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On the instability of solutions of some fifth order nonlinear delay differential equations

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In this paper, by using the Lyapunov-Krasovskii functional approach, we study a kind of fifth order delay differential equations and obtain some new sufficient conditions for the instability of the zero solution of the equations considered. By this work, we improve some instability results obtained in the literature for a fifth order nonlinear differential equation without delay to the instability of solutions of certain fifth order nonlinear differential equations with delay.

Keywords: Instability; Lyapunov-Krasovskii functional; delay differential equation; fifth order.

1 Introduction

In a recent paper Li and Duan [8] proved some instability theorems for the fifth order nonlinear differential equation without delay,

$$x^{(5)} + f_5(x''')x^{(4)} + f_4(x'')x''' + f_3(x'') + f_2(x') + f_1(x) = 0. \tag{1}$$

In this paper, instead of Eq. (1), we consider the fifth order nonlinear delay differential equations of the form

$$\begin{aligned} &x^{(5)} + f_5(x''')x^{(4)} + f_4(x'')x''' + f_3(x, x(t-r), ..., x^{(4)}, x^{(4)}(t-r))x'' \\ &+f_2(x'(t-r)) + f_1(x(t-r)) = 0. \end{aligned} \tag{2}$$

We write Eq. (2) in system form as

$$\begin{aligned} &x' = y, y' = z, z' = w, w' = u, \\ &u' = -f_5(w)u - f_4(z)w - f_3(x, x(t-r), ..., u, u(t-r))z - f_2(y) - f_1(x) \\ &+ \int_{t-r}^{t} f_2'(y(s))z(s)ds + \int_{t-r}^{t} f_1'(x(s))y(s)ds, \end{aligned} \tag{3}$$

where r is a positive constant, the primes in Eq. (2) denote differentiation with respect to $t,\, t \in \Re_+,\ \ \Re_+ = [0, \infty)$; $f_5,\, f_4,\, f_3,\, f_2$ and f_1 are continuous functions on $\Re$, $\Re$, $\Re^{10}$, $\Re$ and $\Re$, respectively, with $f_1(0) = f_2(0) = 0$, and satisfy a Lipschitz condition in their respective arguments. Hence, the existence and uniqueness of the solutions of Eq. (2) are guaranteed (see Èl'sgol'ts [2, pp.14, 15]). We assume in what follows that the functions f_1 and f_2 are also differentiable, and $x(t),\ y(t),\, z(t),\ w(t)$ and $u(t)$ are abbreviated as $x,\ y,\ z,\ w$ and u, respectively.

It should be noted that, since 1978 by now, the instability of solutions of various fifth order nonlinear scalar and vector differential equations without delay has been investigated and is still being studied by some authors (see Ezeilo [3-5], Li and Yu [9], Sadek [11], Sun and Hou [12], Tiryaki [13], Tunç [14-16], Tunç and Erdoğan [17], Tunç and Karta [18], Tunç and Şevli [19]). In the all mentioned papers, the authors used suitable Lyapunov's functions and based on the Krasovskii's properties (seeKrasovskii [7]) to discuss the instability of solutions of the equations considered therein.

Further, the qualitative theory of nonlinear differential equations of higher order has wide applications in science and technology (see, for example, Chlouverakis and Sprott [1] and Linz [10]). It is also crucial to obtain information on the qualitative behaviors of solutions of differential equations while there is no analytical expression for solutions. For this purpose, the theory of Lyapunov functions and functionals is a global and the most effective approach toward determining qualitative behaviors of solutions of higher order nonlinear differential equations. On the other hand, from the past by now, the construction and definition of suitable Lyapunov functions and functionals for higher order differential equations without delay and with delay remain as a general problem in the literature. In this paper, by defining two appropriate Lyapunov functionals we prove our results. The motivation for this paper comes from the above mentioned papers and that of Ko [6]. Our aim is to carry out some results established by Li and Duan [8] to nonlinear delay differential equations of the form (2) for the instability of solutions. To the best of our observations, there exists no paper establishing conditions for the instability of the solutions of delay differential equations of fifth order in the literature. By this work, we improve the mentioned results from the case of without delay to the case of delay. Thus, our results are completely different from those mentioned above. Here, we only study the theoretical aspect of the subject and give two examples to illustrate the theoretical analysis in this work.

