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Computing Technique for Recruitment Process via Nano Topology

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Abstract: The objective of this study is to develop a new multigranulation nano topological model, called Multi*-Granular nano topology. We study this model from three aspects, which are lower, upper approximation and their properties, to incorporate the types of Multi*-granular nano topology based on the intrinsic configuration of sets and then the conjunction and controversy were delibrated in the midst of Nano topological space and among the two types of Multi-granular nano topological spaces. Further more, to emphasize our Multi*-granular nano topological model some examples are considered, which are efficacious for implementing this hypothesis in practical applications.

Keywords: Multi*-Granular nano topology, Multi*-lower approximation, Multi*-upper approximation, Multi*-boundary, Nano-accuracy.

1 Introduction

Lellis Thivagar et al [2] interjected a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. But certain nano terms are satisfied simply to mean ”very small”. It originates from the Greek word ’Nanos’ which means ’dwarf’ in its modern scientific sense, an order to magnititude - one billionth of something. Nano car is an example. The topology recommended here is named so because of its size, since it has atmost five elements in it. In view of granular computing [7], nano topological space is based on a single granulation (only one indiscreniblity relation). But this concept of Multi*-granular nano topological model where the set approximations are defined by Multiple indiscreniblity relations on the universe. More over, several significant measures were found, and their conjunction and controversy were delibrated precisely among nano topological space and between two types of MGNT, and also some examples are considered which are beneficial in solving practical problems.

2 Preliminaries

The following recalls necessary concepts and preliminaries required in the sequel of our work.

Definition 2.1:[2]: Let $\mathscr{U}$ be a non-empty finite set of objects called the universe $\mathscr{R}$ be an equivalence relation on $\mathscr{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathscr{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathscr{U}$.

(i)The Lower appproximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup_{x\in\mathscr{U}} \{R(x) : R(x) \subseteq X\} \right\}$, where R(x) denotes the equivalence class determined by x.

(ii)The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{ \bigcup_{X\in\mathscr{U}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.

(iii)The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by

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$B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2:[2]: Let $\mathscr{U}$ be the universe, R be an equivalence relation on $\mathscr{U}$ and $\tau_R(X) = \{\mathscr{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathscr{U}$. $\tau_R(X)$ satisfies the following axioms:

(i) $\mathscr{U}$ and $\emptyset \in \tau_R(X)$
(ii) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathscr{U}$ called as the nano topology on $\mathscr{U}$ with respect to X. We call $\{\mathscr{U}, \tau_R(X)\}$ as the nano topological space.

Definition 2.3:[3]: Let $\mathscr{U}$ be the universe and P, Q be any two equivalence relations on $\mathscr{U}$ and $\tau_{P+Q}(X) = \{\mathscr{U}, \emptyset, L_{P+Q}(X), U_{P+Q}(X), B_{P+Q}(X)\}$ where $X \subseteq \mathscr{U}$.

(i) $L_{P+Q}(X) = \bigcup_{x \in \mathscr{U}} \{[x] : P(x) \subseteq X \text{ or } Q(x) \subseteq X\}$.
(ii) $U_{P+Q}(X) = \bigcup_{x \in \mathscr{U}} \{[x] : P(x) \cap X \neq \emptyset \text{ and } Q(x) \cap X \neq \emptyset\}$.
(iii) $B_{P+Q}(X) = U_{P+Q}(X) - L_{P+Q}(X)$.

That is, $\tau_{P+Q}(X)$ forms a topology on $\mathscr{U}$ called as the Multi-granular nano topology on $\mathscr{U}$ with respect to X. We call $(\mathscr{U}, \tau_{P+Q}(X))$ as the Multi-granular nano topological space.

Definition 2.4:[3]: Let $\mathscr{U}$ be the universe. $\mathscr{U}/P$, $\mathscr{U}/Q$ be any two equivalence relations defined on $\mathscr{U}$. Then union of two equivalence relations is defined $\mathscr{U}/P \cup Q$ as

$$\mathscr{U}/P \cup Q = \{P_i \cap Q_j : P_i \in \mathscr{U}/P, Q_j \in \mathscr{U}/Q \,\&\, P_i \cap P_j \neq \emptyset\}.$$

Definition 2.5:[2]:: Let $\mathscr{U}$ be the universe of objects and P, Q be any two equivalence relations on $\mathscr{U}$ and $\tau_{P \cup Q}(X) = \{\mathscr{U}, \emptyset, L_{P \cup Q}(X), U_{P \cup Q}(X), B_{P \cup Q}(X)\}$ where $X \subseteq \mathscr{U}$.

(i) $L_{P \cup Q}(X) = \bigcup_{y \in \mathscr{U}} \{[Y] \in \mathscr{U}/P \cup Q : [Y] \subseteq X\}$.
(ii) $U_{P \cup Q}(X) = \bigcup_{y \in \mathscr{U}} \{[Y] \in \mathscr{U}/P \cup Q : [Y] \cap X \neq \emptyset\}$.
(iii) $B_{P \cup Q}(X) = U_{P \cup Q}(X) - L_{P \cup Q}(X)$.

That is, $\tau_{P \cup Q}(X)$ forms a topology on $\mathscr{U}$ called as the multi-granular nano topology in terms of union on $\mathscr{U}$ with respect to X. We call $(\mathscr{U}, \tau_{P \cup Q}(X))$ as the Multi-granular nano topological space in terms of union.

At every moment, in this paper $\mathscr{U}$ is a non-empty, finite universe, $X \subseteq \mathscr{U}$; P and Q are two equivalence relations on $\mathscr{U}$ respectively. $L_{P*Q}(X), U_{P*Q}(X), B_{P*Q}(X)$ and $\tau_{P*Q}(X)$ are the Multi*-lower, Multi*-upper approximations, Multi*-boundary and the Multi*-granular nano topological space based on the Multi*-granulation.

3 Multi*-Granular Nano Topology (M*GNT)

In this section we insinuate a new nano topology called Multi*-granular nano topology in terms of the Multi*-lower and Multi*-upper approximations of a set and its Multi*-boundary region.

Definition 3.1: Let $\mathscr{U}$ be the universe. P,Q be any two equivalence relations on $\mathscr{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. Let $X \subseteq \mathscr{U}$.

(i) The Multi*- lower approximation of X with respect to P and Q is the set of all objects which and it is denoted by $L_{P*Q}(X)$. That is, $L_{P*Q}(X) = \bigcup_{x \in \mathscr{U}} \{[x] : P(x) \subseteq X \text{ and } Q(x) \subseteq X\}$, where P(x) and Q(x) denotes the equivalence class determined by x.
(ii) The Multi*- upper approximation of X with respect to P and Q is the set of all objects which can be possibly classified as X with respect to P and Q and it is denoted by $U_{P*Q}(X)$. That is, $U_{P*Q}(X) = \bigcup_{x \in \mathscr{U}} \{[x] : P(x) \cap X \neq \emptyset \text{ or } Q(x) \cap X \neq \emptyset\}$, where P(x) and Q(x) denotes the equivalence class determined by x.
(iii) The Multi*-boundary region of X with respect to P and Q is the set of all objects which can be classified neither as X nor as not X with respect to P and Q and it is denoted by $B_{P*Q}(X)$. That is, $B_{P*Q}(X) = U_{P*Q}(X) - L_{P*Q}(X)$.

