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Soft Supra Strongly Generalized Closed Sets via Soft Ideals

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Abstract: In this paper, we generalize the notions of soft supra strongly g-closed sets and soft supra strongly g-open sets [2] by using the soft ideals notion [15] in supra soft topological spaces (X,μ,E) and study their basic properties. Here, we used the concept of soft ideals, soft closure, soft interior and supra open soft sets to define soft supra strongly $\tilde{I}g$-closed sets. The relationship between soft supra strongly $\tilde{I}g$-closed sets and other existing soft sets have been investigated. Furthermore, the union and intersection of two soft supra strongly $\tilde{I}g$-closed (resp. open) sets have been obtained. It has been pointed out in this paper that many of these parameters studied have, in fact, applications in real world situations and therefore I believe that this is an extra justification for the work conducted in this paper.

Keywords: Soft sets, Soft topological space, Supra soft topological space, Soft supra strongly $\tilde{I}g$-closed sets, Soft supra strongly $\tilde{I}g$-open sets and soft functions.

1 Introduction

In 1970, Levine [17] introduced the notion of g-closed sets in topological spaces as a generalization of closed sets. Indeed ideals are very important tools in general topology. In 1983, Mashhour et al. [18] introduced the supra topological spaces, not only, as a generalization to the class of topological spaces, but also, these spaces were easier in the application as shown in [6]. In 2001, Popa et al. [20] generalized the supra topological spaces to the minimal spaces and generalized spaces as a new wider classes. In 2007, Arpad Szaz [7] succeed to introduce an application on the minimal spaces and generalized spaces. In 1987, Abd El-Monsef et al. [5] introduced the fuzzy supra topological spaces. In 2001, El-Sheikh success to use the fuzzy supra topology to study some topological properties to the fuzzy bitopological spaces.

The notions of supra soft topological space were first introduced by El-Sheikh et al. [8]. Recently, Abd El-latif [2] introduced the concept of soft supra strongly g-closed sets in supra soft topological spaces. Here, we introduce and study the concept of soft supra strongly $\tilde{I}g$-closed sets, which is the extension of the concept of soft supra strongly g-closed sets [2]. Also, we study the relationship between soft strongly $\tilde{I}g$-closed sets and other existing soft sets have been investigated.

2 Preliminaries

In this section, we present the basic definitions and results of (supra) soft set theory which will be needed in this paper. For more details see [1,3,4,8,13,15,16,19,21].

Definition 2.1.[19] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X, where F is a mapping given by $F:A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \varphi$ i.e
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2.[21] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if
(1)$\tilde{X}, \tilde{\varphi} \in \tau$, where $\tilde{\varphi}(e) = \varphi$ and $\tilde{X}(e) = X$, $\forall e \in E$,

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(2)the union of any number of soft sets in τ belongs to τ,
(3)the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

Definition 2.3.[11] Let (X,τ,E) be a soft topological space. A soft set (F,E) over X is said to be closed soft set in X, if its relative complement $(F,E)^{\tilde{c}}$ is an open soft set.

Definition 2.4.[11] Let(X,τ,E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X,\tau,E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition 2.5.[21] Let (X,τ,E) be a soft topological space, $(F,E)\in SS(X)_E$ and Y be a non-null subset of X. Then the sub soft set of (F,E) over Y denoted by (F_Y,E), is defined as follows:

$$F_Y(e)=Y\cap F(e)\ \forall e\in E.$$

In other words $(F_Y,E)=\tilde{Y}\tilde{\cap}(F,E)$.

Definition 2.6.[21] Let (X,τ,E) be a soft topological space and Y be a non-null subset of X. Then

$$\tau_Y=\{(F_Y,E):(F,E)\in\tau\}$$

is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Definition 2.7.[15] Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with the same set of parameters E. Then $\tilde{I}\subseteq SS(X)_E$ is called a soft ideal on X with the same set E if

(1)$(F,E)\in\tilde{I}$ and $(G,E)\in\tilde{I}\Rightarrow(F,E)\tilde{\cup}(G,E)\in\tilde{I}$,
(2)$(F,E)\in\tilde{I}$ and $(G,E)\tilde{\subseteq}(F,E)\Rightarrow(G,E)\in\tilde{I}$.
i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Lemma 2.8.[14] If $\tilde{I}$ is a soft ideal on X and $Y\subseteq X$, then $\tilde{I}_Y=\{(Y,E)\tilde{\cap}(I,E):(I,E)\in\tilde{I}\}$ is a soft ideal of Y.

