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Purity loss for a Cooper pair box interacting dispersively with a nonclassical field under phase damping

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A system of a Cooper pair box interacting with a field in the binomial state is considered in the dispersive regime. The system is coupled to the environment through phase damping. The effects of the different parameters on the purity of the states of the total system and the subsystems are considered. It is found that damping does not affect the state of the box. While it affects the state of the field and the total system. The initial state of the box has the dominant effect on the heights of oscillations on both the field and the box states.

Keywords: Multipartite quantum entanglement, negativity.

1 Introduction

Recent developments of quantum computation have inspired many interesting ideas in a variety of fields. Ideas for novel cavity quantum electrodynamics (CQED) analogs have been recently suggested by using nanomechanical resonators [1,2] discrete LC circuits [3],large Josephson-junction [4] and Cooper pair boxes coupled to transmission line resonators [5]. It is found that a solid state mesoscopic system may bear many features common to CQED in quantum optics [5]. The role of the atom is played by the Cooper pair box as an artificial atom and the cavity is replaced by the transmission line resonator. At large detuning , $i.e$ when the detuning parameter is larger than the coupling parameter between the field and the artificial atom, the dispersive regime takes place [6-8] whereas the interaction Hamiltonian can be considered as small perturbation. An alternative point of view is by applying a unitary transformation on the Hamiltonian and keeping to the first order of the ratio between the coupling and the detuning parameters [6,7]. Use of the effective Hamiltonian in this regime facilitates the calculations a great deal and in some cases closed form solutions are obtained. The environment would affect the considered system, because dissipations and fluctuations make the primary state of the system collapse from a correlated state into a statistical mixture state irreversibly [9]. Such effect amounts to decoherence or purity loss. Pure dephasing can be incorporated by introducing the appropriate Liadblad phase damping operators in the master equation for the total density operator [10].

In this article we shall investigate the evolution of a Cooper pair box model under phase damping and a nonclassical state of the photons. The organization of this article runs as follows. In section **2**, we setup the Hamiltonian of the model of a Cooper pair box in a resonator. We consider the dispersive regime and dissipation through phase damping. The solution of the master equation for the density operator is given. The linear entropy is investigated to discuss purity loss and decoherence. When the field initial state is considered to be a nonclassical state, namely a binomial state. This is exhibited in section **3**. Section **4** comprises the discussion and conclusion.

2 The model and the dispersive regime

We consider a Cooper pair box which is a mesoscopic super conducting island connected to a larger reservoir through a Josephson junction whose energy is E_J and capacitance C_J. It is voltage baised from a lead with a capacitance C_g to the island. When the charging energy $E_c = \frac{e^2}{2(C_J+C_g)}$ is much larger than $\frac{E_J}{4}$ and the gate charge representing the total polarization charge injected into the island $N_g = \frac{C_g V_g}{2e}$ is restricted to the range [0,1], then only a pair of the adjacent charge states are relevant, and the Hamiltonian in this case can be mapped to the one of a pseudo-spin-$\frac{1}{2}$ particle. At the charge degenercy point (where $N_g = \frac{1}{2}$), the Hamiltonian for the box coupled to a single mode of the resonator can be written in the form [5,11]

$$H = \hbar\omega(\hat{a}^\dagger \hat{a} + \frac{1}{2}) + \hbar\frac{\Omega}{2}\sigma_z - \frac{eC_g}{C_g + C_J}\sqrt{\frac{\hbar\omega}{LC}} \quad \sigma_x(\hat{a}^\dagger + \hat{a}) \tag{2.1}$$

when $\omega, \hat{a}(\hat{a}^\dagger)$ and Ω are the frequency, the annihilation (creation) operator for the field mode and $\Omega = \frac{E_J}{\hbar}$ the energy splitting of the qubit, σ_x and σ_z are the Pauli matrices describing the spin $\frac{1}{2}$ particle. When we neglect rapidly oscillating terms, the Hamiltonian (2.1) reduces to the standard Jaynes-Cummings Hamiltonian [12] with $\hbar g = \frac{-eC_g}{C_g+C_J}\sqrt{\frac{\hbar\omega}{LC}}$. The dispersive limit for this Hamiltonian is obtained when the interaction Hamiltonian can be considered as a perturbation to the non-interacting subsystems. It comes out due to large detuning such that $\frac{g}{\Delta} \ll 1$ where $\Delta = \Omega - \omega$ is the detuning parameter. The Hamiltonian of the system in the dispersive regime takes the form.