Definition 3.2: Let $\mathscr{U}$ be the universe and P,Q be any two equivalence relations on $\mathscr{U}$ and $\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, L_{P*Q}(X), U_{P*Q}(X), B_{P*Q}(X)\}$ where $X \subseteq \mathscr{U}$. Then $\tau_{P*Q}(X)$ satisfies the following axioms:

(i) $\mathscr{U}$ and $\emptyset \in \tau_{P*Q}(X)$.
(ii) The union of elements of any subcollection of $\tau_{P*Q}(X)$ is in $\tau_{P*Q}(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_{P*Q}(X)$ is in $\tau_{P*Q}(X)$.

That is, $\tau_{P*Q}(X)$ forms a topology on $\mathscr{U}$ called as the Multi*-granular nano topology on $\mathscr{U}$ with respect to X. We call $(\mathscr{U}, \tau_{P*Q}(X))$ as the Multi*-granular nano topological space.

Example 3.3: Let $\mathscr{U} = \{a,b,c,d,e\}$ and $\mathscr{U}/P = \{\{a\},\{b,c,d\},\{e\}\}$ and $\mathscr{U}/Q = \{\{b\},\{a\},\{c,d,e\}\}$ be two equivalence relations on $\mathscr{U}$ and let $X = \{a,c,d\} \subseteq \mathscr{U}$. Then $L_{P*Q}(X) = \{a\}, U_{P*Q}(X) = \mathscr{U}$ and $B_{P*Q}(X) = \{b,c,d,e\}$, hence the Multi*-granular nano topology $\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{a\}, \{b,c,d,e\}\}$.

Proposition 3.4: If $[\tau_{P*Q}(X)]$ is the Multi*-nano topology on $\mathscr{U}$ with respect to x, then the

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$\mathscr{B} = \{\mathscr{U}, \emptyset, L_{P*Q}(X), B_{P*Q}(X)\}$ is the basis for $[\tau_{P*Q}(X)]$.

*Proof.*i) $\bigcup_{A\in\mathscr{B}} A = \mathscr{U}$.
(ii) Consider $\mathscr{U}$ and $L_{P*Q}(X)$ from $\mathscr{B}$. Let W= $L_{P*Q}(X)$. since $\mathscr{U} \bigcap Ł_{P*Q}(X) = L_{P*Q}(X)$, $\subset \mathscr{U} \bigcap L_{P*Q}(X)$ and every X in $\mathscr{U} \bigcap L_{P*Q}(X)$ belongs to W. If we consider $\mathscr{U}$ and $B_{P*Q}(X)$ from $\mathscr{B}$, taking W= $B_{P*Q}(X)$, W$\subset \mathscr{U} \bigcap B_{P*Q}(X)$ and every x in $\mathscr{U} \bigcap B_{P*Q}(X)$ belongs to W, since $\mathscr{U} \bigcap B_{P*Q}(X) = B_{P*Q}(X)$. And when we consider $L_{P*Q}(X) \bigcap B_{P*Q}(X) = \emptyset$. Thus $\mathscr{B}$ is a basis for $\tau_{P*Q}(X)$.

Proposition 3.5: Let $\mathscr{U}$ be non-empty, finite universe and $X \subseteq \mathscr{U}$, Let $\tau_{P*Q}(X)$ be the Multi*-nano topology on $\mathscr{U}$ with respect to X. Then $[\tau_{P*Q}(X)]^c$ whose elements are A^c for $A \in \tau_{P*Q}(X)$ is a topology on $\mathscr{U}$.

*Proof.*The Multi*-granular nano topology on $\mathscr{U}$ with respect to X is given by $\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, L_{P*Q}(X), U_{P*Q}(X), B_{P*Q}(X)\}$. Therefore $[\tau_{P*Q}(X)]^c = \{\mathscr{U}, \emptyset, [L_{P*Q}(X)]^c, [U_{P*Q}(X)]^c, [B_{P*Q}(X)]^c\}$. Consider $[L_{P*Q}(X)]^c \cup [U_{P*Q}(X)]^c = [L_{P*Q}(X)] \bigcap [U_{P*Q}(X)]^c = [L_{P*Q}(X)]^c \in [\tau_{P*Q}(X)]^c$. $[L_{P*Q}(X)]^c \cup [B_{P*Q}(X)]^c = [L_{P*Q}(X)] \bigcap B_{P*Q}(X)]^c = \emptyset^c = \mathscr{U} \in [\tau_{P*Q}(X)]^c$. And $[U_{P*Q}(X)]^c \cup [B_{P*Q}(X)]^c = [U_{P*Q}(X)] \bigcap [B_{P*Q}(X)]^c = [B_{P*Q}(X)]^c \in [\tau_{P*Q}(X)]^c$. Also,$[L_{P*Q}(X)]^c \cup [U_{P*Q}(X)]^c \cup [B_{P*Q}(X)]^c = \emptyset^c = \mathscr{U} \in [\tau_{P*Q}(X)]^C$. Thus, arbitrary union of members of $[\tau_{P*Q}(X)]^c$ are in $[\tau_{P*Q}(X)]^c$. Also, $[L_{P*Q}(X)]^c \bigcap [U_{P*Q}(X)]^c = [L_{P*Q}(X) \cup U_{P*Q}(X)]^c = [U_{P*Q}(X)]^c \in [\tau_{P*Q}(X)]^c$. $[L_{P*Q}(X)]^c \bigcap [B_{P*Q}(X)]^c = [L_{P*Q}(X)] \cup [B_{P*Q}(X)]^c \in [\tau_{P*Q}(X)]^C$. since, $[L_{P*Q}(X)] \cup [B_{P*Q}(X)]^c \in [\tau_{P*Q}(X)]^C$ and $[U_{P*Q}](X)]^c \bigcap [B_{P*Q}(X)]^c = [U_{P*Q}(X)] \cup [B_{P*Q}(X)]^c = [U_{P*Q}(X)]^c \in [\tau_{P*Q}(X)]^c$. That is, finite intersection of members of $[\tau_{P*Q}(X)]^c$ belongs to $[\tau_{P*Q}(X)]^c$. Thus, $[\tau_{P*Q}(X)]^c$ is a topology on $\mathscr{U}$.

