Applied Mathematics & Information Sciences
An International Journal

The Non-equidistant New Information Optimizing MGM(1,n) Based on a Step by Step Optimum Constructing Background Value

Ruibiao Zou

College of Sciences, Hunan Agriculture University, Changsha, 410128, China

Abstract: Grey system is a theory which studies poor information specially, and it possesses wide suitability. Applying a step by step optimum new information modeling method to construct new information background value of multi-variable non-equidistance new information Grey model MGM(1,n), taking the mth component of each variable as initial value of response function, taking the mean relative error as objective function, and taking the modified values of response function initial value as design variables, the multi-variable non-equidistance new information Grey model MGM(1,n) was built. The proposed model can be used to build model in non-equal interval and equal interval time series. It enlarges the scope of application and has high precision and easy to use. Example validates the practicability and reliability of the proposed model.

Keywords: Multivariable, background value, non-equidistance sequence, a step by step optimum modeling, new information, optimizing, non-equidistance MGM(1,n), least square method.

1. Introduction

The theory of Grey system is the study of the grey system analysis, modeling, prediction, decision making and control theory. Gray model is an important content of the grey system theory, In the search for laws between data, it makes up for the lack of available data mining method, and provides a new scientific method for data mining.

Since professor Deng Ju Long bring about the grey system theory in 1982, gray model is widely applied in many areas [1,2]. Grey model has more types, mainly including GM (1,1), GM (1,N), MGM (1,N) etc. where GM (1,1) has been used widely and researched deeply , GM(1,N) Can only be used for the qualitative analysis and cannot be used to predict. Being an extension of GM (1,1) model in case of n variable, MGM(1,N) model is neither a simple combination of GM (1,1) model ,also differs from GM(1,n) model.There are n differential equations contained n elements In MGM(1,N) model ,but just a single first order differential equation contained n element in GM(1,N) model, then we can find their simultaneous solution , and parameters in model MGM (1,N) can reflect the interre-

lating and interacting relationship among multiple variables. Because Study on MGM (1,N) model is much less than that on GM (1,1) model so far, studying deeply on it has important theory significance and application value. Literature [1] have corrected and established optimization MGM(1,N) model that regard the first component of the sequence as the initial conditions of grey differential equation. According to the new information priority principle of grey system theory, Literature [4] have corrected and established multivariate variables new information optimization MGM (1,N) model that regard the nth component of the sequence $X^{(1)}$ as the initial conditions of grey differential equation. Literature [4] have established multivariate new information MGM (1, N) model that regard the nth component of $X^{(1)}$ as initial conditions of the grey differential equation, and made optimal correction for the initial value and background value coefficient(background value is introduced in the form of $z_i^{(1)} = qx_i^{(1)}(k + 1) + (1 - q)x_i^{(1)}(k), (q \in [0, 1])$, but these MGM (1,N) model are equidistant model. Literature [6] built non equidistant multivariable MGM (1,N) by means of homogeneous ex-

ponential function fitting background value. However non-homogeneous exponential function is more universal, then, there are inherent defects in mechanism of the modeling. Literature [7] built non equidistant multivariable MGM (1,N). But its background value was generated by using of the mean value, so the precision of the model needs to be further improved. Literature [8] built non equidistant multivariable MGM(1,N) through non homogeneous exponential function fitting background value, improving the accuracy of the model. However the parameters only use least square method without making a response function optimization. Literature [9] analyzed the structure method of background value of multivariable grey model MGM (1,m), putting forward to utilizing rational interpolation and numerical integration of trapezoid formula and extrapolation to reconstruct background value by using of the vector valued continued fraction theory, and improving the precision of simulating and forecasting model effectively for multivariate interval MGM (1,m) model. Literature [10] proposed a gradual optimization new information equidistant GM (1,1) model by using of grey system modeling method and new information principle. Based on the GM (1,1) modeling, the model used the nth component of the original data as initial conditions of the grey differential equation. Parameters of the model were estimated through the optimization of background value and difference adjustment coefficient.