$$H_{eff} = \hbar(\omega + \frac{g^2}{\Delta}\sigma_z)\hat{a}^\dagger\hat{a} + \frac{\hbar}{2}(\Omega + \frac{g^2}{\Delta})\sigma_z \tag{2.2}$$

It is clear from this expression that the atom transition is Stark shifted by $\frac{g^2}{\Delta}(n+\frac{1}{2})$, with n the number of the photons in the field mode. Also it is apparent that $\hat{\sigma_z}$ and $\hat{n} = \hat{a}^\dagger\hat{a}$ are constants of motion of the effective Hamiltonian(2.2). Thus eigen states of either σ_z or $\hat{n}$ would be eigen states of H_{eff}.

The system is coupled to a reservoir through phase decay. We may understand this damping in the framework of the spin-depolarization observed in nuclear magnetic resonance experiments [13,14]. Besides the importance of the phase-damping model in describing physical situations, it gives insight in the problems involved by allowing for analytical treatments for different quantities that are needed of discussing related phenomena [15,16]. The master equation for the density operator $\rho(t)$ is given under phase-damping dissipation by the expression [10]

$$\frac{\partial\rho}{\partial t} = -i[H_{eff}, \rho] + \gamma\{2\hat{a}^\dagger\hat{a}\rho\hat{a}^\dagger\hat{a} - (\hat{a}^\dagger\hat{a})^2\rho - \rho(\hat{a}^\dagger\hat{a})^2\} \tag{2.3}$$

where H_{eff} is given by (2.2) and γ is the rate of dissipation.

We assume that at $t = 0$, the density operator for the system is given by

$$\rho(0) = \rho_B(0) \otimes \rho_F(0) \tag{2.4}$$

where ρ_B is the box initial density operator and $\rho_F(0)$is the initial density operator for the field, showing that the box is decoupled from the field initially. We further assume that both the box and the field are initially in pure states $i.e.$

$$\begin{aligned} \rho_B(0) &= |\Psi_B(0)\rangle\langle\Psi_B(0)| \qquad , |\Psi_B(0)\rangle = b_e|e\rangle + b_g|g\rangle \\ \rho_F(0) &= |\Psi_F(0)\rangle\langle\Psi_F(0)| \qquad , |\Psi_F(0)\rangle = \sum_n \beta_n|n\rangle \end{aligned} \tag{2.5}$$

where $|b_e|^2 + |b_g|^2 = 1$,$|e\rangle$ ($|g\rangle$) is the excited (ground) state of the box and β_n is the amplitude for the n<u>th</u> state of the field,$\sum_n |\beta_n|^2 = 1$. By writing $\lambda = \frac{g^2}{2\Delta}$, we obtain the solution of Eq.(2.3) under the conditions (2.4,2.5)in the form

$$\begin{aligned} \rho(t) = \sum_{m,n}\{&A(m,n)|e,m\rangle\langle e,n| + B(m,n)|e,m\rangle\langle g,n| \\ &+B^*(m,n)|g,m\rangle\langle e,n| + C(m,n)|g,m\rangle\langle g,n|\} \end{aligned} \tag{2.6}$$

where

$$\begin{aligned} A(m,n) &= |b_e|^2\beta_m\beta_n^* \quad e^{-\gamma t(m-n)^2 - 2i(m-n)\lambda t} \\ B(m,n) &= b_e b_g^*\beta_m\beta_n^* \quad e^{-\gamma t(m-n)^2 - 2i(m+n+1)\lambda t} \\ C(m,n) &= |b_g|^2\beta_m\beta_n^* \quad e^{-\gamma t(m-n)^2 + 2i(m-n)\lambda t} \end{aligned} \tag{2.7}$$