Example 3.6: Let $\mathscr{U} = \{a,b,c,d,e\}$. and $\mathscr{U}/P = \{\{a,b\},\{c,d\},\{e\}\}$ and $\mathscr{U}/Q = \{\{a,c\},\{b,e\},\{d\}\}$ be two equivalence relations on $\mathscr{U}$ and let $X = \{c,d\} \subseteq \mathscr{U}$.Then $L_{P*Q}(X) = \{d\}, U_{P*Q}(X) = \{a,c,d\}$ and $B_{P*Q}(X) = \{a,c\}$, hence the Multi*-granular nano topology $\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{a\}, \{b,c,d,e\}\}$. Thus, $\mathscr{B}_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{d\}, \{a,c\}\}$ is the basis for $\tau_{P*Q}(X)$.
Theorem 3.7: Let $(\mathscr{U}, \mathscr{R})$ be the approximation space $\mathscr{U}/P, \mathscr{U}/Q \in R$ be two equivalence relations defined on $\mathscr{U}$ respectively. Let $X \subseteq \mathscr{U}$, then the following properties hold:

(i)$L_{P*Q}(X) = L_P(X) \cap L_Q(X)$
(ii)$U_{P*Q}(X) = U_P(X) \cup U_Q(X)$

Proof.(i): For any [x]$\in L_{P*Q}(X)$, we have [x]$\in L_{P*Q}(X) \iff [P(X)] \subseteq X, [Q(X)] \subseteq X$. $\iff [x] \in L_P(X), [x] \in L_Q(X), \iff [x] \in L_P(X) \cap L_Q(X)$. Hence, $L_{P*Q}(X) = L_P(X) \cap L_Q(X)$.
(ii): For any [x]$\in U_{P*Q}(X)$, we have [x]$\in U_{P*Q}(X) \iff [P(x)] \bigcap X \neq \emptyset or [Q(x)] \bigcap X \neq \emptyset$. $\iff [x] \in U_P(X) or [x] \in U_Q(X)$. $\iff [x] \in U_P(X) \cup U_Q(X)$.

Theorem 3.8: Let $(\mathscr{U}, \mathscr{R})$ be the approximation space $\mathscr{U}/P, \mathscr{U}/Q \in R$ be two equivalence relations defined on $\mathscr{U}$ respectively. Let $X \subseteq \mathscr{U}$, then the following properties hold:

(i)$L_{P*Q}(X) \subseteq X$.
(ii)$U_{P*Q}(X) \supseteq X$.
(iii)$L_{P*Q}(X^c) = [U_{P*Q}(X)]^c$.
(iv)$U_{P*Q}(X^c) = [L_{P*Q}(X)]^c$.
(v)$L_{P*Q}(\emptyset) = \emptyset$.
(vi)$U_{P*Q}(\emptyset) = \emptyset$.
(vii)$L_{P*Q}(\mathscr{U}) = \mathscr{U}$.
(viii)$U_{P*Q}(\mathscr{U}) = \mathscr{U}$.
(ix)$L_{P*Q}(X) = L_{Q*P}(X)$.
(x)$U_{P*Q}(X) = U_{Q*P}(X)$.

Proof.(i). For any $[x] \in L_{P*Q}(X)$ it can be known that $[P(x)] \subseteq X$ and $[Q(x)] \subseteq X$, however $[x] \in [P(x)]$ and $X \in [Q(x)]$. So we can have $[x] \in X$ and hence $L_{P*Q}(X) \subseteq X$.
(ii). For any $[x] \in X$, we have $[x] \in [P(x)]$ and $[x] \in [Q(x)]$. So $P[x] \cap X \neq \emptyset$ or $Q[x] \cap X \neq \emptyset$, hence $[x] \in U_{P*Q}(X)$. Hence $X \subseteq U_{P*Q}(X)$.
(iii). For any $[x] \in L_{P*Q}(X^c)$, then $[x] \in L_{P*Q}(X^c) \Leftrightarrow P[x] \subseteq X^c$, $Q[x] \subseteq X^c \Leftrightarrow [P(x)] \cap X = \emptyset$ *or* $[Q(x)] \cap X = \emptyset \Leftrightarrow [x] \notin U_{P*Q}(X) \Leftrightarrow [x] \in [U_{P*Q}(X)]^c$. Hence $L_{P*Q}(X^c) = [U_{P*Q}(X)]^c$.
(iv). By the above, we have $L_{P*Q}(X) = [U_{P*Q}(X^c)]^c$. So it can be obtain that $[L_{P*Q}(X)]^c = [U_{P*Q}(X^c)]$.
(v). From (1) we have $L_{P*Q}(\emptyset) \subseteq \emptyset$, besides it is well known that $\emptyset \subseteq L_{P*Q}(\emptyset)$. So $L_{P*Q}(\emptyset) = \emptyset$.
(vi). If $U_{P*Q}(\emptyset) \neq \emptyset$, then there must exist a $[x] \in U_{P*Q}(\emptyset)$, so we can find that $P(x) \cap \emptyset \neq \emptyset$ and $Q(x) \cap \emptyset \neq \emptyset$, hence a contradiction. Thus, $U_{P*Q}(\emptyset) = \emptyset$.
(vii). $L_{P*Q}(\mathscr{U}) = L_{P*Q}(\emptyset)^c = [U_{P*Q}(\emptyset)]^c = [\emptyset]^c = \mathscr{U}$.
(viii). $U_{P*Q}(\mathscr{U}) = U_{P*Q}(\emptyset)^c = [L_{P*Q}(\emptyset)]^c = [\emptyset]^c = \mathscr{U}$.
(ix) & (X). It can be easily proved from the definition (or) directly.

4 Comparision and Classification

In this section we provide a comparative study of the two types of Multigranulations and classify the types of M*GNT.

Theorem 4.1: Let $(\mathscr{U}, \mathscr{R})$ be the approximation space $\mathscr{U}/P, \mathscr{U}/Q \in R$ be two equivalence relations defined on $\mathscr{U}$ respectively. Let $X \subseteq \mathscr{U}$, then the following properties hold:

(i)$L_{P*Q}(X) \subseteq L_{P+Q}(X) \subseteq L_{P\cup Q}(X)$.
(ii)$U_{P*Q}(X) \supseteq U_{P+Q}(X) \supseteq U_{P\cup Q}(X)$.

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Proof.(i): Since $L_{P+Q}(X) \subseteq L_{P\cup Q}(X)$, and also $L_{P*Q}(X) \subseteq L_{P\cup Q}(X)$, and also $L_{P+Q}(X) = L_P(X) \cup L_Q(X)$, $L_{P*Q}(X) = L_P(X) \cap L_Q(X)$, hence
$L_{P*Q}(X) \subseteq L_{P+Q}(X) \subseteq L_{P\cup Q}(X)$.
(ii): We know that $U_{P*Q}(X) \supseteq U_{P\cup Q}(X)$, and $U_{P*Q}(X) \supseteq U_{P\cup Q}(X)$,
but $U_{P+Q}(X) = U_P(X) \cup U_Q(X)$. $U_{P*Q}(X) = U_P(X) \cup U_Q(X)$. Therefore, $U_{P*Q}(X) \supseteq U_{P+Q}(X)$, hence $U_{P*Q}(X) \supseteq U_{P+Q}(X) \supseteq U_{P\cup Q}(X)$.