In this paper, taking the mth component of the data as the initial value of solution of grey differential equation, absorbing new information modeling and a step by step optimization method from literature [10], constructing new information background value of multivariable non equidistant grey new-information model MGM(1,n), taking minimum relative error as objective function and the correction of initial value as design variables, multivariable non-equidistance grey new information optimization model MGM(1,n) is established. The model is not only suitable for the equidistant model, as well as including non equidistance modeling. It extends the scope of application of the grey model. In addition to high precision, the model has great theoretical value and applied value.

2. Non-equidistant multivariate new information optimization model based on a step by step optimization modeling to construct new information background values

Definition 1: Suppose the sequence

$$X_i^{(0)} = [x_i^{(0)}(t_1), x_i^{(0)}(t_2), \cdots, x_i^{(0)}(t_m)],$$

where $i = 1, 2, \cdots, n$. If

$$\triangle t_j = t_i - t_{j-1} \neq cost, (2 \leq j \leq m),$$

then $X_i^{(0)}$ is called non-equidistant sequence.

Definition 2: Let $X_i^{(1)}$ denoted by

$$X_i^{(1)} = [x_i^{(1)}(t_1), x_i^{(1)}(t_2), \cdots, x_i^{(1)}(t_m)],$$

the sequence $X_i^{(1)}$ is called the first-order accumulated generating operation of non equidistant sequence $X_i^{(0)}$, if

$$x_i^{(1)}(t_1) = x_i^{(0)}(t_1), x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j)\triangle t_j,$$

where $i = 1, 2, \cdots, n, \ j = 2, 3, \cdots, m, \ \triangle t_j$ is same to Definition 1.

Set the original data matrix of multivariable

$$
\begin{aligned}
X^{(0)} &= [X_1^{(0)}, X_2^{(0)}, \cdots, X_n^{(0)}]^T \\
&= \begin{bmatrix}
x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m) \\
x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m) \\
\cdots & \cdots & \cdots & \cdots \\
x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m)
\end{bmatrix},
\end{aligned} \tag{1}
$$

where $x_i^{(0)}(t_j)(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$ is the observed values of variable $X_i^{(0)}$ in the moments of t_j, $[x_i^{(0)}(t_1), x_i^{(0)}(t_2), \cdots, x_i^{(0)}(t_m)]$ is non equidistant sequence, namely, the distance of $t_j - t_{j-1}(j = 2, 3, \cdots, m)$ is not constant.

In order to establish model, first taking the accumulated raw data, a new matrix can be obtained, namely

$$
\begin{aligned}
X^{(1)} &= [X_1^{(1)}, X_2^{(2)}, \cdots, X_n^{(1)}]^T \\
&= \begin{bmatrix}
x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m) \\
x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m) \\
\cdots & \cdots & \cdots & \cdots \\
x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m)
\end{bmatrix},
\end{aligned} \tag{2}
$$

where $x_i^{(1)}(t_j) \ (i = 1, 2, \cdots, n)$ meet the definition 2, namely,

$$
x_i^{(1)}(t_j) =
\begin{cases}
x_i^{(0)}(t_1) + \sum_{k=2}^{j} x_i^{(0)}(t_k)\Delta t_k, \ (j = 2, \cdots, m), \\
x_i^{(0)}(t_1), \quad (j = 1).
\end{cases} \tag{3}
$$

Multivariable non equidistant MGM (1, n) model is first order differential equations containing n element

$$
\begin{cases}
\frac{dx_1^{(1)}}{dt} = p_{11}x_1^{(1)} + p_{12}x_2^{(1)} + \cdots + p_{1n}x_n^{(1)} + q_1, \\
\frac{dx_2^{(1)}}{dt} = p_{21}x_1^{(1)} + p_{22}x_2^{(1)} + \cdots + p_{2n}x_n^{(1)} + q_2, \\
\cdots \cdots \\
\frac{dx_n^{(1)}}{dt} = p_{n1}x_1^{(1)} + p_{n2}x_2^{(1)} + \cdots + p_{nn}x_n^{(1)} + q_n.
\end{cases} \tag{4}
$$