We wish to draw some remarks about the solution (2.7). If we take the box initially in one of its eigenstates excited (or ground)state $i.e.$ we take b_e(or b_g) $= 1$; then it will stay in this state and never jump to the other state, if the field assumes a Fock state at the beginning, then the diagonal terms in ρ will be time-independent, while the off diagonal terms are oscillatory. In both of these cases, which are eigenstates of either $\hat{\sigma_z}$ or $\hat{n}$, and hence eigenstates of H_{eff} as mentined before, the phase damping never exhibits any effect. Therefore for the phase damping to be effective for this model, both the box and the field should have neither Fock state nor eigenstates of the box but should have superposition states. Further, we note that the damping appears through the factor $\gamma(m-n)^2$ in the exponent, which means that the diagonal terms in the field states will show no damping. This shows up when we consider the density for the subsystem of either the box or the field. This will be discussed in the following section.

3 Purity loss

Once decoherence is introduced, pure states are to be changed into mixed states. However, one requires a state of high purity and large amount of entanglement in many cases of quantum information processing. Thus it is necessary to investigate purity losses in the system displayed in the previous section. We use the idempotency defect, defined by the linear entropy as a measure of the degree of the purity of the state, in analogy to the way the entanglement is treated in terms of the von Neumann entropy which has similar behavior [17,18]. In order to analyze what happens to the purity loss in the Cooper pair box we trace over the field states to get $\rho_B(t) = Tr_F\rho(t)$, while we trace over the box states to get $\rho_F(t) = Tr_B\rho(t)$ to discuss what happens to the field. The idempotency defect as a measure of purity loss is defined by

$$S_{B(F)} = Tr\rho_{B(F)}(1-\rho^2_{B(F)}) \tag{3.1}$$

while for the total system we use $S = Tr\rho(1-\rho)$. The state is pure when S or $S_{B(F)}$is zero, and purity is lost if it does not vanish. When we use the formula for $\rho(t)$ of Eqn.(2.6), we obtain

$$\begin{aligned}
\rho_B(t) &= \sum_n |\beta_n|^2\{|b_e|^2\,|e\rangle\langle e| + |b_g|^2\,|g\rangle\langle g| \\
&+ b_e b_g^* e^{-2i\lambda t(2n+1)}\,|e\rangle\langle g| + b_e^* b_g e^{2i\lambda t(2n+1)}\,|g\rangle\langle e|\} \\
\rho_F(t) &= \sum_n |\beta_n|^2\,|n\rangle\langle n| + \sum_{m\neq n}\beta_m\beta_n^* \quad e^{-\gamma t(m-n)^2} \\
&\times \{|b_e|^2 e^{-2i(m-n)\lambda t} + |b_g|^2 e^{2i(m-n)\lambda t}\}\,|m\rangle\langle n|
\end{aligned} \tag{3.2}$$

We can look at $\rho_B(t)$ as its $|\Psi_B\rangle = b_e\,|e\rangle + b_g\,|g\rangle$ has evolved in the n$\underline{th}$ sector of the field space as $b_e e^{-i\lambda t(2n+1)}\,|e\rangle + b_g e^{i\lambda t(2n+1)}\,|g\rangle$. These pure states with changes of phases are superposed with the weight $|\beta_n|^2$ for each n state of the field. Thus superposition gives rise to a loss of purity in the state of the box.

The remarks above mentioned after Eq.(2.7) appear here very clearly. The diagonal terms do not depend on the damping parameter. The diagonal and off-diagonal of ρ_B also do not depend on γ, while the off-diagonal terms oscillate regularly. The parameter γ appears only in the off-diagonal terms of the reduced density of the field.