Definition 2.9.[16] A soft set (F,E) is called supra generalized closed soft with respect to a soft ideal $\tilde{I}$ (supra-$\tilde{I}g$-closed soft) in a supra soft topological space (X,μ,E) if $cl^sF_E\setminus G_E\in\tilde{I}$ whenever $F_E\tilde{\subseteq}G_E$ and $G_E\in\mu$.

Definition 2.10.[8] Let (X,τ,E) be a soft topological space and (X,μ,E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau\subseteq\mu$.

Definition 2.11.[8] Let (X,μ,E) be a supra soft topological space over X, then the members of μ are said to be supra open soft sets in X. We denote the set of all supra open soft sets over X by $supra-OS(X,\mu,E)$, or when there can be no confusion by $supra-OS(X)$ and the set of all supra closed soft sets by $supra-CS(X,\mu,E)$, or $supra-CS(X)$.

Definition 2.12.[8] Let (X,μ,E) be a supra soft topological space over X and $(F,E)\in SSE(X)_E$. Then, the supra soft interior of (F,E), denoted by $int^s(F,E)$ is the soft union of all supra open soft subsets of (F,E). i.e
$int^s(F,E)=\tilde{\cup}\{(G,E):(G,E)$ is supra open soft set and $(G,E)\tilde{\subseteq}(F,E)\}$.

Definition 2.13.[8] Let (X,μ,E) be a supra soft topological space over X and $(F,E)\in SSE(X)_E$. Then, the supra soft closure of (F,E), denoted by $cl^s(F,E)$ is the soft intersection of all supra closed super soft sets of (F,E). i.e
$cl^s(F,E)=\tilde{\cap}\{(H,E):(H,E)$ is supra closed soft set and $(F,E)\tilde{\subseteq}(H,E)\}$.

Definition 2.14.[2] A soft set (F,E) is called soft supra strongly generalized closed set (soft supra strongly g-closed) in a supra soft topological space (X,μ,E) if $cl^s(int^s(F,E))\tilde{\subseteq}(G,E)$ whenever $(F,E)\tilde{\subseteq}(G,E)$ and (G,E) is supra open soft in X.

3 Soft supra strongly $\tilde{I}g$-closed sets

Abd El-latif [2] introduced the notion of soft supra strongly generalized closed sets in supra soft topological spaces. In this section, we generalize this notion by using the soft ideal notion [15].

Definition 3.1. A soft set $F_E\in SS(X,E)$ is called soft supra strongly generalized closed set with respect to soft ideal $\tilde{I}$ (soft supra strongly $\tilde{I}g$-closed set) in a supra soft topological space (X,μ,E) if $cl^s(int^s(F_E))\setminus G_E\in\tilde{I}$ whenever $F_E\tilde{\subseteq}G_E$ and G_E is supra open soft in X.

Example 3.2. Suppose that there are three phones in the universe X given by $X=\{a,b,c\}$. Let $E=\{e_1,e_2\}$ be the set of decision parameters which are stands for ”expensive” and ”beautiful” respectively.
Let $(G_1,E),(G_2,E)$ be two soft sets over the common universe X, which describe the composition of the cars, where:
$G_1(e_1)=\{b,c\}$ $G_1(e_2)=\{a,b\}$,
$G_2(e_1)=\{a,b\}$ $G_2(e_2)=\{a,c\}$.
Then, $\mu=\{\tilde{X},\tilde{\varphi},(G_1,E),(G_2,E)\}$ is a supra soft topology over X. Let

$$\tilde{I}=\{\varphi,F_E,G_E,H_E\}$$

be a soft ideal on X, where:
$F(e_1)=\{b\}$ $F(e_2)=\varphi$,
$G(e_1)=\{b\}$ $G(e_2)=\{a\}$,
$H(e_1)=\varphi$ $H(e_2)=\{a\}$.
Hence, the soft sets $(F_1,E),(F_2,E)$, where:
$F_1(e_1)=\{b,c\}$ $F_1(e_2)=\{a,c\}$,
$F_2(e_1)=\{a,b\}$ $F_2(e_2)=\{a,b\}$.
are supra strongly $\tilde{I}g$-closed soft sets in (X,μ,E), but the soft sets $(G_1,E),(G_2,E)$ are not soft supra strongly $\tilde{I}g$-closed in (X,μ,E).