Note

$$
A = \begin{bmatrix}
p_{11} & p_{12} & \cdots & p_{1n} \\
p_{21} & p_{22} & \cdots & p_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
p_{n1} & p_{n2} & \cdots & p_{nn}
\end{bmatrix}, B = \begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_n
\end{bmatrix},
$$

then (4) may be written as

$$\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + B. \tag{5}$$

According to new information priority principle of grey system theory, we regard the first component of the sequence $x_i^{(1)}(t_j)(j = 1, 2, \cdots, m)$ as initial conditions of the grey differential equation, causing that new information cannot be applied fully, If we regard mth component of the sequence $x_i^{(1)}(t_j)(j = 1, 2, \cdots, m)$ as initial conditions of the grey differential equation, causing that new information can be applied fully, in which continuous time response of (5) is

$$X^{(1)}(t) = e^{At}X^{(1)}(t_m) + A^{-1}(e^{At} - I)B, \tag{6}$$

where $e^{At} = I + \sum_{k=1}^{\infty} \frac{A^k}{k!} t^k$, I is unit matrix.

In order to obtain A and B, we integral the both side on the interval $[t_{j-1}, t_j]$,

$$x_i^{(0)}(t_j)\Delta t_j = \sum_{k=1}^{n} p_{ik} \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt + q_i\Delta t_j, \tag{7}$$

$$x_i^{(0)}(t_j) = \sum_{k=1}^{n} p_{ik} \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j} + q_i. \tag{8}$$

Let

$$\bar{z}_i^{(1)}(t_j) = \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j},$$

The traditional background value calculation formula actually use trapezoidal area $z_i^{(1)}(t_j)\Delta t_j$, thus, there is big error. But we obtain Parameter matrix $\hat{A}$ and $\hat{B}$ by means of regarding $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$ as background value calculation $x_{t_j}^{(1)}$ in the interval $[t_{j-1}, t_j]$ that is more suitable for whitening equation (4). According to exponential rule of gray forecast model and modeling theory and method [10] of a step by step optimization new information non-equidistant GM (1, 1) model, we take $x_i^{(1)}(t) = a_i e^{b_i t} + c_i$, where a_i, b_i, c_i are the undetermined coefficient. The literature [10] analyzed thought and method of a step by step optimization new information non-equidistance GM (1, 1) model and the key of modeling is whitening grey derivative $\frac{dx_i^{(1)}(t_j)}{dt}$. If we choose reasonable whitening grey derivative, accuracy of modeling will be improved. The method to obtain whitening grey derivative with the most intuitionist and easy to understand is a differential quotient instead of derivative.

$$\frac{dx_i^{(1)}(t_{j+1})}{dt} \approx \frac{x_i^{(1)}(t_{j+1}) - x_i^{(1)}(t_j)}{t_{j+1} - t_j}. \tag{9}$$

$$\frac{dx_i^{(1)}(t_j)}{dt} \approx \frac{x_i^{(1)}(t_{j+1}) - x_i^{(1)}(t_j)}{t_{j+1} - t_j}. \tag{10}$$

In fact, on the premise of X(t) being a derivative, the Lagrange mean value theorem shows, $\frac{x_i^{(1)}(t_{j+1}) - x_i^{(1)}(t_j)}{t_{j+1} - t_j}$ is a derivative value in a point of the interval (t_k, t_{k+1}), that is , derivative value can be thought as being known value, the corresponding variable values are interval gray numbers (t_j, t_{j+1}). For the introduction of grey derivative correction coefficient correction of grey derivative, we adopt grey derivative correction coefficient ρ and ξ to correct grey derivative and construct modeling $\rho_i \frac{x_i^{(1)}(t_{j+1}) - x_i^{(1)}(t_j)}{t_{j+1} - t_j}$.