4 Discussion and conclusion

In what follows we set to study the effects due to the field being in a nonclassical state. This state is taken to be the binomial state [19]

$$|\eta, M\rangle = \sum_{n=0}^{M} \beta_n |n\rangle, \qquad with \qquad \beta_n = \eta^n(\sqrt{1-|\eta|^2} \quad)^{M-n}\sqrt{\binom{M}{n}}$$

with mean photon number= $M|\eta|^2$ and we consider different values of η, M. It tends to the coherent state as $M \to \infty, \eta \to 0$ such that$M|\eta|^2$ is finite. Also we consider the effect of different values of the damping parameters γ. The effect of the atomic coherence is taken into consideration. These effects are illustrated in Figs. (4.1-4.4).

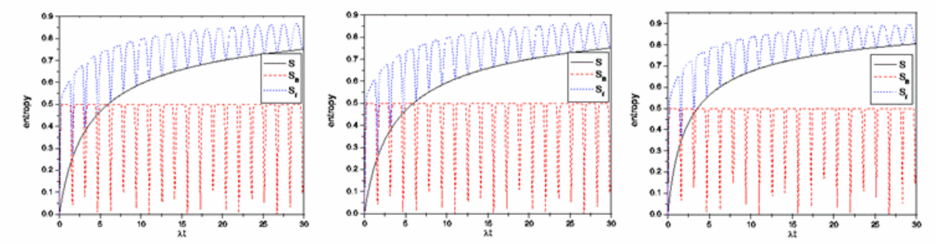

Figure 4.1: The linear entropy S (solid line), the box entropy S_B (dash line) and the field entropy S_F (dot line) as functions of the scaled time λt for (a): $M = 10$, $\eta^2 = 0.3$, $\gamma = 0.01$ and $b_e = b_g = \frac{1}{\sqrt{2}}$, (b): $M = 30$, $\eta^2 = 0.3$, $\gamma = 0.01$ and $b_e = b_g = \frac{1}{\sqrt{2}}$ and (c): $M = 50$, $\eta^2 = 0.3$, $\gamma = 0.01$ and $b_e = b_g = \frac{1}{\sqrt{2}}$.

We plot the linear entropy for the whole system S which shows monotonic increase as the time increases to settle to the fully mixed state with $S(\infty) = 1 - \sum_{n=0}^{M} \mid \beta_n \mid^4$. Also, we consider the linear entropy for the field $S_F(t)$. We note that this function oscillates with the $S(t)$ curve as its lower envelope. As time increases the amplitudes of the oscillations die out until the curve coincides asymptotically with $S(\infty)$ of the whole system. On contrast the curves of the atomic linear entropy have periodic behavior since it is of the form $S_A(t) = 2|b_e|^2|b_g|^2(1 - \sum_{m,n} |\beta_n|^2|\beta_m|^2 \cos 4\lambda t(m-n))$, which is periodic with period $\frac{\pi}{2}$ in the scaled time(λt). The effects of the different parameters are discussed in what follows in some detail.

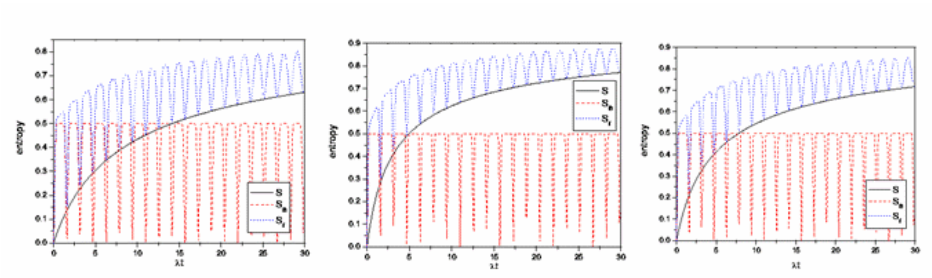

Figure 4.2: The linear entropy S (solid line), the box entropy S_B (dash line) and the field entropy S_F (dot line) as functions of the scaled time λt for (a): $M = 30, \eta^2 = 0.1, \gamma = 0.01$ and $b_e = b_g = \frac{1}{\sqrt{2}}$, (b): $M = 30, \eta^2 = 0.5, \gamma = 0.01$ and $b_e = b_g = \frac{1}{\sqrt{2}}$ and (c): $M = 30, \eta^2 = 0.8, \gamma = 0.01$ and $b_e = b_g = \frac{1}{\sqrt{2}}$.