Theorem 3.3. Every soft supra g-closed set is a soft supra strongly $\tilde{I}g$-closed.

Proof. Let $F_E\tilde{\subseteq}G_E$ and $G_E\in\mu$. Since F_E is soft supra g-closed, then $cl^s(F_E)\tilde{\subseteq}G_E$ and $cl^s(int^s(F_E))\tilde{\subseteq}cl^s(F_E)\tilde{\subseteq}G_E$. Hence, $cl^s(int^s(F_E))\setminus G_E=\tilde{\varphi}\in\tilde{I}$. Therefore, F_E is soft supra strongly $\tilde{I}g$-closed.

Remark 3.4. The converse of the above theorem is not true in general. The following example supports our claim.

Example 3.5. Suppose that there are two suits in the universe X given by $X = \{a,b\}$. Let $E = \{e_1(cotton), e_2(woollen)\}$ be the set of parameters showing the material of the dresses.
Let A_E, B_E, C_E be three soft sets over the common universe X, which describe the composition of the dresses, where:
$A(e_1) = \{a\} \quad A(e_2) = X$,
$B(e_1) = \{a\} \quad B(e_2) = \{b\}$,
$C(e_1) = \{a\} \quad C(e_2) = \{a\}$.
Then, $\mu = \{\varphi, \tilde{X}, A_E, B_E, C_E\}$ is a supra soft topology over X. Let

$$\tilde{I} = \{\varphi, F_E, G_E, H_E\}$$

be a soft ideal on X, where:
$F(e_1) = \{a\} \quad F(e_2) = \varphi$,
$G(e_1) = \{a\} \quad G(e_2) = \{a\}$,
$H(e_1) = \varphi \quad H(e_2) = \{a\}$.
The soft set V_E is soft supra strongly $\tilde{I}g$-closed but not supra soft g-closed, where:
$V(e_1) = \varphi \quad V(e_2) = \{b\}$.

Theorem 3.6. Every supra closed soft set is a soft supra strongly $\tilde{I}g$-closed.

Proof. Let $F_E \tilde{\sqsubseteq} G_E$ and $G_E \in \mu$. Since F_E is supra closed soft, then $cl^s(int^s(F_E)) \tilde{\sqsubseteq} cl^s(F_E) = F_E \tilde{\sqsubseteq} G_E$. Hence, $cl^s(int^s(F_E)) \setminus G_E = \tilde{\varphi} \in \tilde{I}$. Therefore, F_E is a soft supra strongly $\tilde{I}g$-closed.

Remark 3.7. The converse of the above theorem is not true in general as shall shown in the following example.

Example 3.8. In Example 3.2, the soft set T_E is soft supra strongly $\tilde{I}g$-closed but not supra closed soft set where:
$T(e_1) = X \quad T(e_2) = \{a\}$.

Theorem 3.9. Every soft supra $\tilde{I}g$-closed set is a soft supra strongly $\tilde{I}g$-closed.

Proof. Let $F_E \tilde{\sqsubseteq} H_E$ and $H_E \in \mu$. Then, $cl^s(int^s(F_E)) \setminus I_E \tilde{\sqsubseteq} cl^s(F_E) \setminus I_E \in \tilde{I}$ for some $I_E \in \tilde{I}$. Therefore, F_E is soft supra strongly $\tilde{I}g$-closed.

Remark 3.10. The converse of the above theorem is not true in general, as shown in the following example.

Example 3.11. In Example 3.5, the soft set Y_E is soft supra strongly $\tilde{I}g$-closed but not soft supra $\tilde{I}g$-closed, where:
$Y(e_1) = \varphi \quad Y(e_2) = \{b\}$.

Proposition 3.12. If a soft subset F_E of a supra soft topological space (X, μ, E) is supra open soft, then it is soft supra strongly $\tilde{I}g$-closed if and only if it is soft supra $\tilde{I}g$-closed.

