009	0.73720
.09	0.92147	0.91361	0.89673	0.71638
.10	0.92076	0.91196	0.89355	0.69671

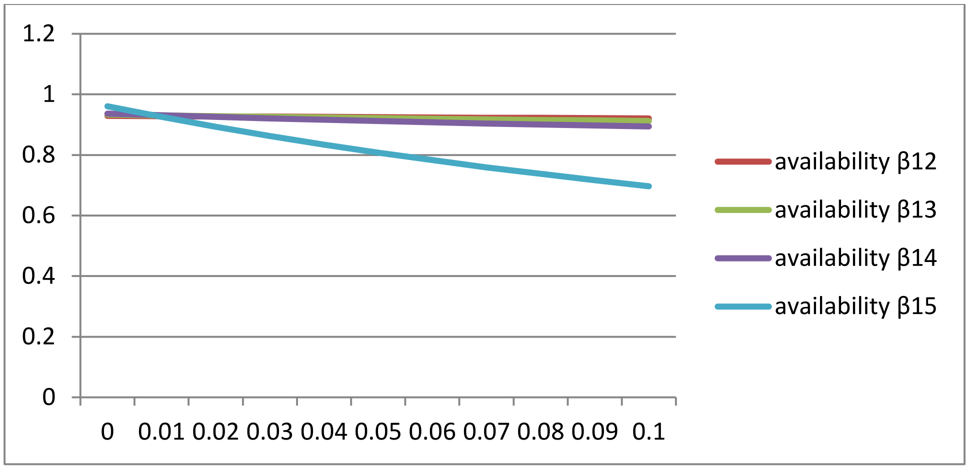

Figure (5): Effect of failure rates on steady-state availability.

Table (5): Variation of availability with different values of repair rates.

Repair rate	availability α_{21}	availability α_{31}	availability α_{41}	availability α_{51}
0	0.91241	90361	0.8913	0.0
.1	0.92017	0.91754	0.91787	8.6121×10^{-2}
.2	0.92303	0.92272	0.92543	9.2543×10^{-2}
.3	0.92452	0.92543	0.92901	0.28471
.4	0.92543	0.92709	0.93109	0.38451
.5	0.92605	0.92822	0.93245	0.48439
.6	0.92649	0.92903	0.93342	0.58416
.7	0.92682	0.92964	0.93413	0.68425
.8	0.92709	0.93012	0.93469	0.7842
.9	0.9273	0.93051	0.93513	0.88417
1.0	0.92747	0.93082	0.93549	0.98414

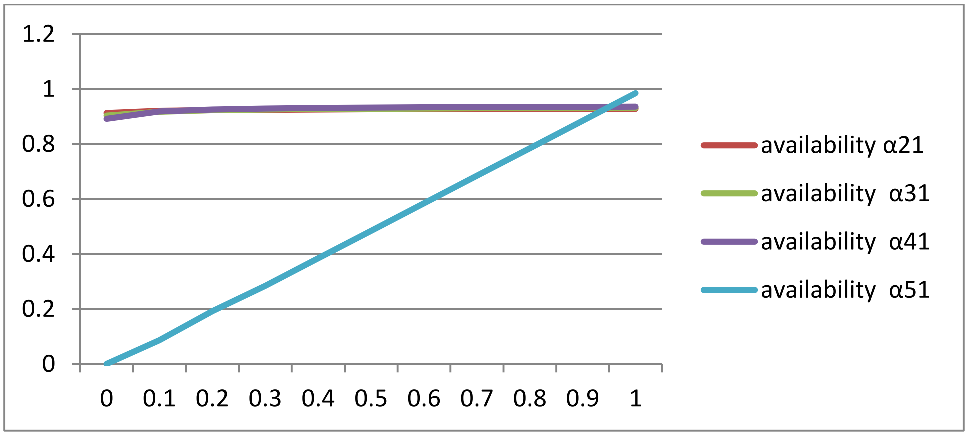

Figure (6): Effect of repair rates on steady-state availability.