(i) **The binomial parameter M:-**

In Figs. 4.1 we display the effect of this parameter by fixing the other parameters as follows $\frac{\gamma}{\lambda} = 0.01, b_e = b_g = \frac{1}{\sqrt{2}}$ and $\eta^2 = 0.3$ and we take $M = 10$ (Fig 4.1a),$M = 30$ (Fig 4.1b) and $M = 50$ (Fig 4.1c). We note that the amplitudes of the box linear entropy are not affected by the change in the number M of the photons in the state. For the field linear entropy, we note that by increasing the parameter M, the amplitudes of the function decrease and consequently settle to the stationary limit faster(compare Fig. 4.1a with 4.1b,4.1c).On the other hand, we find that the linear entropy for the total system tends faster to the stationary state by increasing the parameter M.

(ii) **The parameter η:-**

In Figs. 4.2 we exhibit the effect of the parameter η where we fix the values of γ, b_e, b_g as before, and fix $M = 30$ while η^2 takes the values 0.1(Fig. 4.2a),0.5(Fig. 4.2b),0.8(Fig. 4.2c). As before the box state is affected slightly by this parameter. The fluctuation in the fields entropy are affected by the change in η^2. The amplitudes of these fluctuations decrease as η^2 increases until it reaches $\frac{1}{2}$, then these amplitudes increase. These are due to the fact that as η^2 increases towards the value 1, the few states near the state $|M\rangle$ are the most effective . The effect of η^2 on the total entropy is almost the same as the parameter M.

(iii) **The box coherence:-**

The effect of changing b_e, b_g on the idempotency defect, is displayed in Figs. 4.3. By writing $|b_e| = \sin\theta$ and $|b_g| = \cos\theta$, we note that the dependence on $|b_e b_g|^2$ $i.e$ $(sin^2 2\theta)$ which has its maximum values when $\theta = \frac{\pi}{4}$. The amplitudes of the oscillations in both

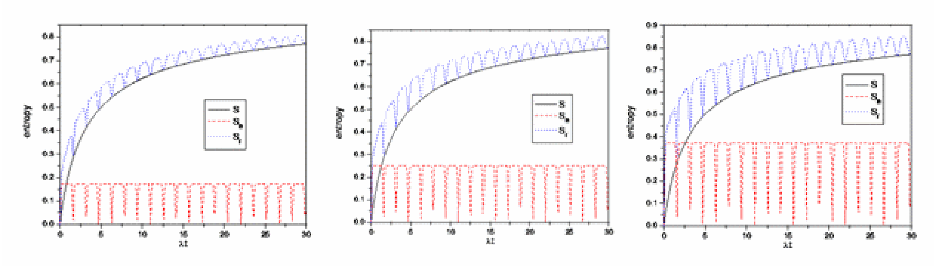

Figure 4.3: The linear entropy S (solid line), the box entropy S_B (dash line) and the field entropy S_F (dot line) as functions of the scaled time λt for (a): $M = 30$, $\eta^2 = 0.5$, $\gamma = 0.01$, $b_e = \sin\frac{\pi}{10}$ and $b_g = \cos\frac{\pi}{10}$, (b): $M = 30$, $\eta^2 = 0.5$, $\gamma = 0.01$, $b_e = \sin\frac{\pi}{8}$ and $b_g = \cos\frac{\pi}{8}$ and (c): $M = 30$, $\eta^2 = 0.5$, $\gamma = 0.01$, $b_e = \sin\frac{\pi}{6}$ and $b_g = \cos\frac{\pi}{6}$.

the box and field entropies dependence on $(sin^2 2\theta)$ is clearly shown in these Figs. The maximum amplitudes exist at $\theta = \frac{\pi}{4}$ as comparison of the figures in 4.3 with Fig. 4.2b shows. As θ moves away from this value the amplitudes decrease.