Let $r \geq 0$ be given, and let $C = C([-r, 0],\ \ \Re^n)$ with

$$\|\phi\| = \max_{-r \leq s \leq 0} |\phi(s)|\,, \phi \in C.$$

For $H > 0$ define $C_H \subset C$ by

$$C_H = \{\phi \in C : \|\phi\| < H\}.$$

If $x\,:\,[-r,\ a] \to \Re^n$ is continuous, $0 < A \leq \infty$, then, for each t in $[0, A)$, x_t in C is defined by

$$x_t(s) = x(t+s), -r \leq s \leq 0, t \geq 0.$$

Let G be an open subset of C and consider the general autonomous delay differential system with finite delay

$$\dot{x} = F(x_t), x_t = x(t+\theta), -r \leq \theta \leq 0, t \geq 0,$$

where $F(0) = 0$, $F : G \to \Re^n$ is continuous and maps closed and bounded sets into bounded sets. It follows from the conditions on F that each initial value problem

$$\dot{x} = F(x_t), x_0 = \phi \in G$$

has a unique solution defined on some interval $[0, A)$, $0 < A \leq \infty$. This solution will be denoted by $x(\phi)(.)$ so that $x_0(\phi) = \phi$.

Definition. The zero solution, $x = 0$, of $\dot{x} = F(x_t)$ is stable if for each $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon) > 0$ such that $\|\phi\| < \delta$ implies that $|x(\phi)(t)| < \varepsilon$ for all $t \geq 0$. The zero solution is said to be unstable if it is not stable.

Theorem A. Suppose there exists a Lyapunov function $V : G \to \Re_+$ such that $V(0) = 0$ and $V(x) > 0$ if $x \neq 0$. If either

(i) $\dot{V}(\phi) > 0$ for all ϕ in G for which

$$V[\phi(0)] = \max_{-s \leq t \leq 0} V[\phi(s)] > 0$$

or **(ii)** $\dot{V}(\phi) > 0$ for all ϕ in G for which

$$V[\phi(0)] = \min_{-s \leq t \leq 0} V[\phi(s)] > 0 \ ,$$

then the solution $x = 0$ of $\dot{x} = F(x_t)$ is unstable (see Ko [6]).

2 Main results

Our first main result is the following theorem.

Theorem 1. In addition to all the assumptions imposed on the functions f_5, f_4, f_3, f_2 and f_1 in Eq. (2), assume that there exist constants a_3, a_4, $\bar{a}_5$ and a_5 such that the following conditions hold:

$$f_1(0) = f_2(0) = 0, f_1(x) \neq 0,\ (x \neq 0), f_2(y) \neq 0, (y \neq 0),$$

$$\bar{a}_5 \leq f_1'(x) \leq a_5 < 0, 0 \leq |f_2'(y)| \leq a_4, (a_4 > 0), f_5(w) \leq 0 \text{ for arbitrary } x,\ y,\ w$$

and

$$f_3(x, x(t-r), ..., u, u(t-r)) \geq a_3 > 0 \text{ for arbitrary } x,\ x(t-r), ...,\ u,\ u(t-r).$$

Then, the zero solution, $x = 0$, of Eq. (2) is unstable provided that

$$r < \min\left\{\frac{2a_5}{\bar{a}_5},\ \frac{2a_3}{2a_4 - \bar{a}_5}\right\}.$$

Remark. To prove Theorem 1, we follow a standard pattern. Namely, we need to show that, subject to the conditions in the theorem, there exists indeed a Lyapunov functional $V = V(x_t, y_t, z_t, w_t, u_t)$ which satisfies the three Krasovskii properties (K_1), (K_2) and (K_3) (see Krasovskii [7]): (K_1) In every neighborhood of $(0,0,0,0,0)$, there exists a point $(\xi, \eta, \zeta, \mu, \sigma)$ such that $V(\xi, \eta, \zeta, \mu, \sigma) > 0$, (K_2) the time derivative $\dot{V} = \frac{d}{dt}V(x_t, y_t, z_t, w_t, u_t)$ along solution paths of the corresponding equivalent differential system for Theorem 1 is positive semi-definite, (K_3) the only solution (x, y, z, w, u) = $(x(t), y(t), z(t), w(t), u(t))$ of (3) which satisfies $\frac{d}{dt}V(x_t, y_t, z_t, w_t, u_t) = 0$ is the trivial solution $(0,0,0,0,0)$. It worth mentioning that the satisfaction of the three Krasovskii properties (K_1), (K_2) and (K_3) is a sufficient condition for the instability of the zero solution of Eq. (2). That is, the Krasovskii properties guarantee the instability of the zero solution of Eq. (2).