8.5 Steady-state Busy Period due to maintenance

If we put: β_{12}=.01, .02,.03,……, at constant $\beta_{13}=.03, \beta_{14}=.02, \beta_{15}=.01, \beta_{23}=.05, \beta_{25}=.025, \beta_{34}=.045, \beta_{35}=.035, \beta_{45}=.055, \alpha_{21}=.4, \alpha_{31}=.3, \alpha_{41}=.2, \alpha_{51}=.1$ in Equations (7 – 11), and so on for $\beta_{13}, \beta_{14}, \beta_{15}$, we get Table 6. Variation of steady state Busy Period due to maintenance with respect to failure rates is shown in Figure 7.

Table (6): Variation of steady-state busy period with respect to failure rates

failure rates	busy period β_{12}	busy period β_{13}	busy period β_{14}	busy period β_{15}
0	0.1359	7.2579×10^{-2}	8.3347×10^{-2}	0.13957
.01	0.13553	9.4386×10^{-2}	0.10988	0.13448
.02	0.13516	0.11499	0.13448	0.12975
.03	0.13482	0.13448	0.15735	0.12534
.04	0.13448	0.15296	0.17867	0.12122
.05	0.13416	0.17049	0.19858	0.11736
.06	0.13386	0.18715	0.21722	0.11374
.07	0.13356	0.203	0.23472	0.11034
.08	0.13327	0.2181	0.25117	0.10713
.09	0.1330	0.2325	0.26666	0.1041
.10	0.13273	0.24625	0.28127	0.10125

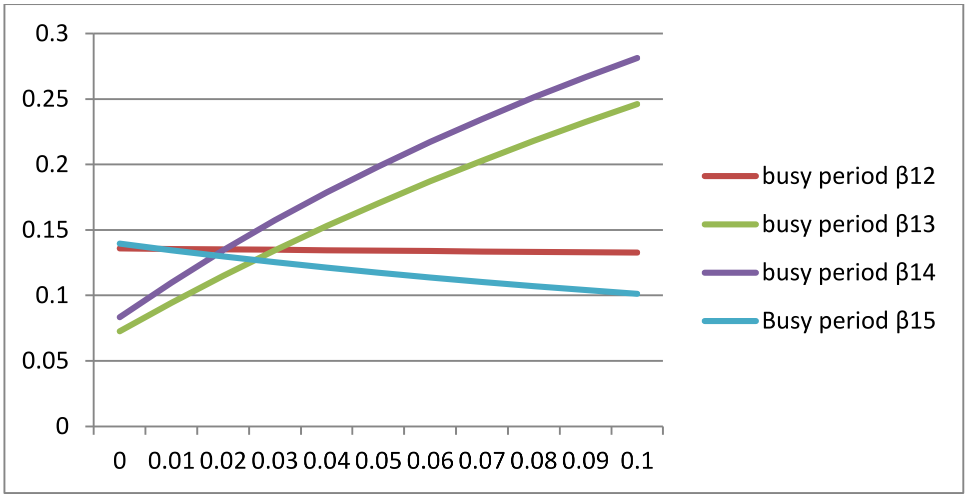

Figure (7): Effect of failure rates on steady-state busy period.

Table (7): Variation of busy period with different values of repair rates.

Repair rate	busy period α_{21}	busy period α_{31}	busy period α_{41}	busy period α_{51}
0	0.12961	0.31546	0.33822	0.0
.1	0.13251	0.19991	0.22743	0.18232
.2	0.13359	0.15693	0.19592	0.19592
.3	0.13414	0.13448	0.18101	0.20091
.4	0.13448	0.12070	0.17232	0.2035
.5	0.13471	0.11137	0.16662	0.20509
.6	0.13488	0.10464	0.16261	0.20617
.7	0.13501	9.9554×10^{-2}	0.15962	0.20694
.8	0.1351	9.5575×10^{-2}	0.15732	0.20752
.9	0.13518	9.2378×10^{-2}	0.15548	0.20798
1.0	0.13525	8.9752×10^{-2}	0.15370	0.20835