Actually, we don't know $\xi_i(t_j)$ and $\rho_i(t_j)$. We take step by step optimization method, and, the steps are as follows:

1) Obtain original data $x_i^{(0)}$.

2)Take initial value of the iteration step number repeatedly $s = 0, a_{i(s)} = a_{i(0)} = 0$, so,

$$\xi_{i(s)} = \frac{e^{a_{i(s)}}}{e^{a_{i(s)}} - 1} - \frac{1}{a_{i(s)}} = \frac{1}{2},$$

$$\rho_{i(s)} = \frac{a_{i(s)}(1 + e^{-a_{i(s)}})}{2(1 - e^{-a_{i(s)}})} = 1.$$

Taking linear regression to whitening values $x_i^{(1)}(t_j) = ([1 - \xi_i]x_i^{(1)}(t_j) + \xi_i)x_i^{(1)}(t_{j+1})$, We obtain the whitening values of parameter:

$$\hat{a}_{i(s+1)} = -\rho_{i(s)} \frac{S_{i(s)xy}}{S_{i(s)xx}}, \tag{11}$$

where

$$x_{i(s)}^{(1)}(t_j) = [1 - \xi_{i(s)}x_{i(s)}^{(1)}(t_j)] + \xi_{i(s)}x_{i(s)}^{(1)}(t_{j+1}),$$

$$\bar{x}_{i(s)}^{(1)} = \frac{1}{m-1} \sum_{j=1}^{m-1} x_{i(s)}^{(1)}(t_j),$$

$$y_{i(s)}(t_j) = \frac{x_{i(s)}^{(1)}(t_{j+1}) - x_{i(s)}^{(1)}(t_j)}{t_{j+1} - t_j},$$

$$\bar{y}_{i(s)} = \frac{1}{m-1} \sum_{j=1}^{m-1} y_{i(s)}(t_j),$$

$$S_{i(s)xx} = \sum_{j=1}^{m-1} [x_{i(s)}^{(1)}(t_j) - \bar{x}_{i(s)}^{(1)}]^2,$$

$$S_{i(s)xy} = \sum_{j=1}^{m-1} [x_{i(s)}^{(1)} - \bar{x}_{i(s)}^{(1)}][y_{i(s)}(t_j) - \bar{y}_{i(s)}].$$

Then, we construct the linear regression to get index model M_{s+1}:

$$\hat{x}_{i(s)}^{(1)}(t_{j+1}) = \hat{c}_{i(s+1)}e^{-\hat{a}_{i(s+1)}(t_j - t_m)} + \hat{b}_{i(s+1)}, \tag{12}$$

where

$$
\hat{c}_{i(s+1)}
= \frac{1}{\sum_{j=1}^{m}[e^{-\hat{a}_{i(s+1)}(t_j-t_m)} - \frac{1}{m}\sum_{j=1}^{m}e^{-\hat{a}_{i(s+1)}(t_j-t_m)}]^2}
$$
$$
\times \{ \sum_{j=1}^{m}([e^{-\hat{a}_{i(s+1)}(t_j-t_m)} - \frac{1}{m}\sum_{j=1}^{m}e^{-\hat{a}_{i(s+1)}(t_j-t_m)}]
$$
$$
\times [x_{i(s)}^{(1)}(t_j) - \tfrac{1}{m}\sum_{j=1}^{m}x_{i(s)}^{(1)}(t_j)]) \}, \tag{13}
$$

$$
\hat{b}_{i(s+1)} = \tfrac{1}{m}\sum_{j=1}^{m}x_{i(s)}^{(1)}(t_j) -
$$
$$
\hat{c}_{i(s+1)}\tfrac{1}{m}\sum_{j=1}^{m}e^{-\hat{a}_{i(s+1)}(t_j-t_m)}. \tag{14}
$$

After finding out $\hat{a}_i, \hat{b}_i, \hat{c}_i, z_i^{(1)}(t_j)$ can be solved, and then get

$$
\bar{z}_i^{(1)}(t_j) = \frac{\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt}{\Delta t_j} = x_i^{(1)}(t_j).
$$