Proof. It is obvious.

Theorem 3.13. A soft set A_E is soft supra strongly $\tilde{I}g$-closed in a supra soft topological space (X, μ, E) if and only if $F_E \tilde{\sqsubseteq} cl^s(int^s(A_E)) \setminus A_E$ and F_E is supra closed soft implies $F_E \in \tilde{I}$.

Proof. $(\Rightarrow)$Let $F_E \tilde{\sqsubseteq} cl^s(int^s(A_E)) \setminus A_E$ and F_E is supra closed soft. Then, $A_E \tilde{\sqsubseteq} F_E^{\tilde{c}}$. By hypothesis, $cl^s(int^s(A_E)) \setminus F_E^{\tilde{c}} \in \tilde{I}$. But, $F_E \tilde{\sqsubseteq} cl^s(int^s(A_E)) \setminus F_E^{\tilde{c}}$, then $F_E \in \tilde{I}$ from Definition 2.7.
$(\Leftarrow)$ Assume that $A_E \tilde{\sqsubseteq} G_E$ and $G_E \in \mu$. Then, $cl^s(int^s(A_E)) \setminus G_E = cl^s(int^s(A_E)) \tilde{\sqcap} G_E^{\tilde{c}}$ is a supra closed soft set and $cl^s(int^s(A_E)) \setminus G_E \tilde{\sqsubseteq} cl^s(int^s(A_E)) \tilde{\sqcap} G_E^{\tilde{c}}$. By assumption, $cl^s(int^s(A_E)) \setminus G_E \in \tilde{I}$. Therefore, A_E is soft supra strongly $\tilde{I}g$-closed.

Theorem 3.14. If F_E is soft supra strongly $\tilde{I}g$-closed in a supra soft topological space (X, μ, E) and $F_E \tilde{\sqsubseteq} G_E \tilde{\sqsubseteq} cl^s(int^s(F_E))$, then G_E is soft supra strongly $\tilde{I}g$-closed.

Proof. Let $G_E \tilde{\sqsubseteq} H_E$ and $H_E \in \mu$. Then, $F_E \tilde{\sqsubseteq} H_E$. Since F_E is soft supra strongly $\tilde{I}g$-closed, then $cl^s(int^s(F_E)) \setminus H_E \in \tilde{I}$. Now, $G_E \tilde{\sqsubseteq} cl^s(int^s(F_E))$, implies that $cl^s(G_E) \tilde{\sqsubseteq} cl^s(int^s(F_E))$. So, $cl^s(int^s(G_E)) \setminus H_E \tilde{\sqsubseteq} cl^s(G_E) \setminus H_E \tilde{\sqsubseteq} cl^s(int^s(F_E)) \setminus H_E$. Thus, $cl^s(int^s(G_E)) \setminus H_E \in \tilde{I}$ from Definition 2.7. Consequently, G_E is soft supra strongly $\tilde{I}g$-closed.

Remark 3.15. The soft intersection (resp. union) of two soft supra strongly $\tilde{I}g$-closed sets need not to be a soft supra strongly $\tilde{I}g$-closed as shown by the following examples.

Examples 3.16.

(1) Let $X = \{a,b\}$ be the set of two cars under consideration and, $E = \{e_1(costly), e_2(Luxurious)\}$. Let A_E, B_E be two soft sets representing the attractiveness of the phone which Mr. X and Mr. Y are going to buy, where:
$A(e_1) = \varphi \quad A(e_2) = \{a\}$,
$B(e_1) = \{b\} \quad B(e_2) = \{a\}$.
Then, A_E and B_E are soft sets over X and

$$\mu = \{\tilde{X}, \tilde{\varphi}, A_E, B_E\}$$

is the supra soft topology over X. Let

$$\tilde{I} = \{\tilde{\varphi}, F_E, G_E, H_E\}$$

be a soft ideal on X, where:
$F(e_1) = \{b\} \quad F(e_2) = \varphi$,
$G(e_1) = \{b\} \quad G(e_2) = \{a\}$,
$H(e_1) = \varphi \quad H(e_2) = \{a\}$.
The soft sets M_E, N_E are soft supra strongly $\tilde{I}g$-closed, where:
$M(e_1) = \varphi \quad M(e_2) = X$,
$N(e_1) = X \quad N(e_2) = \{a\}$.
But, $P_E = M_E \tilde{\sqcap} N_E$ is not soft strongly $\tilde{I}g$-closed, where:
$P(e_1) = \varphi \quad P(e_2) = \{a\}$.