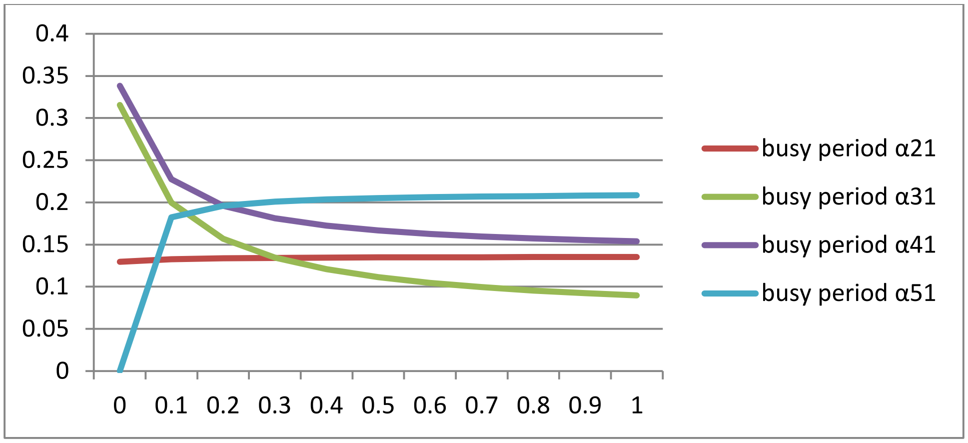

Figure (8): Effect of repair rates on steady-state busy period.

8.6 Steady-state Busy Period due to failure

If we put: β_{12}=.01, .02,.03,……, at constant $\beta_{13} = .03, \beta_{14} = .02, \beta_{15} = .01, \beta_{23} = .05, \beta_{25} = .025, \beta_{34} = .045, \beta_{35} = .035, \beta_{45} = .055, \alpha_{21} = .4, \alpha_{31} = .3, \alpha_{41} = .2, \alpha_{51} = .1$in Equations (7 – 12), and so on for $\beta_{13}, \beta_{14}, \beta_{15}$, we get Table 8. Variation of steady state Busy Period due to failure with respect to failure rates is shown in Figure 9.

Table (8): Variation of steady-state busy period with respect to failure rates.

failure rates	busy period β_{12}	busy period β_{13}	busy period β_{14}	busy period β_{15}
0	7.0777×10^{-2}	6.7107×10^{-2}	6.3465×10^{-2}	3.9536×10^{-2}
.01	7.1786×10^{-2}	6.9736×10^{-2}	6.9228×10^{-2}	0.07457
.02	7.2753×10^{-2}	0.072220	0.07457	0.10714
.03	7.3680×10^{-2}	0.074570	7.9536×10^{-2}	0.13749
.04	7.4570×10^{-2}	7.6797×10^{-2}	8.4165×10^{-2}	0.16585
.05	7.5425×10^{-2}	7.8911×10^{-2}	8.8489×10^{-2}	0.1924
.06	7.6247×10^{-2}	8.0919×10^{-2}	9.2538×10^{-2}	0.21731
.07	7.7037×10^{-2}	0.08283	9.6337×10^{-2}	0.24074
.08	7.7799×10^{-2}	8.4651×10^{-2}	9.9908×10^{-2}	0.2628
.09	7.8532×10^{-2}	8.6387×10^{-2}	0.10327	0.28362
.10	7.9239×10^{-2}	8.8044×10^{-2}	0.10645	0.30329

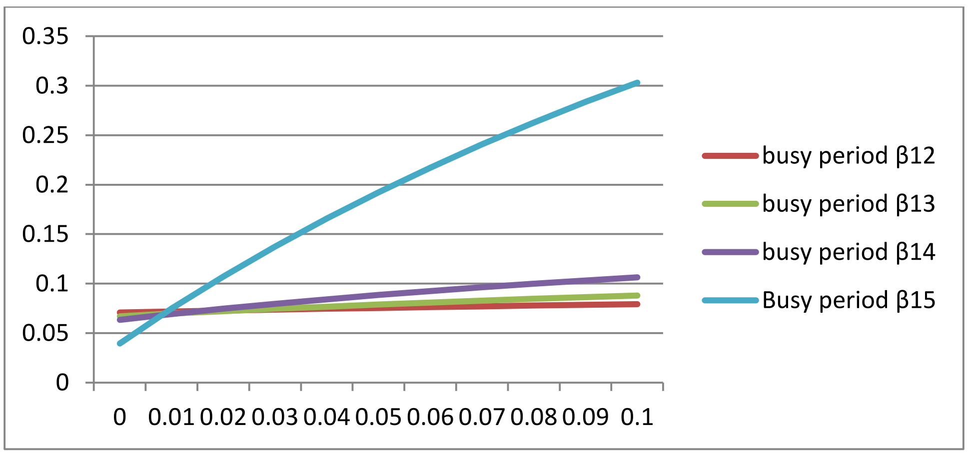

Figure (9): Effect of failure rates on steady-state busy period.

