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Integral Formulas Involving Product of General Class of Polynomials and Generalized Bessel Function

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Abstract: The paper presents two new generalized integral formulae involving product of generalized Bessel function of the first kind $w_v(z)$ and general class of polynomials $S_n^m[x]$ which are presented in terms of the generalized Wright hypergeometric function. Some interesting special cases of the main results are also considered. The results presented here are of general character and easily reducible to new and known integral formulae. The results are obtained with the help of an interesting integral due to Oberhettinger.

Keywords: Gamma function, Generalized hypergeometric function, Generalized (Wright) hypergeometric functions, generalized Bessel function of the first kind, general class of polynomials and Oberhettinger integral formula

1 Introduction

In view of importance of the Bessel function a large number of integral formulae involving this function have been developed by many authors. For example, Choi and Agarwal [8] derived unified integrals involving Bessel functions. Further, Ali [5] gave unified integrals associated the hypergeometric function. Recently, many useful integral formulae associated with the generalized Bessel functions have been studied by Agarwal [1]-[3], Agarwal *et al.* [4] and Choi and Agarwal [7].
Many integral formulae involving products of Bessel functions have been developed and play an important role in several physical problems. In fact, Bessel functions are associated with a wide range of problems in diverse areas of mathematics. These connections of Bessel functions with various other research areas have led many researchers to the field of special functions. Among many properties of Bessel functions, they also have investigated some possible extensions of the Bessel functions.
In this paper, two generalized integral formulae have been established involving product of generalized Bessel function of the first kind $w_v(z)$ and general class of polynomials $S_n^m[x]$, in terms of the generalized Wright hypergeometric function.

For this we recall following known functions.
The general class of polynomials $S_n^m[x]$ defined by (cf. [13]):

$$S_n^m[x] = \sum_{k=0}^{[\frac{n}{m}]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k \qquad (n = 0, 1, 2, \dots), \tag{1}$$

where m is an arbitrary positive integer and the coefficient $A_{n,k}(n,k \geq 0)$ are arbitrary constants, real or complex. The polynomial family $S_n^m[x]$ gives a number of known polynomials as its special cases on suitably specializing the coefficients $A_{n,k}$.
A useful generalization $w_v(z)$ of the Bessel function has been introduced and studied in [6]. The generalized Bessel function of the first kind, $w_v(z)$ is defined for $z \in \mathbb{C} \setminus \{0\}$ and $b, c, v \in \mathbb{C}$ with $\Re(v) > -1$ by the following series ([12]):

$$w_v(z) = \sum_{l=0}^{\infty} \frac{(-1)^l c^l \left(\frac{z}{2}\right)^{v+2l}}{l!\,\Gamma\left(v + l + \frac{1+b}{2}\right)}, \tag{2}$$

where $\Gamma(z)$ is the Gamma function [14] and $\mathbb{C}$ denotes set of complex numbers.

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A unification of the generalized hypergeometric series ${}_pF_q\,(.)$ is due to Wright [16]-[18] and Fox [10] who gave the generalized (Wright) hypergeometric function [15]:

$$
{}_p\Psi_q\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_p,A_p)\ ;z\\(\beta_1,B_1),\ldots,(\beta_q,B_q)\ ;\end{matrix}\right]=
$$

$$
\sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma\,(\alpha_j+A_jk)}{\prod_{j=1}^{q}\Gamma\,(\beta_j+B_jk)}\frac{z^k}{k!}\,, \tag{3}
$$

where the coefficients $A_1,\ldots,A_p$ and $B_1,\ldots,B_q$ are real positive numbers such that

$$
1+\sum_{j=1}^{q}B_j-\sum_{j=1}^{p}A_j\geqq 0. \tag{4}
$$

A special case of (3) is

$$
{}_p\Psi_q\left[\begin{matrix}(\alpha_1,1),\ldots,(\alpha_p,1)\ ;z\\(\beta_1,1),\ldots,(\beta_q,1)\ ;\end{matrix}\right]=
$$

$$
\frac{\prod_{j=1}^{p}\Gamma\,(\alpha_j)}{\prod_{j=1}^{q}\Gamma\,(\beta_j)}\,{}_pF_q\left[\begin{matrix}\alpha_1,\ldots,\alpha_p\ ;z\\\beta_1,\ldots,\beta_q\ ;\end{matrix}\right], \tag{5}
$$