Theorem 4.2: Let $(\mathscr{U},\mathscr{R})$ be the approximation space $\mathscr{U}/P, \mathscr{U}/Q \in R$ be two equivalence relations defined on $\mathscr{U}$ respectively. Let $X \subseteq \mathscr{U}$, then the following properties hold:

(i)$L_{P*Q}(X) \subseteq L_P(X)$ or $L_Q(X) \subseteq L_{P+Q}(X)$.
(ii)$U_{P*Q}(X) \supseteq U_P(X)$ or $U_Q(X) \supseteq U_{P+Q}(X)$.

Theorem 4.3: Let $(\mathscr{U},\mathscr{R})$ be the approximation space $\mathscr{U}/P, \mathscr{U}/Q \in R$ be two equivalence relations defined on $\mathscr{U}$ respectively. Let $X \subseteq \mathscr{U}$, then the following properties hold:

(i)$B_{P*Q}(X) \supseteq B_P(X)$ or $B_Q(X) \supseteq B_{P+Q}(X)$.
(ii)$B_{P*Q}(X) \supseteq B_{P+Q}(X) \supseteq B_{P\bigcup Q}(X)$.

Remark 4.4: The above theorem reveals us that the Multi-* lower approximation is smaller than lower approximation in nano topology while the Multi-* upper approximation is greater than upper approximation in nano topology. Moreover, the Multi-* lower approximation is smaller than Multi-lower approximation while the Multi-* upper approximation is greater than Multi-upper approximation.

From the above theorem, it can be seen that the "distance" the Multi*-lower approximation and its upper approximation is largest, which leads to larger boundary region for a given subset X than Multi-granular nano topology and nano topology. The following example illustrates the results of Theorem 4.1., proposition 4.2. and Theorem 4.3.

Example 4.5: Let $\mathscr{U} = \{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$ and $\mathscr{U}/P = \{\{u_1,u_7\},\{u_2,u_8\},\{u_3,u_4\},\{u_5,u_6\}\}$ and $\mathscr{U}/Q = \{\{u_1,u_2\},\{u_3,u_4,u_5\},\{u_6,u_7,u_8\}\}$ be two equivalence relations on $\mathscr{U}$ and
let X $= \{u_1,u_2,u_8,u_3\} \subseteq \mathscr{U}$. $\mathscr{U}/P\cup Q = \{\{u_1\},\{u_2\},\{u_3,u_4\},\{u_5\},\{u_6\},\{u_7\},\{u_8\}\}$. Then $L_{P*Q}(X) = \{u_2\}, U_{P*Q}(X) = \mathscr{U}$, $B_{P*Q}(X) = \{u_1,u_3,u_4,u_5,u_6,u_7,u_8\}$, and also $L_{P\cup Q}(X) = \{u_1,u_2,u_8\}$, $U_{P\cup Q}(X) = \{u_1,u_2,u_8,u_3,u_4\}$.
$L_{P+Q}(X) = \{u_1,u_2,u_8\}$,
$U_{P+Q}(X) = \{u_1,u_2,u_3,u_4,u_7,u_8\}$,
$B_{P+Q}(X) = \{u_3,u_4,u_7\}$,
hence we can conclude that $L_{P*Q}(X) \subseteq L_{P+Q}(X) \subseteq L_{P\cup Q}(X) \subseteq X \subseteq U_{P\cup Q}(X) \subseteq U_{P+Q}(X) \subseteq U_{P*Q}(X)$.
$B_{P*Q}(X) \supseteq B_{P+Q}(X) \supseteq B_{P\cup Q}(X)$.

Topological property generally deals with the intrinsic structure of sets, based on this we can incorporate our Nano topological space as follows:

Definition 4.6: Let $\mathscr{U}$ be a non-empty finite universe and let $X \subseteq \mathscr{U}$, $\mathscr{U}/\mathscr{R}$ be an indiscreniblity relation on $\mathscr{U}$.
Nano Type-1 ($\mathscr{N}T_1$):
If $L_R(X) \neq U_R(X)$ or $L_R(X) = U_R(X)$, where $L_R(X) \neq \emptyset$ and $U_R(X) \neq \mathscr{U}$, then either $\tau_R(X) = \{\mathscr{U},\emptyset,L_R(X),U_R(X),B_R(X)\}$ or $\tau_R(X) = \{\mathscr{U},\emptyset,L_R(X)\}$.
Nano Type-2 ($\mathscr{N}T_2$):
If $L_R(X) = \emptyset$ and $U_R(X) \neq \mathscr{U}$, then $\tau_R(X) = \{\mathscr{U},\emptyset,U_R(X)\}$.
Nano Type-3 ($\mathscr{N}T_3$):
If $L_R(X) \neq \emptyset$ and $U_R(X) = \mathscr{U}$, then $\tau_R(X) = \{\mathscr{U},\emptyset,L_R(X),B_R(X)\}$.
Nano Type-4 ($\mathscr{N}T_4$):
If $L_R(X) = \emptyset$ and $U_R(X) = \mathscr{U}$, then $\tau_R(X) = \{\mathscr{U},\emptyset\}$.

Remark 4.7: Based on the classification of nano topology, we have considered the unification of different types of nano topology and investigated their type in Multi*-Granular nano topological space. The following subsection contributes the type of Multi*-Granular nano topology of X with respect to P*Q.

Table 1: Table for type of X with respect to $P*Q$

$P \backslash Q$	NT_1	NT_2	NT_3	NT_4
NT_1	$NT_1/NT_2/NT_3$	NT_2/NT_4	NT_3/NT_4	NT_4
NT_2	NT_2/NT_4	NT_2/NT_4	NT_4	NT_4
NT_3	NT_3/NT_4	NT_4	NT_3/NT_4	NT_4
NT_4	NT_4	NT_4	NT_4	NT_4

Table for type of X with respect to $P*Q$.

Example 4.8: Verification for the entry (1,1) in the table.
Let $\mathscr{U} = \{a,b,c,d,e\}, \mathscr{U}/P = \{\{a\},\{b,c,d\},\{e\}\}$, $\mathscr{U}/Q = \{\{c,d,e\},\{b\},\{a\}\}$ be any two equivalence relations defined on $\mathscr{U}$
Case [i(a)]: Let $X = \{a\} \subseteq \mathscr{U}$.
$L_P(X) = \{a\} \neq \emptyset, U_P(X) = \{a\} \neq \mathscr{U}, B_P(X) = \emptyset$ and $\tau_P(X) = \{\mathscr{U},\emptyset,\{a\}\}$. Therefore X is of $\mathscr{N}T_1$,with respect to'P'.
$L_Q(X) = \{a\} \neq \emptyset, U_Q(X) = \{a\} \neq \mathscr{U}, B_Q(X) = \emptyset$ and $\tau_Q(X) = \{\mathscr{U},\emptyset,\{a\}\}$. Therefore X is of $\mathscr{N}T_1$ with

respect to 'Q'.