(2) Suppose that there are four alternatives in the universe of houses $X = \{a,b,c,d\}$ and consider $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "quality of houses" and "green surroundings" respectively.
Let $(F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E)$ be twelve

soft sets over the common universe X which describe the goodness of the houses, where:
$F_1(e_1)=\{a\} \quad F_1(e_2)=\{d\}$,
$F_2(e_1)=\{a,d\} \quad F_2(e_2)=\{a,d\}$,
$F_3(e_1)=\{d\} \quad F_3(e_2)=\{a\}$,
$F_4(e_1)=\{a,b\} \quad F_4(e_2)=\{b,d\}$,
$F_5(e_1)=\{b,d\} \quad F_5(e_2)=\{a,b\}$,
$F_6(e_1)=\{a,b,c\} \quad F_6(e_2)=\{a,b,c\}$,
$F_7(e_1)=\{b,c,d\} \quad F_7(e_2)=\{b,c,d\}$,
$F_8(e_1)=\{a,b,d\} \quad F_8(e_2)=\{a,b,d\}$,
$F_9(e_1)=X \quad F_9(e_2)=\{a,b,c\}$,
$F_{10}(e_1)=\{b,c,d\} \quad F_{10}(e_2)=X$,
$F_{11}(e_1)=\{a,b,c\} \quad F_{11}(e_2)=X$,
$F_{12}(e_1)=X \quad F_{12}(e_2)=\{b,c,d\}$.
Hence, $\mu = \{\tilde{X},\tilde{\varphi},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E),(F_9,E),(F_{10},E),(F_{11},E),(F_{12},E)\}$ is a supra soft topology over X. Let

$$\tilde{I}=\{\varphi,F_E,G_E,H_E\}$$

be a soft ideal on X, where:
$F(e_1)=\{a\} \quad F(e_2)=\{b\}$,
$G(e_1)=\{a\} \quad G(e_2)=\varphi$,
$H(e_1)=\varphi \quad H(e_2)=\{b\}$.
Therefore, the soft sets $(G,E),(H,E)$, where:
$G(e_1)=\{d\} \quad G(e_2)=\{d\}$,
$H(e_1)=\{a\} \quad H(e_2)=\{a\}$.
are soft supra strongly $\tilde{I}g$-closed sets in (X,μ,E), but their soft union $(G,E)\tilde{\cup}(H,E)=(A,E)$ where:
$A(e_1)=\{a,d\} \quad A(e_2)=\{d,a\}$ is not soft supra strongly $\tilde{I}g$-closed.

Theorem 3.17. If A_E is soft supra strongly $\tilde{I}g$-closed and F_E is supra closed soft in a supra soft topological space (X,μ,E). Then, $A_E\tilde{\cap}F_E$ is soft supra strongly $\tilde{I}g$-closed.

Proof. Assume that $A_E\tilde{\cap}F_E\tilde{\sqsubseteq}G_E$ and $G_E\in\mu$. Then, $A_E\tilde{\sqsubseteq}G_E\tilde{\cup}F_E^{\tilde{c}}$. Since A_E is soft supra strongly $\tilde{I}g$-closed. So, $cl^s(int^s(A_E))\setminus(G_E\tilde{\cup}F_E^{\tilde{c}})\in\tilde{I}$. Now, $cl^s(int^s((A_E\tilde{\cap}F_E)))\tilde{\sqsubseteq}cl^s(int^s(A_E))\tilde{\cap}$ $cl^s(int^s(F_E))$ $=$ $cl^s(int^s(A_E))\tilde{\cap}int^s(F_E)\tilde{\sqsubseteq}cl^s(int^s(A_E))\tilde{\cap}F_E$ $=$ $[cl^s(int^s(A_E))\tilde{\cap}F_E]\setminus F_E^{\tilde{c}}$. Therefore, $cl^s(int^s((A_E\tilde{\cap}F_E)))\setminus G_E\tilde{\sqsubseteq}[cl^s(int^s(A_E))\tilde{\cap}F_E]\setminus[F_E^{\tilde{c}}\tilde{\cup}G_E]\tilde{\sqsubseteq}cl^s(int^s(A_E))\setminus[G_E\tilde{\cup}F_E^{\tilde{c}}]\in\tilde{I}$. Hence, $A_E\tilde{\cap}F_E$ is soft supra strongly $\tilde{I}g$-closed.