Table (9): Variation of steady-state busy period with respect to repair rates.

Repair rate	busy period α_{21}	busy period α_{31}	busy period α_{41}	busy period α_{51}
0	0.57259	0.61173	0.63412	0.75618
.1	0.634624	0.71126	0.73804	0.75975
.2	0.73171	0.74828	0.76761	0.76761
.3	0.75397	0.76761	0.78159	0.52478
.4	0.76761	0.77948	0.78975	0.40449
.5	0.77682	0.78751	0.79508	0.32142
.6	0.78347	0.79331	0.79885	0.26926
.7	0.78848	0.79769	0.76761	0.23154
.8	0.79241	0.80112	0.80382	0.20327
.9	0.79556	0.80387	0.80554	0.18108
1.0	0.79814	0.80613	0.80545	0.16326

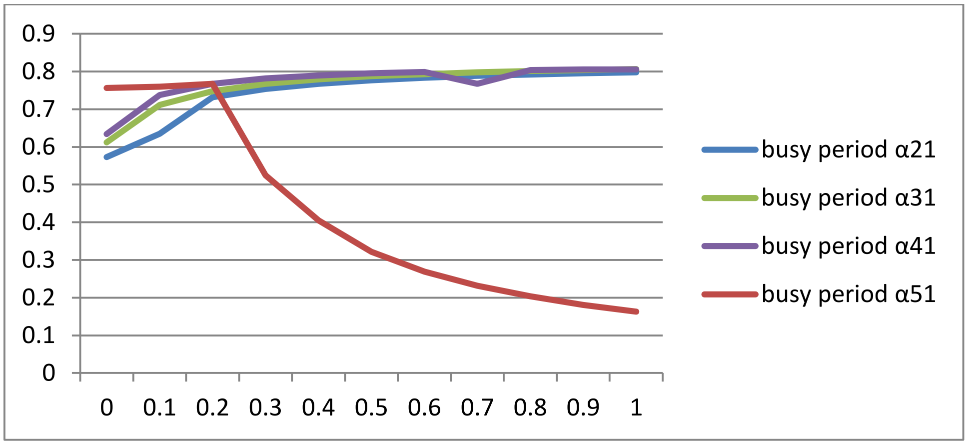

Figure (10): Effect of repair rates on steady-state busy period.

8.7 The expected total profit incurred to the system in the steady-state

The expected total profit per unit time incurred to the system in the steady-state is:

$$PF = R\mu_{up} - C_1 . B_M(\infty) - C_2 . B_F(\infty)$$

If we put: $\beta_{12} = .01, .02, .03, \dots\dots$, at constant , $\beta_{13} = .03, \beta_{14} = .02, \beta_{15} = .01, \beta_{23} = .05, \beta_{25} = .025, \beta_{34} = .045, \beta_{35} = .035, \beta_{45} = .055, \alpha_{21} = .4, \alpha_{31} = .3, \alpha_{41} = .2, \alpha_{51} = .1$, R = 1000, $C_1 = 50$, $C_2 = 100$ in equations (7 – 13), and so on for $\beta_{13}, \beta_{14}, \beta_{15}$ we get Table 10. Variation of steady state profit w.r.t. failure rates are shown in Figure 11.

Table (10): Variation of steady-state profit with respect to failure rates.

failure rates	steady-state profit β_{12}	steady-state profit β_{13}	steady-state profit β_{14}	steady-state profit β_{15}
0	912.09	922.28	925.02	944.53
.01	911.07	917.33	916.32	908.25
.02	910.10	912.67	908.25	874.53
.03	909.15	908.25	900.75	843.1
.04	908.25	904.06	893.76	813.74
.05	907.39	900.1	887.23	786.24
.06	906.55	896.32	881.11	760.45
.07	905.75	892.73	875.37	736.19
.08	904.98	889.31	869.98	713.35
.09	904.24	884.67	864.9	691.79
.10	903.53	884.31	860.1	671.42