Now,
$L_{P*Q}(X) = \{a\} \neq \emptyset, U_{P*Q}(X) = \{a\} = \mathscr{U}, B_{P*Q}(X) = \emptyset$ and
$\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{a\}\}$. Thus X is of $\mathscr{N}T_1$ with respect to 'P*Q'.

Case [i(b)]: Let $\mathscr{U} = \{a,b,c,d,e\}, \mathscr{U}/P = \{\{a,b\},\{c,d\},\{e\}\}$, $\mathscr{U}/Q = \{\{a,c\},\{b,e\},\{d\}\}$ be any two equivalence relations defined on $\mathscr{U}$. Let $X = \{c,d\} \subseteq \mathscr{U}$.

$L_P(X) = \{c,d\} \neq \emptyset, U_P(X) = \{c,d\} \neq \mathscr{U}, B_P(X) = \emptyset$ and $\tau_P(X) = \{\mathscr{U}, \emptyset, \{c,d\}\}$. Therefore X is of $\mathscr{N}T_1$,with respect to'P'.

$L_Q(X) = \{d\} \neq \emptyset, U_Q(X) = \{a,c,d\} \neq \mathscr{U}, B_Q(X) = \{a,c\}$ and
$\tau_Q(X) = \{\mathscr{U}, \emptyset, \{d\}, \{a,c,d\}, \{a,c\}\}$. Therefore X is of $\mathscr{N}T_1$ with respect to 'Q'.

Now, $L_{P*Q}(X) = \{d\} \neq \emptyset, U_{P*Q}(X) = \{a,c,d\} = \mathscr{U}, B_{P*Q}(X) = \{a,c\}$ and
$\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{d\}, \{a,c,d\}, \{a,c\}\}$. Thus X is of $\mathscr{N}T_1$ with respect to 'P*Q'.

Case [ii]: Let $\mathscr{U} = \{a,b,c,d,e\}, \mathscr{U}/P = \{\{a\},\{b,c,d\},\{e\}\}$, $\mathscr{U}/Q = \{\{c,d,e\},\{b\},\{a\}\}$ be any two equivalence relations defined on $\mathscr{U}$
Let $X = \{a,c,d\} \subseteq \mathscr{U}$.

$L_P(X) = \{a\} \neq \emptyset, U_P(X) = \{a,b,c,d\} \neq \mathscr{U}, B_P(X) = \{b,c,d\}$ and
$\tau_P(X) = \{\mathscr{U}, \emptyset, \{a\}, \{a,b,c,d\}, \{b,c,d\}\}$. Therefore X is of $\mathscr{N}T_1$,with respect to'P'.

$L_Q(X) = \{a\} \neq \emptyset, U_Q(X) = \{a,c,d,e\} \neq \mathscr{U}, B_Q(X) = \{c,d,e\}$ and
$\tau_Q(X) = \{\mathscr{U}, \emptyset, \{a\}, \{a,c,d,e\}, \{c,d,e\}\}$. Therefore X is of $\mathscr{N}T_1$ with respect to 'Q'.

Now, $L_{P*Q}(X) = \{a\} \neq \emptyset, U_{P*Q}(X) = \{a,b,c,d,e\} = \mathscr{U}, B_{P*Q}(X) = \{b,c,d,e\}$ and
$\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{a\}, \{b,c,d,e\}\}$. Thus X is of $\mathscr{N}T_3$ with respect to 'P*Q'.

Case [iii]: Let $\mathscr{U} = \{a,b,c,d,e\}, \mathscr{U}/P = \{\{a,b\},\{c,d\},\{e\}\}$, $\mathscr{U}/Q = \{\{a,c\},\{b,e\},\{d\}\}$ be any two equivalence relations defined on $\mathscr{U}$
Let $X = \{d,e\} \subseteq \mathscr{U}$.

$L_P(X) = \{e\} \neq \emptyset, U_P(X) = \{c,d,e\} \neq \mathscr{U}, B_P(X) = \{c,d\}$ and
$\tau_P(X) = \{\mathscr{U}, \emptyset, \{e\}c,d,e\}, \{c,d\}$. Therefore X is of $\mathscr{N}T_1$,with respect to'P'.

$L_Q(X) = \{d\} \neq \emptyset, U_Q(X) = \{b,d,e\} \neq \mathscr{U}, B_Q(X) = \{b,e\}$ and
$\tau_Q(X) = \{\mathscr{U}, \emptyset, \{d\}, \{b,d,e\}, \{b,e\}\}$. Therefore X is of $\mathscr{N}T_1$ with respect to 'Q'.

Now, $L_{P*Q}(X) = \emptyset, U_{P*Q}(X) = \{b,c,d,e\}, B_{P*Q}(X) = \{b,c,d,e\}$ and
$\tau_{P*Q}(X) = \{\mathscr{U}, \emptyset, \{b,c,d,e\}\}$. Thus X is of $\mathscr{N}T_2$ with respect to 'P*Q'.

5 Proportion Based on Measures

In this section, we have defined a new measure nano accuracy in terms of knowledge granulation and investigated the difference and relation ship among MGNT and M*GNT based on their approximations and also by means of calculating their nano accuracy and degree of dependence of decision attribute in Recruitment process of an software concern.

Definition 5.1: Let $(\mathscr{U}, A)$ be an information system where $\mathscr{U}$ is an non-empty finite set of objects, A is a finite set of attributes and A is divided into a set C of conditional attributes and a set D of decision attributes.

Definition 5.2: Let $(\mathscr{U}, A)$ be an information system, let $(\mathscr{U}, \tau_R(X))$ be a nano topological space and $X \subseteq \mathscr{U}$, then the nano accuracy of X is defined as $\mathscr{N}\mathscr{A}_R(X) = 1 - \xi_R(X)GK(R)$, where $\xi_R(X) = 1 - \frac{|L_R(X)|}{|U_R(X)|}$ and GK(R)= $\frac{1}{|\mathscr{U}|^2}[X_1^2 + X_2^2 + \ldots X_i^2]$.

Definition 5.3: Let $(\mathscr{U}, \tau_{P+Q}(X))$ be a Multi-granular nano topological space and let $X \subseteq \mathscr{U}$, then the Multi-nano accuracy of X is defined as
$\mathscr{N}\mathscr{A}_{P+Q}(X) = 1 - \xi_{P+Q}(X)GK(P+Q)$, where $\xi_{P+Q}(X) = 1 - \frac{|L_{P+Q}(X)|}{|U_{P+Q}(X)|}$ and GK(P+Q) = GK(P) + GK(Q).