Theorem 3.18. Let (Y,τ_Y,E) be a soft subspace of a supra soft topological space (X,μ,E), $F_E\tilde{\sqsubseteq}Y_E$ and F_E is soft supra strongly $\tilde{I}g$-closed in (X,μ,E). Then, F_E is soft supra strongly $\tilde{I}_Y g$-closed in (Y,τ_Y,E).

Proof. Assume that $F_E\tilde{\sqsubseteq}B_E\tilde{\cap}Y_E$ and $B_E\in\mu$. So, $B_E\tilde{\cap}Y_E\in\mu_Y$ and $F_E\tilde{\sqsubseteq}B_E$. Since F_E is soft supra strongly $\tilde{I}g$-closed in (X,μ,E), then $cl^s(int^s(F_E))\setminus B_E\in\tilde{I}$. Now, $[cl^s(int^s(F_E))\tilde{\cap}Y_E]\setminus[B_E\tilde{\cap}Y_E]=[cl^s(int^s(F_E))\setminus B_E]\tilde{\cap}Y_E\in\tilde{I}_Y$. Therefore, F_E is soft supra strongly $\tilde{I}_Y g$-closed in (Y,τ_Y,E).

4 Soft supra strongly $\tilde{I}g$-open sets

Definition 4.1. A soft set $F_E\in SS(X,E)$ is called soft supra strongly generalized open set with respect to soft ideal $\tilde{I}$ (soft strongly $\tilde{I}g$-open) in a supra soft topological space (X,μ,E) if and only if its relative complement $F_E^{\tilde{c}}$ is soft supra strongly $\tilde{I}g$-closed in (X,μ,E).

Example 4.2. In Example 3.2, The soft sets $F_{1E}^{\tilde{c}},F_{2E}^{\tilde{c}}$ are soft supra strongly $\tilde{I}g$-open where $F_{1E}^{\tilde{c}},F_{2E}^{\tilde{c}}$ are defined by:
$F_1^{\tilde{c}}(e_1)=\{a\} \quad F_1^{\tilde{c}}(e_2)=\{b\}$,
$F_2^{\tilde{c}}(e_1)=\{c\} \quad F_2^{\tilde{c}}(e_2)=\{c\}$.

Theorem 4.3. A soft set A_E is soft supra strongly $\tilde{I}g$-open in a supra soft topological space (X,μ,E) if and only if $F_E\setminus B_E\tilde{\sqsubseteq}int^s(cl^s(A_E))$ for some $B_E\in\tilde{I}$, whenever $F_E\tilde{\sqsubseteq}A_E$ and F_E is supra closed soft in (X,μ,E).

Proof. $(\Rightarrow)$ Suppose that $F_E\tilde{\sqsubseteq}A_E$ and F_E is supra closed soft. We have $A_E^{\tilde{c}}\tilde{\sqsubseteq}F_E^{\tilde{c}},A_E^{\tilde{c}}$ is soft supra strongly $\tilde{I}g$-closed and $F_E^{\tilde{c}}\in\mu$. By assumption, $cl^s(int^s((A_E^{\tilde{c}})))\setminus F_E^{\tilde{c}}\in\tilde{I}$. Hence, $cl^s(int^s((A_E^{\tilde{c}})))\tilde{\sqsubseteq}F_E^{\tilde{c}}\tilde{\cup}B_E$ for some $B_E\in\tilde{I}$. So, $(F_E^{\tilde{c}}\tilde{\cup}B_E)^{\tilde{c}}\tilde{\sqsubseteq}[cl^s(int^s((A_E^{\tilde{c}})))]^{\tilde{c}} = int^s(cl^s(A_E))$ and therefore,

$$F_E\setminus B_E=F_E\tilde{\cap}B_E^{\tilde{c}}\tilde{\sqsubseteq}int^s(cl^s(A_E)).$$