Set $p_i = (p_{i1}, p_{i2}, \cdots, p_{in}, q_i)^T, \quad (i = 1, 2, \cdots, n)$, p_i can be to achieved the value $\hat{p}_i$ by the least square method.

$$
\hat{p}_i = (\hat{p}_{i1}, \hat{p}_{i2}, \cdots, \hat{p}_{in}, \hat{q}_i)^T
$$
$$
= (L^T L)^{-1} L^T Y_i, \qquad (i = 1, 2, \cdots, n), \tag{15}
$$

where,

$$
L = \begin{bmatrix}
\bar{z}_1^{(1)}(t_2) & \bar{z}_2^{(1)}(t_2) & \cdots & \bar{z}_n^{(1)}(t_2) & 1 \\
\bar{z}_1^{(1)}(t_3) & \bar{z}_2^{(1)}(t_3) & \cdots & \bar{z}_n^{(1)}(t_3) & 1 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\bar{z}_1^{(1)}(t_m) & \bar{z}_2^{(1)}(t_m) & \cdots & \bar{z}_n^{(1)}(t_m) & 1
\end{bmatrix}, \tag{16}
$$

$$
Y_i = [x_i^{(0)}(t_2), x_i^{(0)}(t_3), \cdots, x_i^{(0)}(t_m)]^T. \tag{17}
$$

We can get discrimination value of A and B:

$$
\hat{A} = \begin{bmatrix}
\hat{p}_{11} & \hat{p}_{12} & \cdots & \hat{p}_{1n} \\
\hat{p}_{21} & \hat{p}_{22} & \cdots & \hat{p}_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
\hat{p}_{n1} & \hat{p}_{n2} & \cdots & \hat{p}_{nn}
\end{bmatrix}, \quad
\hat{B} = \begin{bmatrix}
\hat{q}_1 \\
\hat{q}_2 \\
\vdots \\
\hat{q}_n
\end{bmatrix}.
$$

Computation value of new information MGM (1, n) mode is

$$
\hat{X}_i^{(1)}(t_j) = e^{\hat{A}(t_j-t_m)} X_i^{(1)}(t_m) +
$$
$$
\hat{A}^{-1}(e^{\hat{A}(t_j-t_m)} - I)\hat{B}. \tag{18}
$$

We use the mth component of the data as initial value of the grey differential equation and modify initial value, namely, $X^{(0)}(t_m) + \beta$ instead of $X^{(0)}(t_m)$, which β is vector $\beta = [\beta_1, \beta_2, \cdots, \beta_n]^T$. The fitting values of the original data are obtained.

$$
\hat{X}_i^{(0)}(t_j)
= \begin{cases}
\lim_{\Delta t \to 0} \dfrac{X_i^{(1)}(t_1) - X_i(1)(t_1 - \Delta t)}{\Delta t}, & (j = 1), \\[2mm]
\dfrac{\hat{X}_i^{(1)}(t_j) - \hat{X}_i^{(1)}(t_{j-1})}{t_j - t_{j-1}}, & (j = 2, 3, \cdots, m).
\end{cases} \tag{19}
$$

The absolute error of ith variable is $\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)$. Relative error of ith variables is

$$
e_i(t_j) = \frac{\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)}{i^{(0)}(t_j)} \times 100.
$$

The mean value of relative error of ith variable is

$$
\frac{1}{m}\sum_{j=1}^{m} | e_i(t_j) | .
$$

The average error of all the data is

$$
\bar{e} = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} | e_i(t_j) | .
$$

Taking the average error f as objective function and as design variables, using the optimization function of Matlab 7.5 optimization method or other method for solving, all parameters are obtained.

3. Precision inspecting for the model

The inspecting means contain residual analysis, correlation degree analysis, and post-error analysis [7, 12, 13]. The displacement relative degree,

the speed related degree, the acceleration degree, and the total related degree are calculated simultaneity. These kinds of related degrees are called related degrees of C-type [14], which can be used to both of the whole analysis and the dynamic analysis. The following related degree inspection of C-type is employed in this paper.