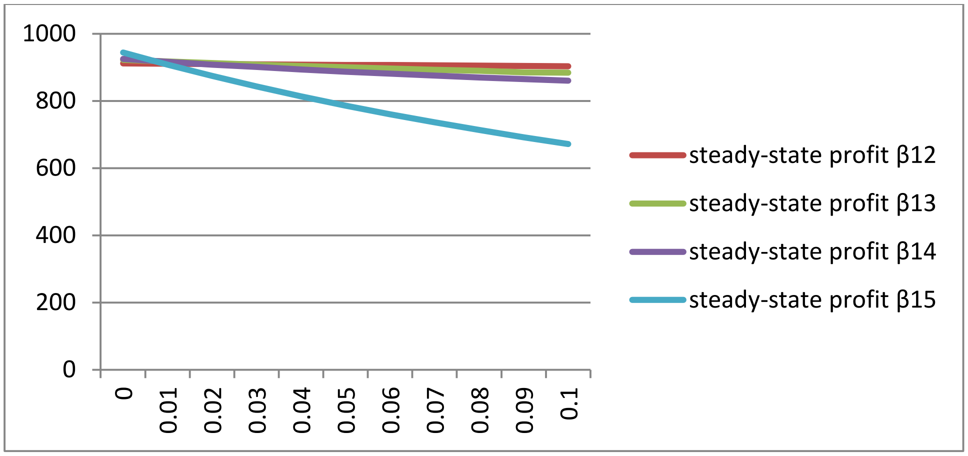

Figure (11): Effect of failure rates on steady-state profit.

Table (11): Variation of steady-state profit with different values of repair rate.

Repair rate	steady-state profit α_{21}	steady-state profit α_{31}	steady-state profit α_{41}	steady-state profit α_{51}
0	870.82	841.48	825.77	25.089
.1	889.61	861.99	858.23	29.902
.2	873.09	869.61	867.46	34.571
.3	873.41	873.60	871.83	238.38
.4	873.60	876.05	874.37	343.94
.5	873.74	877.71	876.03	447.81
.6	873.83	878.90	877.22	550.08
.7	873.90	879.80	879.79	651.98
.8	873.96	880.51	878.77	753.28
.9	874.00	881.08	879.31	854.32
1.0	874.04	881.54	879.85	955.14

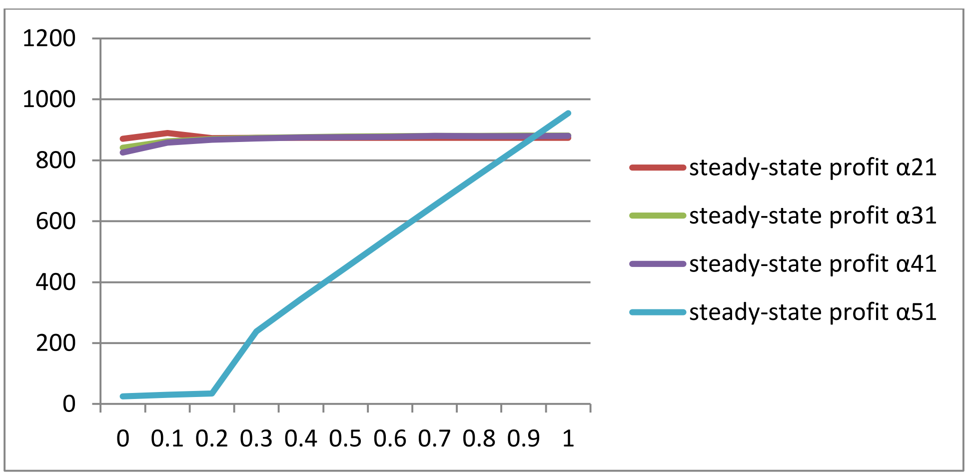

Fig. (12): Effect of repair rates on steady-state profit.

9 Conclusion

In this paper, we developed the explicit expressions for the availability, busy period due to maintenance, busy period due to failure of the system, and profit function. It is evident from Figures 11 that the increase in deterioration and failure rates induces the decrease in profit, while from Figures 12, the increase in maintenance and repair rates induces increase in profit. We concluded that: The effect of both maintenance, and repair rates have been captured. The results have shown that both failure and deterioration rates decrease the availability, MTSF and profit while maintenance and repair rates increase the availability and profit.

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