$(\Leftarrow)$ Conversely, assume that A_E be a supra closed soft set. We want to prove that A_E is a soft supra strongly $\tilde{I}g$-open. It is sufficient to prove that, $A_E^{\tilde{c}}$ is soft supra strongly $\tilde{I}g$-closed. So, let $A_E^{\tilde{c}}\tilde{\subseteq}G_E$ such that $G_E\in\mu$. Hence, $G_E^{\tilde{c}}\tilde{\subseteq}A_E$. By assumption, $G_E^{\tilde{c}}\setminus\tilde{I}_E\tilde{\subseteq}int^s(cl^s(A_E))=[cl^s(cl^s(A_E))^{\tilde{c}}]^{\tilde{c}}$ for some $\tilde{I}_E\in\tilde{I}$. Hence,
$cl^s(int^s(A_E^{\tilde{c}}))=cl^s(cl^s(A_E))^{\tilde{c}}\tilde{\subseteq}[G_E^{\tilde{c}}\setminus\tilde{I}_E]^{\tilde{c}}=G_E\tilde{\cup}\tilde{I}_E$. Thus, $cl^s(int^s((A_E^{\tilde{c}})))\setminus G_E\tilde{\subseteq}[G_E\tilde{\cup}\tilde{I}_E]\setminus G_E=[G_E\tilde{\cup}\tilde{I}_E]\tilde{\cap}G_E^{\tilde{c}}=\tilde{I}_E\tilde{\cap}G_E^{\tilde{c}}\tilde{\subseteq}\tilde{I}_E\in\tilde{I}$. This shows that, $cl^s(int^s(A_E^{\tilde{c}}))\setminus G_E\in\tilde{I}$. Therefore, $A_E^{\tilde{c}}$ is soft supra strongly $\tilde{I}g$-closed and hence A_E is soft supra strongly $\tilde{I}g$-open. This completes the proof.

Theorem 4.4. Every supra open soft set is a soft supra strongly $\tilde{I}g$-open.

Proof. Let A_E be supra open soft set such that $F_E\tilde{\sqsubseteq}A_E$ and $A_E\in\mu^{\tilde{c}}$. Then, $F_E\tilde{\sqsubseteq}A_E\tilde{\sqsubseteq}int^s(A_E)$ $\tilde{\sqsubseteq}int^s(cl^s(A_E))$. Hence, $F_E\setminus int^s(cl^s(A_E))=\tilde{\varphi}\in\tilde{I}$. Therefore, A_E is soft supra strongly $\tilde{I}g$-open.

Remark 4.5. The converse of the above theorem is not true in general as shall shown in the following example.

Example 4.6. In Example 3.2, the soft set Q_E is soft supra strongly $\tilde{I}g$-open but not supra open soft set where:
$Q(e_1)=\varphi \quad Q(e_2)=\{b\}$.

Proposition 4.7. Every soft supra $\tilde{I}g$-open set is soft supra strongly $\tilde{I}g$-open.

Proof. Obvious from Theorem 3.9.

Remark 4.8. The converse of the above theorem is not true in general, as shown in the following example.

Example 4.9. In Example 3.5, the soft set Z_E is soft supra strongly $\tilde{I}g$-open but not soft supra $\tilde{I}g$-open, where:
$Z(e_1) = X \quad Z(e_2) = \{a\}$.

Remark 4.10. The soft intersection (resp. union) of two soft supra strongly $\tilde{I}g$-open sets need not to be soft supra strongly $\tilde{I}g$-open as shown by the following examples.

Examples 4.11.

(1)In Examples 3.16 (1), the soft sets $(F_1,E),(F_2,E)$ are soft supra strongly $\tilde{I}g$-open in (X,μ,E), where:
$F_1(e_1) = X \quad F_1(e_2) = \varphi$,
$F_2(e_1) = \varphi \quad F_2(e_2) = \{b\}$.
But, their soft union $(F_1,E)\tilde{\cup}(F_2,E) = (S,E)$ where:
$S(e_1) = X \quad S(e_2) = \{b\}$ is not soft supra strongly $\tilde{I}g$-open.