1) To calculate the three-layer related degrees.
Displacement related degree $d^{(0)}(t_j)$:

$$
d^{(0)}(t_j) = \frac{x^{(0)}(t_j)}{\hat{x}^{(0)}(t_j)}, \tag{20}
$$

where, $j = 1, 2, \cdots, m$.
Speed related degree $d^{(1)}(t_j)$:

$$
d^{(1)}(t_j) = \frac{x^{(0)}(t_{j+1}) - x^{(0)}(t_j)}{\hat{x}^{(0)}(t_{j+1}) - \hat{x}^{(0)}(t_j)}, \tag{21}
$$

where $j = 1, 2, \cdots, m - 1$.

Acceleration related degree $d^{(2)}(t_j)$:

$$d^{(2)}(t_j) = \frac{x^{(0)}(t_{j+1}) - 2x^{(0)}(t_j) + x^{(0)}(t_{j-1})}{\hat{x}^{(0)}(t_{j+1}) - 2\hat{x}^{(0)}(t_j) + \hat{x}^{(0)}(t_{j-1})}, \quad (22)$$

where $j = 1, 2, \cdots, m-1$.

2) To calculate the three-layer related comprehensive degree at t_j:

$$D(t_1) = \frac{d^{(1)}(t_1) + d^{(0)}(t_1)}{2}, D(t_m) = d^{(0)}(t_m), \quad (23)$$

$$D(t_j) = \frac{d^{(0)}(t_j) + d^{(1)}(t_j) + d^{(2)}(t_j)}{3}, \quad (24)$$

where $j = 2, 3, \cdots, m-1$.

3) To calculate the total related degree of the model $\hat{x}^{(0)}(t_j)$:

$$D = \frac{1}{m} \sum_{j=1}^{m} D(t_j), j = 1, 2, \cdots, m. \quad (25)$$

When $0.6 < D \leq \frac{5}{3}$, the precision of the model is "Good". When $-0.30 \leq D \leq 0.60$, the precision of the model is "better". When $D < -0.30$ or $D > \frac{5}{3}$, the precision of the model is "bad" [14].

4. Model applications

Example 1: On the contacting strength calculation, principal curvature function $F(\rho)$ and the coefficient m_a, m_b, with the point contacting ellipse length and short radius a, b. Parameters are generally processed by consulting table .The data extracted from the table 1 [11].

Table1   The values of $F(\rho), m_a$, and m_b

No.	1	2	3	4	5
$F(\rho)$	0.9995	0.9990	0.9980	0.9970	0.9960
m_a	23.95	18.53	14.25	12.26	11.02
m_b	0.163	0.185	0.212	0.228	0.241
No.	6	7	8	9	10
$F(\rho)$	0.9950	0.9940	0.9930	0.9920	0.9910
m_a	10.15	9.46	8.92	8.47	8.10
m_b	0.251	0.260	0.268	0.275	0.281
No.	11	12	13	14	15
$F(\rho)$	0.9900	0.9890	0.9880	0.9870	0.9860
m_a	7.76	7.49	7.25	7.02	6.84
m_b	0.287	0.292	0.297	0.301	0.305
No.	16	17	18	19	20
$F(\rho)$	0.9850	0.9840	0.9830	0.9820	0.9810
m_a	6.64	6.47	6.33	6.19	6.06
m_b	0.310	0.314	0.317	0.321	0.325
No.	21	22	23	24	25
$F(\rho)$	0.9800	0.9790	0.9780	0.9770	
m_a	5.95	5.83	5.72	5.63	
m_b	0.328	0.332	0.335	0.338	