where ${}_pF_q(.)$ is the generalized hypergeometric series [12], defined as:

$$
{}_pF_q\left[\begin{matrix}\alpha_1,\ldots,\alpha_p\ ;z\\\beta_1,\ldots,\beta_q\ ;\end{matrix}\right]=\sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)_n\,z^n}{(\beta_1)_n\cdots(\beta_q)_n n!}\,, \tag{6}
$$

where $(\alpha)_n$ is the Pochhammer symbol defined (for $\lambda\in\mathbb{C}$) by [12]:

$$
(\alpha)_n=\begin{cases}1, & (n=0)\\ \alpha(\alpha+1)\ldots(\alpha+n-1), & (n\in\mathbb{N}=\{1,2,\cdots\})\end{cases}
$$

$$
=\frac{\Gamma\,(\alpha+n)}{\Gamma\,(\alpha)}(\alpha\in\mathbb{C}\setminus\mathbb{Z}_0^-) \tag{7}
$$

and $\mathbb{Z}_0^-$ denotes the set of non positive integers.
We also take the following integral formula given by Oberhettinger [11]:

$$
\int_0^{\infty}x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}dx
$$

$$
=2\,\lambda a^{-\lambda}\left(\frac{a}{2}\right)^{\mu}\frac{\Gamma\,(2\mu)\,\Gamma\,(\lambda-\mu)}{\Gamma\,(1+\lambda+\mu)}, \tag{8}
$$

provided $0<\Re(\mu)<\Re(\lambda)$.

2 Main Results

In this part, two generalized integral formulae involving product of generalized Bessel function of the first kind $w_v(z)$ and general class of polynomials $S_n^m[x]$ are established, which are expressed in terms of the generalized Wright hypergeometric function.

Theorem 2.1 Let $\lambda,b,c,v,\mu\in\mathbb{C}$ with $\Re\,(v)>-1$, $x>0,n,k\geq 0$, and $0<\Re\,(\mu)<\Re\,(\lambda+v)$. Then we obtain

$$
\int_0^{\infty}x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}S_n^m\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right)
$$

$$
\times w_v\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right)dx
$$

$$
=\sum_{k=0}^{[\frac{n}{m}]}\frac{(-n)_{mk}}{k!}A_{n,k}\,y^{k+v}\,2^{1-\mu-v}a^{\mu-\lambda-k-v}\Gamma\,(2\mu)
$$

$$
\times{}_2\Psi_3\left[\begin{matrix}(\lambda+k+v-\mu,2), & (\lambda+k+v+1,2)\\ (v+\frac{1+b}{2},1), & (1+\mu+\lambda+k+v,2),\end{matrix}\right.
$$

$$
\left.\begin{matrix} ; & \\ (\lambda+k+v,2); & \dfrac{-y^2c}{4a^2}\end{matrix}\right]. \tag{9}
$$

Theorem 2.2 Let $\lambda,b,c,v,\mu\in\mathbb{C}$ with $\Re\,(v)>-1$, $x>0,n,k\geq 0$, and $0<\Re\,(\mu)<\Re\,(\lambda+v)$. Then we have

$$
\int_0^{\infty}x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}S_n^m\left(\frac{xy}{x+a+\sqrt{x^2+2ax}}\right)
$$

$$
\times w_v\left(\frac{xy}{x+a+\sqrt{x^2+2ax}}\right)dx
$$

$$
=\sum_{k=0}^{[\frac{n}{m}]}\frac{(-n)_{mk}}{k!}A_{n,k}\,\,y^{k+v}\,2^{1-\mu-k-2v}a^{\mu-\lambda}\Gamma\,(\lambda-\mu)
$$

$$
\times{}_2\Psi_3\left[\begin{matrix}(2\mu+2k+2v,4), & (\lambda+k+v+1,2)\\ (v+\frac{1+b}{2},1), & (1+\mu+\lambda+2k+2v,4),\end{matrix}\right.
$$

$$
\left.\begin{matrix} ; & \\ (\lambda+k+v,2); & \dfrac{-y^2c}{16}\end{matrix}\right]. \tag{10}
$$