(2)In Examples 3.16 (2), the soft sets $(H_1,E),(H_2,E)$ are soft supra strongly $\tilde{I}g$-open in (X,μ,E), where:
$H_1(e_1) = \{a,b,c\} \quad H_1(e_2) = \{a,b,c\}$,
$H_2(e_1) = \{b,c,d\} \quad H_2(e_2) = \{b,c,d\}$.
But, their soft intersection $(H_1,E)\tilde{\cap}(H_2,E) = (W,E)$ where:
$W(e_1) = \{b,c\} \quad W(e_2) = \{b,c\}$ is not soft supra strongly $\tilde{I}g$-open.

Theorem 4.12. If F_E is soft supra strongly $\tilde{I}g$-open in a supra soft topological space (X,μ,E) and $int^s(cl^s(F_E))\tilde{\sqsubseteq}G_E\tilde{\sqsubseteq}F_E$, then G_E is soft supra strongly $\tilde{I}g$-open.

Proof. Let $H_E\tilde{\sqsubseteq}G_E$ and $H_E \in \mu^{\tilde{c}}$. Then, $H_E\tilde{\sqsubseteq}F_E$. Since F_E is soft supra strongly $\tilde{I}g$-open, then $G_E \setminus int^s(cl^s(H_E))\tilde{\sqsubseteq}F_E \setminus int^s(cl^s(H_E)) \in \tilde{I}$. It follows that, $G_E \setminus int^s(cl^s(H_E)) \in \tilde{I}$. Therefore, G_E is soft supra strongly $\tilde{I}g$-open.

Proposition 4.13. If a soft subset F_E of a supra soft topological space (X,μ,E) is supra closed soft, then it is soft supra strongly $\tilde{I}g$-open if and only if it is soft $\tilde{I}g$-open.

Proof. Immediate.

Theorem 4.14. A soft set A_E is soft supra strongly $\tilde{I}g$-closed in a supra soft topological space (X,μ,E) if and only if $cl^s(int^s(A_E)) \setminus A_E$ is soft supra strongly $\tilde{I}g$-open.

Proof. $(\Rightarrow)$ Let $F_E\tilde{\subseteq}cl^s(int^s(A_E)) \setminus A_E$ and F_E is a supra closed soft set, then $F_E \in \tilde{I}$ from Theorem 3.13. Hence, there exists $I_E \in \tilde{I}$ such that $F_E \setminus I_E = \tilde{\varphi}$. Thus, $F_E \setminus I_E = \tilde{\varphi}\tilde{\subseteq}int^s(cl^s[cl^s(int^s(A_E)) \setminus A_E])$. Therefore, $cl^s(int^s(A_E)) \setminus A_E$ is a soft supra strongly $\tilde{I}g$-open from Theorem 4.3.

$(\Leftarrow)$ Let $A_E\tilde{\subseteq}G_E$ such that $G_E \in \mu$. Then, $cl^s(int^s(A_E))\tilde{\cap}G_E^{\tilde{c}}\tilde{\subseteq}cl^s(int^s(A_E))\tilde{\cap}A_E^{\tilde{c}} = cl^s(int^s(A_E)) \setminus A_E$. By hypothesis, $[cl^s(int^s(A_E))\tilde{\cap}A_E^{\tilde{c}}] \setminus I_E\tilde{\subseteq}int^s(cl^s[cl^s(int^s(A_E)) \setminus A_E]) = \tilde{\varphi}$, for some $I_E \in \tilde{I}$ from Theorem 4.3. This implies that, $cl^s(int^s(A_E))\tilde{\cap}G_E^{\tilde{c}}\tilde{\subseteq}I_E \in \tilde{I}$. Therefore, $cl^s(int^s(A_E)) \setminus G_E \in \tilde{I}$. Thus, A_E is a soft supra strongly $\tilde{I}g$-closed.

5 Conclusion

In this paper, the notions of soft supra strongly $\tilde{I}g$-closed sets and soft supra strongly $\tilde{I}g$-open sets have been introduced and investigated. In future, the generalization of these concepts to supra fuzzy soft topological spaces will be introduced and the future research will be undertaken in this direction.

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