The coefficient m_b of the oval short radius b is noted as t_j, principal curvature function $F(\rho)$ is noted as X_1, the coefficient m_a of the oval short radius a is noted as X_2. Establishing non-equidistance new information optimum GM(1,2) model with the proposed method, the model parameters are as follows:

$$A = \begin{bmatrix} -0.3787 & 0.0243 \\ 16.4555 & -8.1528 \end{bmatrix}, B = \begin{bmatrix} 0.7945 \\ 196.8118 \end{bmatrix},$$

$$\beta = \begin{bmatrix} -0.0025205 \\ 0.070706 \end{bmatrix}.$$

Principal curvature function fitting value is

$$\hat{F}(\rho) = [1.00020, 1.00040, 0.99990, 0.99837,$$
$$0.99682, 0.99534, 0.99396, 0.99262,$$
$$0.99136, 0.99022, 0.98912, 0.98808,$$
$$0.98710, 0.98621, 0.98539, 0.98446,$$
$$0.98351, 0.98276, 0.98200, 0.98112,$$
$$0.98034, 0.97955, 0.97875, 0.97806].$$

.

The absolute error of principal curvature function is

$$q = [\ 0.6772, \ 1.4183, \ 1.8995, \ 1.3741,$$
$$0.8205, \ 0.3375, -0.0406, -0.3793,$$
$$-0.6374, -0.7824, -0.8821, -0.9236,$$
$$-0.8975, -0.7944, -0.6079, -0.5405,$$
$$-0.4899, -0.2399, -0.0003, \ 0.1195,$$
$$0.3396, \ 0.5505, \ 0.7532, \ 1.0631].$$

The relative error (percent) of principal curvature function is:

$$e = [\ 0.067753, \ 0.141972, \ 0.190329, \ 0.137826,$$
$$0.082379, \ 0.033920, -0.004080, -0.038197,$$
$$-0.064259, -0.078950, -0.089106, -0.093386,$$
$$-0.090839, -0.080488, -0.061650, -0.054871,$$
$$-0.049785, -0.024408, -0.000026, \ 0.012178,$$
$$0.034655, \ 0.056231, \ 0.077018, \ 0.108810].$$

The average value of relative error is 0.069713%.

The precision of the model is "Good". The average value of relative error of without optimization for new information model is 0.14657%. So, optimization model has very high precision.

Example 2: Refer to reference [7] of data for water absorption rate affecting the pure PA66 mechanics performance, according to mechanics performance test on PA66 samples with different water absorption rate, get the bending strength and flexural modulus of PA66 and the experimental data of the tensile strength changing with the water absorption rate. t_j is water absorption rate, $X_1^{(0)}$ is the bending strength (Mpa), $X_2^{(0)}$ is the flexural modulus (Gpa), $X_3^{(0)}$ is the tensile strength (Mpa). The data is shown in table 2.

Table2 Absorption rate's influence for mechanics performance of pure PA66

No.	1	2	3	4	5
t_j	0	0.0607	0.1071	0.1662	0.2069
$X_1^{(0)}$	83.4	84.9	84.5	84.2	84.4
$X_2^{(0)}$	2.63	2.64	2.61	2.65	2.66
$X_3^{(0)}$	84.2	84.4	86.3	84.3	81.3

No.	5	6	7	8	9
t_j	0.4344	0.5243	0.8524	0.9756	
$X_1^{(0)}$	78.4	75.4	59.5	54.1	
$X_2^{(0)}$	2.52	2.32	1.90	1.72	
$X_3^{(0)}$	74.9	75.7	73.2	66.9	

According to the method mentioned in this paper, set up the non-equidistant model MGM (1,2). The model parameters are shown as follows:

$$A = 10^4 \times \begin{bmatrix} 0.1155 & -4.2174 & 0.0172 \\ 0.0046 & -0.1662 & 0.0007 \\ -0.0291 & 1.0613 & -0.0043 \end{bmatrix},$$

$$B = \begin{bmatrix} 185.1780 \\ 6.1876 \\ 97.9652 \end{bmatrix}, \rho = \begin{bmatrix} -1.16073 \\ -0.03880 \\ 1.4395 \end{bmatrix}.$$