Proof. Consider the Lyapunov functional $V = V(x_t, y_t, z_t, w_t, u_t)$ defined by

$$\begin{aligned} V \; &= \tfrac{1}{2}w^2 - yf_1(x) - zu - z\int_0^w f_5(s)ds - \int_0^z f_4(s)sds - \int_0^y f_2(s)ds \\ &-\lambda_1 \int_{-r}^0 \int_{t+s}^t y^2(\theta)d\theta ds - \lambda_2 \int_{-r}^0 \int_{t+s}^t z^2(\theta)d\theta ds, \end{aligned} \tag{4}$$

where s is a real variable such that the integrals $\int_{-r}^0 \int_{t+s}^t y^2(\theta)d\theta ds$ and $\int_{-r}^0 \int_{t+s}^t z^2(\theta)d\theta ds$ are non-negative, and λ_1 and λ_2 are some positive constants which will be determined later in the proof.

It is clear that

$$V(0,0,0,\varepsilon,0) = \frac{1}{2}\varepsilon^2 > 0$$

for all sufficiently small ε. Hence, in every neighborhood of the origin, $(0,0,0,0,0)$, there exists a point $(0,0,0,\varepsilon,0)$ such that $V(0,0,0,\varepsilon,0) > 0$, which shows that V has the property (K_1).

By an elementary differentiation, time derivative of the functional $V(x_t, y_t, z_t, w_t, u_t)$ in (4) along the solutions of (3)gives that

$$\begin{aligned} \tfrac{d}{dt}V(x_t, y_t, z_t, w_t, u_t) \; &= -f_1'(x)y^2 + f_3(x, x(t-r), ..., u, u(t-r))z^2 - w\int_0^w f_5(s)ds \\ &-z\int_{t-r}^t f_2'(y(s))z(s)ds - z\int_{t-r}^t f_1'(x(s))y(s)ds \\ &-\lambda_1 r y^2 + \lambda_1 \int_{t-r}^t y^2(s)ds - \lambda_2 r z^2 + \lambda_2 \int_{t-r}^t z^2(s)ds. \end{aligned}$$

The assumptions of Theorem 1 and the estimate $2|mn| \leq m^2 + n^2$ imply that

$$\begin{aligned} -z\int_{t-r}^{t} f_1'(x(s))y(s)ds &\geq |z| \int_{t-r}^{t} f_1'(x(s))\ |y(s)|\, ds \\ &\geq \tfrac{1}{2}\int_{t-r}^{t} f_1'(x(s))\ (z^2(t)+y^2(s))ds \\ &\geq \tfrac{1}{2}\bar{a}_5 r z^2 + \tfrac{1}{2}\bar{a}_5 \int_{t-r}^{t} y^2(s)ds, \end{aligned}$$

$$\begin{aligned} -z\int_{t-r}^{t} f_2'(y(s))z(s)ds &\geq -|z| \int_{t-r}^{t} |f_2'(y(s))|\ |z(s)|\, ds \\ &\geq -\tfrac{1}{2}\int_{t-r}^{t} |f_2'(y(s))|\ (z^2(t)+z^2(s))ds \\ &\geq -\tfrac{1}{2}a_4 r z^2 - \tfrac{1}{2}a_4 \int_{t-r}^{t} z^2(s)ds \end{aligned}$$

so that

$$\begin{aligned} \tfrac{d}{dt}V(x_t,y_t,z_t,w_t,u_t) &\geq (-a_5-\lambda_1 r)y^2 + \{a_3 + (2^{-1}\bar{a}_5 - 2^{-1}a_4 - \lambda_2)r\}z^2 \\ &+ \left(\tfrac{1}{2}\bar{a}_5+\lambda_1\right)\int_{t-r}^{t} y^2(s)ds + \left(\lambda_2 - \tfrac{1}{2}a_4\right)\int_{t-r}^{t} z^2(s)ds. \end{aligned}$$

Let $\lambda_1 = -\frac{1}{2}\bar{a}_5$ and $\lambda_2 = \frac{1}{2}a_4$. Hence

$$\frac{d}{dt}V(x_t,y_t,z_t,w_t,u_t) \geq (-a_5 + 2^{-1}\bar{a}_5 r)y^2 + \{a_3 + (2^{-1}\bar{a}_5 - a_4)r\}z^2 > 0$$

provided that $r < \min\left\{\frac{2a_5}{\bar{a}_5},\ \frac{2a_3}{2a_4-\bar{a}_5}\right\}$, which verifies that V has the property (K_2).