(iv) **The damping rate:-**

Figs 4.4 shows the dependence of the purity loss on the damping rate γ. This is exhibited through considering different values for $\gamma = 0.0001$ while keeping $M = 30, \eta^2 = 0.5$ and $b_e = b_g = \frac{1}{\sqrt{2}}$(Fig. 4.4a),$\gamma = 0.001$(Fig. 4.4b) and $\gamma = 0.1$(Fig. 4.4c). As it has been mentioned before,this parameter does not affect the state of the box. It is demonstrated here, the behavior of the quantity S_B which is the same in the different figures. For the field, we note that increasing γ results in suppressing the oscillations of the quantity S_F and brings it to the stationary limit faster. This fast arrival to the stationary limit also noted in the quantity S of the total system as comparison of the different figures of Figs. 4.4 and fig. 4.2b clearly shows.

A practical schemes for triggering evolution of entanglement between qubits has been presented [20]. The schemes are especially appealing as they require no experimentally difficult dynamical control and addressing of individual atoms. It is shown that the evolution of a stable or frozen entanglement can be triggered by varying the parameters of a given system such as coupling constants between atoms and the field modes or detunings between the atomic and field frequencies. Our discussion can be extended to test such observation. They also addressed the issue of a controlled (steered) evolution of entanglement between desired pairs of qubits that can be achieved by varying the parameters of a given system [21,22].

In conclusion the system of a Cooper pair box in interaction with a field in the binomial state in the dispersive limit shows under phase damping the following features: partial gain

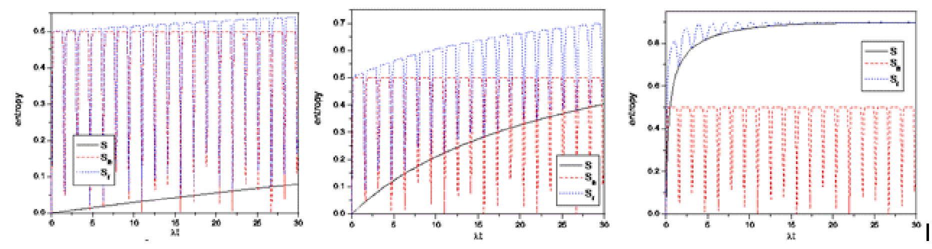

Figure 4.4: The linear entropy S (solid line), the box entropy S_B (dash line) and the field entropy S_F (dot line) as functions of the scaled time λt for (a): $M = 30$, $\eta^2 = 0.5$, $\gamma = 0.0001$ and $b_e = b_g = \frac{1}{\sqrt{2}}$,(b): $M = 30$, $\eta^2 = 0.5$, $\gamma = 0.001$ and $b_e = b_g = \frac{1}{\sqrt{2}}$ and (c): $M = 30$, $\eta^2 = 0.5$, $\gamma = 0.1$ and $b_e = b_g = \frac{1}{\sqrt{2}}$.

of purity for the box state with periodicity $\frac{\pi}{2\lambda}$ and the amount of purity loss is governed by the initial state of the box and slightly by the distribution of the photons in the field. The dependence on the coherency of the box is shown clearly. The box purity loss curves have their maximum heights when $b_e = b_g = \frac{1}{\sqrt{2}}$. The damping rate does not affect the purity of the box state. The field state is affected by the decay rate, however. It oscillates and settles to a mixed state in the limit as t tends to infinity. The heights of the oscillations are governed by the initial state of the box and the field. The dependence of the amplitudes of the oscillations on the field parameters varies according to the increase of M and η^2. These amplitudes decrease by increasing M. However for η^2, amplitudes decrease as η^2 increases until it reaches $\eta^2 = 0.5$; after that they increase. Therefore this may be dependent on the variance of the field state which has its maximum value at $\eta^2 = 0.5$. Correlation to the environment is noted through the idempotency defect of the total system. It is increased monotonically and settles to a statistical mixture as $t \to \infty$ which coincides with the limiting state of the field.

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