Definition 5.4: Let $(\mathscr{U}, \tau_{P*Q}(X))$ be a Multi*-granular nano topological space and $X \subseteq \mathscr{U}$, then the Multi*-nano accuracy of X is defined as $\mathscr{N}\mathscr{A}_{P*Q}(X) = 1 - \xi_{P*Q}(X)GK(P*Q)$, where $\xi_{P*Q}(X) = 1 - \frac{|L_{P*Q}(X)|}{|U_{P*Q}(X)|}$,
GK(P*Q) = GK(P) * GK(Q).

Definition 5.5: Let $(\mathscr{U}, \tau_R(X))$ be an nano topological space and let
$X \subseteq \mathscr{U}$. Let $\mathscr{U}/D = \{D_1, D_2, \ldots D_K\}$ be all decision classes induced by decision attribute D and A is divided into a set C of condition attributes, then the nano approximation of A is called the nano degree of dependence and is defined as

$$\gamma[P,D] = \frac{1}{|\mathscr{U}|}\,[|L_P(D_1)| + |L_Q(D_2)|].$$

Example 5.6: Now, we can consider the problem of finding the difference and relationship based on their approximations and also by finding the nano accuracy and nano degree of dependence in all the cases, of the selection list for Recruitment in a software concern.

Consider the following table giving information about the Selection list for Recruitment in a Software Company. Qualification, Experience, Performance, Technical skill and Salary Expectation are the conditional attributes of the system, where as Decision is the decision attribute.

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In the sequel, C_1, C_2, C_3, C_4, C_5 and D will stand for Qualification, Experience, Performance, Technical skill, Salary Expectation and Decision respectively. The domains are as follows:
$V_{C_1} = \{B.E., M.C.A., M.Sc.,\}$,
$V_{C_2} = \{High, Medium, Low\}$,
$V_{C_3} = \{Excellent, Neutral, Good\}$,
$V_{C_4} = \{V.Good, Good, Bad\}$,
$V_{C_5} = \{Undernorms, Abovenorms, Veryhigh\}$ and
$V_{C_D} = \{Accept, Reject\}$.

Table 2: Information table for recruitment process

Candidates	Qualification	Experience	Performance	Technical Skill	Salary Expectation	Decision
X_1	B.E.,	High	Excellent	V.Good	Undernorms	Accept
X_2	B.E.,	High	Excellent	V.Good	Undernorms	Accept
X_3	M.Sc.,	Low	Neutral	Bad	Veryhigh	Reject
X_4	M.Sc.,	Medium	Good	Bad	Abovenorms	Reject
X_5	M.Sc.,	Low	Neutral	Bad	Veryhigh	Reject
X_6	M.C.A.,	High	Good	Good	Undernorms	Accept
X_7	M.C.A.,	Medium	Neutral	Bad	Veryhigh	Reject
X_8	B.E.,	High	Excellent	V.Good	Undernorms	Accept
X_9	B.E.,	High	Excellent	Good	Abovenorms	Accept
X_{10}	M.Sc.,	Low	Neutral	Bad	Veryhigh	Reject

The columns of the table represent the Key factors evaluated in the Interview and the rows represent the individual ability of the candidates, who attended the Interview. The entries in the table are the attribute values.

Here $\mathscr{U} = \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_3, ..., \mathscr{X}_{10}\}$, the list of candidates those who appear for the Interview in the Software Concern.
$A = \{Qualification, Experience, Performance, Technicalskill, SalaryExpectation, Decision\}$ be the set of attributes.

From the table, we can find that

$$\mathscr{U}/C_1 = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8, \mathscr{X}_9\}, \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_{10}\}, \{\mathscr{X}_6, \mathscr{X}_7\}\}$$
$$\mathscr{U}/C_2 = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8, \mathscr{X}_9\}\{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_{10}\}, \{\mathscr{X}_4, \mathscr{X}_7\}\}$$
$$\mathscr{U}/C_3 = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8, \mathscr{X}_9\}, \{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}, \{\mathscr{X}_4, \mathscr{X}_6\}\}$$
$$\mathscr{U}/C_4 = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8\}, \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}, \{\mathscr{X}_6, \mathscr{X}_9\}\}$$
$$\mathscr{U}/C_5 = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8\}, \{\mathscr{X}_4, \mathscr{X}_9\}, \{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}\}.$$

Let us take $X = \{\mathscr{X}_4, \mathscr{X}_6, \mathscr{X}_7\} \subseteq \mathscr{U}$, then

$$L_{C_1+C_3}(X) = \{\mathscr{X}_4, \mathscr{X}_6, \mathscr{X}_7\},$$
$$U_{C_1+C_3}(X) = \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_6, \mathscr{X}_7, \mathscr{X}_{10}\}\},$$
$$L_{C_1*C_3}(X) = \{\mathscr{X}_6\},$$
$$U_{C_1*C_3}(X) = \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_6, \mathscr{X}_7, \mathscr{X}_{10}\}\}$$

$$\xi_{C_1+C_3}(X) = 1 - \frac{|L_{C_1+C_3}(X)|}{|U_{C_1+C_3}(X)|} = \frac{1}{2}$$
$$\xi_{C_1*C_3}(X) = 1 - \frac{|L_{P*Q}(X)|}{|U_{P*Q}(X)|} = \frac{5}{6}$$
$$GK[C_1+C_3] = GK[C_1] + GK[C_3] = 0.72$$
$$GK[C_1*C_3] = = GK[C_1] * GK[C_3] = 0.13$$
$$\mathscr{NA}_{(C_1+C_3)}(X) = 1 - \xi_{C_1+C_3}[X][GK(C_1+C_3)] = 0.64$$
$$\mathscr{NA}_{(C_1*C_3)}(X) = 1 - \xi_{C_1*C_3}[X][GK(C_1*C_3)] = 0.17$$

Hence by comparing the above measures we can conclude that

$$\mathscr{NA}_{(C_1+C_3)}(X) \geq \mathscr{NA}_{(C_1*C_3)}(X)$$

From the table 2, we have $\mathscr{U}/D = \{D_A, D_R\}$, $D_A = \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8, \mathscr{X}_9\}$ and $D_R = \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}$ where $\mathscr{U}/D = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8, \mathscr{X}_9\}, \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}\}$.
From the table

$$L_{C_1*C_3}(D_A) = \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8, \mathscr{X}_9\}$$
$$L_{C_1+C_3}(D_A) = \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8, \mathscr{X}_9\}$$
$$L_{C_1*C_3}(D_R) = \{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_{10}\}$$
$$L_{C_1+C_3}(D_R) = \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}$$
$$\gamma[C_1+C_3, D] = \frac{1}{|\mathscr{U}|}[|L_{C_1+C_3}(D_A)| + |L_{C_1+C_3}(D_R)|] = \frac{9}{10}$$
$$\gamma[C_1*C_3, D] = = \frac{1}{|\mathscr{U}|}[|L_{C_1*C_3}(D_A)| + |L_{C_1*C_3}(D_R)|] = \frac{7}{10}$$

Hence, from the above it can be found that,

$$\gamma[C_1*C_3, D] \leq \gamma[C_1, D] \leq \gamma[C_1+C_3, D]$$
$$\gamma[C_1*C_3, D] \leq \gamma[C_3, D] \leq \gamma[C_1+C_3, D]$$

Observation 5.7: Therefore the Nano accuracy of X with respect to
$[C_1+C_3](X)$ increases than $[C_1*C_3](X)$ and hence the Multi-Granular nano topology becomes finer than the Multi*-Granular nano topology. From the contribution, it can be found that when two attribute sets in information systems posseses a contradiction or inconsistent relationship, or when efficient computation is required, the two types of MGNT will display their advanatage for rule extraction and knowledge discovery.