On the other hand, $\frac{d}{dt}V(x_t,y_t,z_t,w_t,u_t) = 0$ if and only if $y = z = 0$, which implies that

$$y = z = w = u = 0.$$

Besides, by $f_1(0) = f_2(0) = 0$, $f_1(x) \neq 0$ for all $x \neq 0$, $f_2(y) \neq 0$ for all $y \neq 0$ and the system (3), we can conclude that $\frac{d}{dt}V(x_t,y_t,z_t,w_t,u_t) = 0$ if and only if $x = y = z = w = u = 0$. Thus, the property (K_3) is fulfilled by V relative to (2). By the foregoing discussion, we conclude that the zero solution of Eq. (2) is unstable. The proof of Theorem 1 is completed.

Example 1. Consider nonlinear fifth order delay differential equation of the form

$$\begin{aligned} &x^{(5)} - \tfrac{1}{1+(x''')^2}x^{(4)} + 9x''' + \{2+\exp(-x^2 - x^2(t-r) - ... - u^2 - u^2(t-r))\}x'' \\ &+ \sin x'(t-r) - x(t-r) - 4arctgx(t-r) = 0. \end{aligned} \tag{5}$$

We write Eq. (5) in system form as follows

$$x' = y, y' = z, z' = w, w' = u,$$

$$\begin{aligned} u' &= \tfrac{u}{1+w^2} - 9w - \{2 + \exp(-x^2 - ... - u^2(t-r)\}z \\ &- \sin y + x + 4arctgx - \int\limits_{t-r}^{t} y(s)ds \\ &+ \int\limits_{t-r}^{t} \cos y(s)z(s)ds - 4\int\limits_{t-r}^{t} \tfrac{1}{1+x^2(s)}y(s)ds. \end{aligned}$$

It follows that Eq. (5) is special case of Eq. (2) and

$$f_5(w) = -\frac{1}{1+w^2} \leq 0,$$

$$f_4(z) = 9,$$

$$f_3(x, ..., u(t-r)) = 2 + \exp\{-x^2 - ... - u^2(t-r)\} \geq 2 = a_3,$$

$$f_2(y) = \sin y, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2},$$

$$f_2(0) = 0, f_2'(y) = \cos y, |\cos y| \leq 1 = a_4,$$

$$f_1(x) = -x - 4arctgx, -\frac{\pi}{2} < x < \frac{\pi}{2},$$

$$f_1(0) = 0, f_1'(x) = -1 - \frac{4}{1+x^2},$$

and

$$\bar{a}_5 = -5 \leq -1 - \frac{4}{1+x^2} \leq -1 = a_5.$$

In view of the above estimates, we conclude that all the assumptions of Theorem 1 hold. Hence, we conclude that the zero solution $x = 0$ of Eq. (5) is unstable provided that $r < \frac{2}{5}$.

We now consider the special case of Eq. (2) with $f_2(x'(t-r)) = f_2(x'(t))$, namely, the fifth order delay differential equation

$$\begin{aligned} &x^{(5)} + f_5(x''')x^{(4)} + f_4(x'')x''' + f_3(x, x(t-r), ..., x^{(4)}, x^{(4)}(t-r))x'' \\ &+f_2(x'(t)) + f_1(x(t-r)) = 0. \end{aligned} \tag{6}$$

We write Eq. (6) in system form as

$$\begin{aligned} &x' = y, y' = z, z' = w, w' = u, \\ &u' = -f_5(w)u - f_4(z)w - f_3(x, x(t-r), ..., u, u(t-r))z - f_2(y) \\ &-f_1(x) + \int\limits_{t-r}^{t} f_1'(x(s))y(s)ds. \end{aligned} \tag{7}$$

Our second main result is the following theorem.