Proof. By making use of product of (1) and (2) in the integrand of (9) and interchanging the order of integral sign and summation, which is verified by uniform convergence of the series, we find

$$
\int_0^{\infty}x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}S_n^m\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right)
$$

$$
\times w_v\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right)dx
$$

$$= \sum_{k=0}^{[\frac{n}{m}]} \frac{(-n)_{mk}}{k!} A_{n,k} y^k \cdot \sum_{l=0}^{\infty} \frac{(-1)^l c^l}{l!\,\Gamma\left(v+l+\frac{1+b}{2}\right)} \left(\frac{y}{2}\right)^{v+2l}$$

$$\times \int_0^\infty x^{\mu-1} (x+a+\sqrt{x^2+2ax})^{-\lambda-k-v-2l} dx. \quad (11)$$

By conditions of Theorem 2.1

$$0 < \Re(\mu) < R(\lambda+v) < R(\lambda+v+k+2l),\ \Re(v) > -1,$$

we use the integral formula (8) to the integral in (11) and get following expression:

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda} S_n^m\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right)$$

$$\times w_v\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right) dx$$

$$= \sum_{k=0}^{[\frac{n}{m}]} \frac{(-n)_{mk}}{k!} A_{n,k}\, y^{k+v} 2^{1-\mu-v} a^{\mu-\lambda-k-v} \Gamma(2\mu)$$

$$\times \sum_{l=0}^{\infty} \frac{\Gamma(\lambda+k+v-\mu+2l)}{l!\,\Gamma\left(v+l+\frac{1+b}{2}\right)\Gamma(1+\mu+\lambda+k+v+2l)}$$

$$\times \frac{\Gamma(\lambda+k+v+1+2l)}{\Gamma(\lambda+k+v+2l)} \left(\frac{-cy^2}{4a^2}\right)^{v+2l}.$$

Now we use (3) to get the desired formula (9).
By similar manner as in proof of Theorem 2.1, we can prove the integral formula (10).

3 Special cases

In this section, we consider some special cases of the main results derived in the preceding section.

For example, if we set $n = 0$, then we observe that the general class of polynomials $S_n^m[x]$ reduces to unity, i.e. $S_0^m[x] \to 1$, and we get the following known results due to Choi *et al.* [9]:

Corollary 3.1 The following integral holds

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}$$

$$\times w_v\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right) dx$$

$$= y^v 2^{1-\mu-v} a^{\mu-\lambda-v} \Gamma(2\mu)$$

$$\times {}_2\Psi_3\left[\begin{matrix}(\lambda+v-\mu,2), & (\lambda+v+1,2) & ; \\ (v+\frac{1+b}{2},1), & (1+\mu+\lambda+v,2), & (\lambda+v,2);\end{matrix}\ \frac{-y^2c}{4a^2}\right], \quad (12)$$

provided the conditions of Theorem 2.1 be satisfied.

Corollary 3.2 Suppose the conditions of Theorem 2.2 be satisfied, then we have

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}$$

$$\times w_v\left(\frac{xy}{x+a+\sqrt{x^2+2ax}}\right) dx$$

$$= y^v 2^{1-2v-\mu} a^{\mu-\lambda} \Gamma(\lambda-\mu)$$

$$\times {}_2\Psi_3\left[\begin{matrix}(2\mu+2v,4), & (\lambda+v+1,2) & ; \\ (v+\frac{1+b}{2},1), & (1+\mu+\lambda+2v,4), & (\lambda+v,2);\end{matrix}\ \frac{-y^2c}{16}\right]. \quad (13)$$

Indeed, for $b = c = 1$, the generalized Bessel function defined by (2), reduces to the well known Bessel function of the first kind J_v [6], defined for $z \in \mathbb{C}\setminus\{0\}$ and $v \in \mathbb{C}$ with $\Re(l) > -1$ as:

$$J_v(z) = \sum_{l=0}^{\infty} \frac{(-1)^l \left(\frac{z}{2}\right)^{v+2l}}{l!\,\Gamma(v+l+1)}.$$

Hence, on setting $b = c = 1$, in above corollaries, we obtain the following known results due to Choi and Agarwal [7]:

Corollary 3.3 The following integral holds true

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}$$

$$\times J_v\left(\frac{y}{x+a+\sqrt{x^2+2ax}}\right) dx$$

$$= y^v 2^{1-\mu-v} a^{\mu-\lambda-v} \Gamma(2\mu)$$

$$\times {}_2\Psi_3\left[\begin{matrix}(\lambda+v-\mu,4), & (\lambda+v+1,2) & ; \\ (v+1,1), & (1+\mu+\lambda+v,2), & (\lambda+v,2);\end{matrix}\ \frac{-y^2c}{4a^2}\right]. \quad (14)$$

Corollary 3.4 Let the conditions of Theorem 2.2 be satisfied, then we have

$$\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda}$$

$$\times J_v\left(\frac{xy}{x+a+\sqrt{x^2+2ax}}\right) dx$$

$$= y^v 2^{1-\mu-2v} a^{\mu-\lambda} \Gamma(\lambda-\mu)$$

$$\times {}_2\Psi_3\left[\begin{matrix}(2\mu+2v,4), & (\lambda+v+1,2) & ; \\ (v+1,1), & (1+\mu+\lambda+2v,4), & (\lambda+v,2);\end{matrix}\ \frac{-y^2}{16}\right]. \quad (15)$$

Further, the polynomial family $S_n^m[x]$ gives a number of known polynomials as its special cases on suitably specializing the coefficients $A_{n,k}$. To illustrate this, we give one more example.
If we set $m=2$ and $A_{n,k}=(-1)^k$, then the general class of polynomials

$$S_n^2[x] \to x^{n/2} H_n\left(\frac{1}{2\sqrt{x}}\right), \tag{16}$$

where $H_n(x)$ denotes the well known Hermite polynomials, and defined by

$$H_n(x)=\sum_{k=0}^{[n/2]}(-1)^k\frac{n!}{k!\,(n-2k)!}(2x)^{n-2k}. \tag{17}$$

Now, on putting $m=2, A_{n,k}=(-1)^k$ and taking relation (16) into account, Theorems 2.1 and 2.2 yields to the following results involving the Hermite polynomial and the generalized Bessel function:

Corollary 3.5 Let $\lambda, b, c, v, \mu \in \mathbb{C}$ with $\Re(v)>-1$, $x>0, n, k \geq 0$, and $0<\Re(\mu)<\Re(\lambda+v)$. Then we obtain

$$\begin{aligned}
&\int_0^\infty x^{\mu-1}(x+a+\sqrt{x^2+2ax})^{-\lambda-\frac{n}{2}}\,y^{\frac{n}{2}}\\
&\times H_n\left(\frac{1}{2\sqrt{X}}\right).w_v(X)\,dx\\
&=\sum_{k=0}^{[\frac{n}{2}]}\frac{(-n)_{2k}}{k!}(-1)^k y^{k+v}2^{1-\mu-v}a^{\mu-\lambda-k-v}\Gamma(2\mu)\\
&\times{}_2\Psi_3\left[\begin{matrix}(\lambda+k+v-\mu,2), & (\lambda+k+v+1,2)\\ (v+\frac{1+b}{2},1), & (1+\mu+\lambda+k+v,2),\end{matrix}\right.\\
&\left.\begin{matrix}; \\ (\lambda+k+v,2);\end{matrix}\ \frac{-y^2c}{4a^2}\right],
\end{aligned} \tag{18}$$

where X is defined as:

$$X=\frac{y}{x+a+\sqrt{x^2+2ax}}\,.$$

Corollary 3.6 Let $\lambda, b, c, v, \mu \in \mathbb{C}$ with $\Re(v)>-1$, $x>0, n, k \geq 0$, and $0<\Re(\mu)<\Re(\lambda+v)$. Then the following integral holds:

$$\begin{aligned}
&\int_0^\infty x^{\mu-1+\frac{n}{2}}(x+a+\sqrt{x^2+2ax})^{-\lambda-\frac{n}{2}}\,y^{\frac{n}{2}}\\
&\times H_n\left(\frac{1}{2\sqrt{Y}}\right)\,w_v(Y)\,dx\\
&=\sum_{k=0}^{[\frac{n}{2}]}\frac{(-n)_{2k}}{k!}\,(-1)^n y^{k+v}2^{1-\mu-k-2v}a^{\mu-\lambda}\Gamma(\lambda-\mu)\\
&\times{}_2\Psi_3\left[\begin{matrix}(2\mu+2k+2v,4), & (\lambda+k+v+1,2)\\ (v+\frac{1+b}{2},1), & (1+\mu+\lambda+2k+2v,4),\end{matrix}\right.\\
&\left.\begin{matrix}; \\ (\lambda+k+v,2);\end{matrix}\ \frac{-y^2c}{16}\right],
\end{aligned} \tag{19}$$

where

$$Y=\frac{xy}{x+a+\sqrt{x^2+2ax}}.$$

4 Concluding remarks

Various Bessel functions, trigonometric functions and hyperbolic functions are particular cases of generalized Bessel function defined by (2). Therefore, we observe that our main results can lead to yield numerous other interesting integrals involving various Bessel functions and trigonometric functions by suitable specializations of arbitrary parameters in the theorems. Further, on giving suitable special values to the coefficient $A_{n,k}$, the general class of polynomials give many known classical orthogonal polynomials as its particular cases. These include Hermite, Laguerre, Jacobi, the Konhauser polynomials and so on.

We conclude with the remark that, using our results one can find numerous other interesting integrals involving various Bessel functions, trigonometric functions and orthogonal polynomials by the suitable specializations of arbitrary sequences in the theorems.

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References

[1] P. Agarwal, On a new unified integral involving hypergeometric functions, Advances in Computational Mathematics and its Applications 2(1) (2012), 239-242.
[2] P. Agarwal, On new unified integrals involving Appell series, Advances in Mechanical Engineering and its Applications 2(1) (2012), 115-120.
[3] P. Agarwal, Certain multiple integral relations involving generalized Mellin-Barnes type of contour integral, Acta Univ. Apulensis Math. Inform. 33 (2013), 257-268.
[4] P. Agarwal, S. Jain, S. Agarwal, and M. Nagpal, On a new class of integrals involving Bessel functions of the first kind, Commun. Numer. Anal., 2014 (2014), Article ID cna-00216, 7 pp.
[5] S. Ali, On some new unified integrals, Advances in Computational Mathematics and its Applications, 1(3) (2012), 151-153.

[6] A. Baricz, Geometric properties of generalized Bessel functions of complex order, Mathematica 48(71)(1) (2006), 13-18.

[7] J. Choi and P. Agarwal, Certain unified integrals associated with Bessel functions, Bound. Value Probl., 1(2013), 1-9.

[8] J. Choi, P. Agarwal, Certain unified integrals involving a product of Bessel functions, Honam Mathematical J., 35(4) (2013), 667-677.

[9] J. Choi, P. Agarwal, S. Mathur, S. D. Purohit, Certain new integral formulas involving the generalized Bessel functions, Bull. Korean Math. Soc. 51(4) (2014), 995-1003.

[10] C. Fox, The asymptotic expansion of generalized hypergeometric functions, Proc. London Math. Soc, 27(2) (1928), 389-400.

[11] F. Oberhettinger, Tables of Mellin Transforms, Springer-Verlag, New York, 1974.

[12] F. W. L. Olver, D. W. Lozier, R. F. Boisvert, C. W. Clark, NIST Handbook of Mathematical Functions, Cambridge University Press, (2010).

[13] H. M. Srivastava, A contour integral involving Fox's H-function, Indian J. Math. 14 (1972), 1-6.

[14] H. M. Srivastava, J. Choi, Zeta and q-Zeta Functions and Associated Series and Integrals, Elsevier Science Publishers, Amsterdam, London and New York, (2012).

[15] H. M. Srivastava, P. W. Karlsson, Multiple Gaussian Hypergeometric Series, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane, and Toronto, (1985).

[16] E. M. Wright, The asymptotic expansion of the generalized hypergeometric functions, J. London Math. Soc, 10 (1935), 286-293.

[17] E. M. Wright, The asymptotic expansion of integral functions defined by Taylor series, Philos. Trans. Roy. Soc., London, A 238 (1940), 423-451.

[18] E. M. Wright, The asymptotic expansion of the generalized hypergeometric function II, Proc. London Math., Soc, 46(2) (1940), 389-408.

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