6 Real life application in M*GNT

In this section we are finding the reduct attribute in the recruitment process of a software concern, using

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M*GNT and also we introduce a quantitative measure for significance as follows.

Definition 6.1: Let ($\mathscr{U}$,A) be an information system, where A is divided into a set "C' of condition attributes and a set 'D' of decision attributes. Intuitively, some attributes are not significant in a representation and their removal has no real impact on the value of representation of elements. If it is not significant, one can simply remove an attribute for further consideration called as reduct and it is denoted by Red_A.

Definition 6.2: Let ($\mathscr{U}$,A) be an information system, where A is divided into a set "C' of condition attributes and a set 'D' of decision attributes. Then a core is a minimal subset of attributes which is such that none of its elements can be removed without affecting the classification powered attributes. And it can be found by $\text{Core}\{A\} = A - Red_A$.

Remark 6.3: Since the M*GNT model mainly considers the Multi*-lower approximation and the Multi*-upper approximation of a target concept by multiple equivalence relations in the following, we introduce a measure of importance of condition attributes with respect to decision attributes in a decision system.

Definition 6.4: Let ($\mathscr{U}$,A) be an information system, where A is divided into a set 'C' of condition attributes C = $\{C_1, C_2, ...C_m\}$, then the inner significance measure of C is defined as
$\text{Sign}_{inner}(C_i, C, D) = S_{C-\{c_i\}}(D) - S_C\{D\}$
$\text{Sign}^{inner}(C_i, C, D) = S^{C-\{c_i\}}(D) - S^C\{D\}$ for all i= 1,2,..m.

where

$$S_C\{D\} = \frac{1}{|\mathscr{U}|}\,[|L_{C_1*C_2...*C_n}(D_A)| + |L_{C_1*C_2...*C_n}(D_R)|].$$

$$S^C\{D\} = \frac{1}{|\mathscr{U}|}\,[|U_{C_1*C_2...*C_n}(D_A)| + |U_{C_1*C_2...*C_n}(D_R)|].$$

Definition 6.5: Let ($\mathscr{U}$,A) be an information system, where A is divided into a set 'C' of condition attributes C= $\{C_1, C_2, ...C_m\}$, then the outer significance measure of A is defined as
$\text{Sign}_{outer}(C_i, C_j, D) = S_{\{c_j\}}\{D\} - S_{\{c_j \cup c_i\}}\{D\}$
$\text{Sign}^{outer}(C_i, C_j, D) = S^{\{c_j\}}\{D\} - S^{\{c_j \cup c_i\}}\{D\}$ for all i=1,2,..m.

6.6: An Attribute reduction algorithm in the M*GNT

Step1: Given a finite universe $\mathscr{U}$, a finite set A of attributes that is divided into two classes, C of condition attributes and D of decision attributes, an equivalence relation R on $\mathscr{U}$ corresponding to $C_1, C_2, ...C_n$ represent the data as an information table, coloumns of which are labelled by attributes and rows by objects. The entries of the table are attribute values.

Step2: Generate the M*GNT, $\tau_{C_1*C_2*...*C_m}(X_A)$, $\tau_{C_1*C_2*...*C_m}(X_R)$,$\mathscr{B}_{C_1*C_2*...*C_m}(X_A)$ and $\mathscr{B}_{C_1*C_2*...*C_m}(X_A)$.

Step3: First we find the Multi*-lower appproximation reduct.

Step4: Now, find the inner significance measure for each C_i,i=1,2,...m.

Step5: Put C_k into Red, where $\text{Sign}^{inner}(C_k, A, d)>0$. Choose $\text{Max}\{Sign^{inner}(C_k, A, d), Sign^{inner}(C_r, A, d)\}$. If there is a tie choose arbitrarily.

Step6: Now compute the outer significance measure C_i, i=1,2,...k-1,k+1,...m. Then $Red_A = Red \cup \{C_0\}$, where $Sign(a_0, red, d) = max\{Sign^{outer}(C_k, red, d), C_k \in C - red\}$.

Step7: Next, by using the Multi*-upper approximation find the Multi*-upper approximation reduct by repeating steps 3,4 and 5.

Step8: Now we can generate the Multi*-granular nano topology and its basis with respect to Red_A,where i=1,2,...n $n<m$.

Step9: Then Red_A is the Multi*-granular reduct iff $\mathscr{B}_{C_1*C_2*...*C_m}(X_A) = \mathscr{B}_{C_1*C_2*...*C_n}(X_A)$ And $\mathscr{B}_{C_1*C_2*...*C_m}(X_R) = \mathscr{B}_{C_1*C_2*...*C_n}(X_R)$.

Now, we consider the problem of finding the key environmental factors for the selection of candidates using significance measures and by means of Multi*-granular nano topolological reduction of attributes in complete information systems in terms of basis of Multi*-granular nano topology. Based on the decision attribute, $\mathscr{U}/D = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8, \mathscr{X}_9\}, \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_7, \mathscr{X}_{10}\}\}$.