The fitted value of $X_3^{(0)}$ is

$$\hat{X}_3^{(0)} = [\, 87.3769, \quad 85.6852, \quad 84.1890, \\ 82.7884, \quad 81.5031, \quad 78.2813, \\ 74.7615, \quad 70.7947, \quad 67.0939].$$

The absolute error of $X_3^{(0)}$ is

$$q = [\quad 3.1769, \quad 1.2852, -2.1110, \\ -1.5116, \quad 0.2031, \quad 3.3813, \\ -0.9385, -2.4053, \quad 0.1939].$$

The relative error (percent) of $X_3^{(0)}$ is

$$e = [\quad 3.7730, \quad 1.5228, -2.4461, \\ -1.7931, \quad 0.2499, \quad 4.5144, \\ -1.2398, -3.2859, \quad 0.2899].$$

The average value of relative error is 2.1239%.

The precision of the model is "Good". The average value of relative error of without optimization for new information model is 3.6153%. So, optimization model has very high precision.

5. Conclusion

In view of multivariable non-equidistance sequence that multiple variables affect restrict mutually, we construct the multivariable non-equidistance new information optimization grey model MGM(1,n). In modeling, applying a step by step optimum new information modeling method to construct new information background value of multi-variable non-equidistance new information Grey model MGM(1,n), regarding mth component of the data as initial conditions

of the grey differential equation, taking the minimum relative error as objective function, and taking revising correction values of the initial value as design variables. The new model is not only suitable for equidistance modeling, also suitable for non-equidistant model. It enlarge the scope of application of the grey model and has high precision, easy to use and so on. Actual examples show that the model is practical and reliable with important practical and theoretical significance. It is worth using widely.

References

[1] Y. X. LUO and J. Y. LI, Proc. International Conference on Mechatronics and Automation, 4023(2009).

[2] Y. X. LUO, Wu, M. Li and A. H. Cai, Kybernetes **38**, 435 (2009).

[3] Z. Jun and J. M. Sheng, Systems Engineering Theory and Practive **17**, 109 (1997).

[4] Z. M. HE and Y. X. LUO, Proc. International Conference on Intelligent Computation Technology and Automation, 10(2009).

[5] Y. X. LUO and W. Y. Xiao, Proc. International Conference on Intelligent Computation Technology and Automation, 86(2009).

[6] F. X. WANG, Systems Engineering and Electronics **9**, 388 (2007).

[7] P. P. Xiong, Y. G. Dang and H. Zhu Hui, Control and Decision **26**, 49 (2011).

[8] P. P. Xiong, Y. G. Dang and Y. YANG, Proc. 19th Chinese Conference on Grey Systems, 277(2010).

[9] L. Z. Cui, S. F. Liu, and Z. P. Wu, Systems Engineering **26**, 47 (2008).

[10] Y. X. LUO, Systems Engineering Theory and Practice, **30**, 2254 (2010).

[11] Z. H. Han and X. H. DONG, Journal of Machine Design **25**, 18 (2008).

[12] Y. X. Luo, D. G. Liao, and Q. X. Tang, Machine tool and Hydraulics **6**, 229(2002).

[13] S. F. Liu, T. B. Guo and Y. G. Dang, Grey systems and Applications (Third Edition, in Chinese) (China Science Press, Beijing, 2004).

[14] Q. Y. Wang, Uncertainty mathematical model of forecast and decision-making (in Chinese)(Metallurgical industry Press, Beijing, 2001).
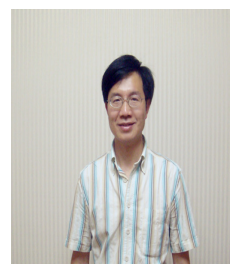
Ruibiao Zou is with college of science, Hunan Agriculture University, Changsha in China. He received the BS degree in 1987 and MS degree in 2005 from Hunan normal University. His current research interests involve grey system theory and its application, fractal theory, system engineering theory and practise and the design and implementation of teaching management system. He published over 20 papers.