Step1:

$$L_{\{C_1*C_2*C_3*C_4*C_5\}}(D) = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8\}, \{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_{10}\}\}$$
$$U_{\{C_1*C_2*C_3*C_4*C_5\}}(D) = \{\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8, \mathscr{X}_9, \mathscr{X}_7, \mathscr{X}_4\}, \{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_6, \mathscr{X}_7, \mathscr{X}_8, \mathscr{X}_9, \mathscr{X}_{10}\}\}$$

$$S_C(D) = \frac{1}{|\mathscr{U}|}[|L_{\{C_1*C_2*C_3*C_4*C_5\}}D_A)| + |L_{\{C_1*C_2*C_3*C_4*C_5\}}(D_R)|] = \frac{6}{10}$$

$$S^C(D) = \frac{1}{|\mathscr{U}|}\,[|U_{\{C_1*C_2*C_3*C_4*C_5\}}(D_A)| + |U_{\{C_1*C_2*C_3*C_4*C_5\}}(D_R)|] = \frac{14}{10}$$

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To compute the inner significance of each condition attribute:

$$Sign_{inner}(C_1,C,D) = S_{C-\{c_1\}}(D) - S_C(D) = \frac{7}{10} - \frac{6}{10} = \frac{1}{10}$$
$$Sign_{inner}(C_2,C,D) = S_{C-\{c_2\}}(D) - S_C(D) = \frac{6}{10} - \frac{6}{10} = 0$$
$$Sign_{inner}(C_3,C,D) = S_{C-\{c_3\}}(D) - S_C(D) = \frac{6}{10} - \frac{6}{10} = 0$$
$$Sign_{inner}(C_4,C,D) = S_{C-\{c_4\}}(D) - S_C(D) = \frac{6}{10} - \frac{6}{10} = 0$$
$$Sign_{inner}(C_5,C,D) = S_{C-\{c_5\}}(D) - S_C(D) = \frac{7}{10} - \frac{6}{10} = \frac{1}{10}$$
$$Sign^{inner}(C_1,C,D) = S^C(D) - S^{C-\{c_1\}}(D) = \frac{14}{10} - \frac{13}{10} = \frac{1}{10}$$
$$Sign^{inner}(C_2,C,D) = S^C(D) - S^{C-\{c_2\}}(D) = \frac{14}{10} - \frac{14}{10} = 0$$
$$Sign^{inner}(C_3,C,D) = S^C(D) - S^{C-\{C_3\}}(D) = \frac{14}{10} - \frac{14}{10} = 0$$
$$Sign^{inner}(C_4,C,D) = S^C(D) - S^{C-\{c_4\}}(D) = \frac{14}{10} - \frac{14}{10} = 0$$
$$Sign^{inner}(C_5,C,D) = S^C(D) - S^{C-\{c_5\}}(D) = \frac{14}{10} - \frac{13}{10} = \frac{1}{10}$$

According to both the Multi*-lower and Multi*-upper inner significance measures and by the algorithm, we select ”C_1” as the first attribute. To Compute the outer significance:

$$\begin{aligned}
Sign_{outer}(C_2,C_1,D) &= S_{c_1}(D) - S_{\{c_1\cup c_2\}}(D) \\
&= \frac{8}{10} - \frac{8}{10} = 0 \\
Sign_{outer}(C_3,C_1,D) &= S_{c_1}(D) - S_{\{c_1\cup c_3\}}(D) \\
&= \frac{8}{10} - \frac{7}{10} = \frac{1}{10} \\
Sign_{outer}(C_4,C_1,D) &= S_{c_1}(D) - S_{\{c_1\cup c_4\}}(D) \\
&= \frac{8}{10} - \frac{8}{10} = 0 \\
Sign_{outer}(C_5,C_1,D) &= S_{c_1}(D) - S_{\{c_1\cup c_5\}}(D) \\
&= \frac{8}{10} - \frac{6}{10} = \frac{2}{10} \\
Sign^{outer}(C_2,C_1,D) &= S^{\{c_1\cup c_2\}}(D) - S^{c_1}(D) \\
&= \frac{12}{10} - \frac{12}{10} = 0 \\
Sign^{outer}(C_3,C_1,D) &= S^{\{c_1\cup c_3\}}(D) - S^{c_1}(D) \\
&= \frac{13}{10} - \frac{12}{10} = \frac{1}{10} \\
Sign^{outer}(C_4,C_1,D) &= S^{\{c_1\cup c_4\}}(D) - S^{c_1}(D) \\
&= \frac{12}{10} - \frac{12}{10} = 0 \\
Sign^{outer}(C_5,C_1,D) &= S^{\{c_1\cup c_5\}}(D) - S^{c_1}(D) \\
&= \frac{14}{10} - \frac{12}{10} = \frac{2}{10}
\end{aligned}$$

By calculating the Multi*-lower and Multi*-upper outer significance measures of the attributes, by algorithm we select the ”C_5” as the next attribute. And also,

$$\begin{aligned}
\tau_{C_1*C_5}(X_A) = \tau_{C_1*C_2*C_3*C_4*C_5}(X_A) &= \{\mathscr{U}, \emptyset, \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8\}, \\
&\{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_6, \mathscr{X}_8, \mathscr{X}_9, \mathscr{X}_7, \mathscr{X}_4\}, \\
&\{\mathscr{X}_6, \mathscr{X}_9, \mathscr{X}_7, \mathscr{X}_4\}\} \\
\tau_{C_1*C_5}(X_R) = \tau_{C_1*C_2*C_3*C_4*C_5}(X_R) &= \{\mathscr{U}, \emptyset, \{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_1 0\}, \\
&\{\mathscr{X}_3, \mathscr{X}_4, \mathscr{X}_5, \mathscr{X}_6, \mathscr{X}_7, \mathscr{X}_8, \mathscr{X}_9, \mathscr{X}_{10}\}, \\
&\{\mathscr{X}_6, \mathscr{X}_9, \mathscr{X}_7, \mathscr{X}_4\}\} \\
\mathscr{B}_{C_1*C_5}(X_A) = \mathscr{B}_{C_1*C_2*C_3*C_4*C_5}(X_A) &= \{\mathscr{U}, \emptyset, \{\mathscr{X}_1, \mathscr{X}_2, \mathscr{X}_8\}, \{\mathscr{X}_6, \mathscr{X}_9, \mathscr{X}_7, \mathscr{X}_4\}\} \\
\mathscr{B}_{C_1*C_5}(X_R) = \mathscr{B}_{C_1*C_2*C_3*C_4*C_5}(X_R) &= \{\mathscr{U}, \emptyset, \{\mathscr{X}_3, \mathscr{X}_5, \mathscr{X}_{10}\}, \{\mathscr{X}_6, \mathscr{X}_9, \mathscr{X}_7, \mathscr{X}_4\}\}
\end{aligned}$$

Hence $Red_A = \{C_1, C_5\} = \{Qualification, SalaryExpectation\}$ is a reduct of these granular structures in the Multi*-granular nano topological space. Thus CORE[A]= $\{C_2, C_3, C_4\}$ = $\{Experience, Performance, TechnicalSkill\}$ is the core of these granular structures in the Multi*-granular topological space.

Observation 6.7: We can conclude that Experience, Performance and Technical Skill are the key factors that have close connection to the selection of a candidate in the Recruitment process for the software concern.

7 Conclusion

Multi*-granular nano topological space is one of desirable directions in nano topological theory, in which Multi*-lower upper approximations are approximated by granular structures induced by multiple binary relations. Moreover, we have discussed the relationship between Multi-granular nano topological spaces and Multi*-granular nano topological spaces. Finally we have developed an approach to find an attribute reduct and CORE from a decision table in the context of Multi*-granular nano topological space model. It provides a new perspective for decision making analysis based on nano topological theory. Further this can be extended with respect to any relation based on the universe, rather than equivalence relation.

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