Theorem 2. In addition to all the assumptions imposed to the functions f_5, f_4, f_3, f_2 and f_1 in Eq. (6), assume that there exist constants a_3, $\bar{a}_5$ and a_5 such that the following conditions hold:

$$f_1(0) = 0, f_1(x) \neq 0, \ (x \neq 0), \bar{a}_5 \geq f_1'(x) \geq a_5 > 0, \ \ (x \neq 0),$$

$$f_2(y) \neq 0, (y \neq 0), f_5(w) \geq 0 \text{ for arbitrary } x,\ w,$$

$$f_3(x, x(t-r), ..., u, u(t-r)) \leq a_3 < 0 \text{ for arbitrary } x,\ x(t-r), ..., u, u(t-r).$$

Then, the zero solution, $x = 0$, of Eq. (6) is unstable provided that

$$r < \frac{2}{\bar{a}_5} \min\{a_5, -a_3\}.$$

Proof. Consider the Lyapunov functional $V_1 = V_1(x_t, y_t, z_t, w_t, u_t)$ defined by

$$\begin{aligned} V_1 &= -\tfrac{1}{2}w^2 + yf_1(x) + zu + z\int_0^w f_5(s)ds + \int_0^z f_4(s)sds + \int_0^y f_2(s)ds \\ &\quad -\lambda_3 \int_{-r}^{0} \int_{t+s}^{t} y^2(\theta)d\theta ds, \end{aligned} \tag{8}$$

where s is a real variable such that the integral $\int_{-r}^{0} \int_{t+s}^{t} y^2(\theta)d\theta ds$ is non-negative, and λ_3 is a positive constant which will be determined later in the proof.

Let $M = \max_{|z|\leq 1} |f_4(z)|$, there exists a positive constant e such that $Me < 1$ and $0 < e < 1$.

Then, it follows that

$$V_1(0, 0, e^2, 0, e) = e^3 + \int_0^{e^2} f_4(s)sds \geq e^3 - \frac{1}{2}Me^4 > 0$$

for all sufficiently small e. Hence, in every neighborhood of the origin, $(0, 0, 0, 0, 0)$, there exist a point $(0, 0, e^2, 0, e)$ such that $V_1(0, 0, e^2, 0, e) > 0$, which shows that V_1 has the property (K_1). By an elementary differentiation, time derivative of the functional $V_1(x_t, y_t, z_t, w_t, u_t)$ in (8) along the solutions of (7)yields that

$$\begin{aligned} \tfrac{d}{dt}V_1(x_t, y_t, z_t, w_t, u_t) &= f_1'(x)y^2 - f_3(x, x(t-r), ..., u, u(t-r))z^2 + w\int_0^w f_5(s)ds \\ &\quad + z\int_{t-r}^{t} f_1'(x(s))y(s)ds - \lambda_3 r y^2 + \lambda_3 \int_{t-r}^{t} y^2(s)ds. \end{aligned}$$

The assumption $\bar{a}_5 \geq f_1'(x) \geq a_5 > 0$ and the estimate $2|mn| \leq m^2 + n^2$ imply that

$$\begin{aligned} z\int_{t-r}^{t} f_1'(x(s))y(s)ds &\geq -|z| \int_{t-r}^{t} f_1'(x(s))\ |y(s)|\, ds \\ &\geq -\tfrac{1}{2}\int_{t-r}^{t} f_1'(x(s))\ (z^2(t) + y^2(s))ds \\ &\geq -\tfrac{1}{2}\bar{a}_5 r z^2 - \tfrac{1}{2}\bar{a}_5 \int_{t-r}^{t} y^2(s)ds, \end{aligned}$$

$$f_1'(x)y^2 \geq a_5 y^2 \geq 0, -f_3(x, x(t-r), ..., u, u(t-r))z^2 \geq -a_3 z^2$$

so that

$$\frac{d}{dt}V_1(x_t,y_t,z_t,w_t,u_t) = (a_5-\lambda_3 r)y^2+\{-a_3-2^{-1}\bar{a}_5 r\}z^2+\left(\lambda_3-\frac{1}{2}\bar{a}_5\right)\int\limits_{t-r}^{t} y^2(s)ds.$$

Let $\lambda_3 = \frac{1}{2}\bar{a}_5$. Hence

$$\frac{d}{dt}V_1(x_t,y_t,z_t,w_t,u_t) = (a_5-2^{-1}\bar{a}_5 r)y^2+\{-a_3-2^{-1}\bar{a}_5 r\}z^2 > 0$$

provided that $r < 2\min\left\{\frac{a_5}{\bar{a}_5},\ -\frac{a_3}{\bar{a}_5}\right\}$, which verifies that V_1 has the property (K_2). The remainder of the proof follows as before, Theorem 1.

Example 2. Consider nonlinear fifth order delay differential equation of the form

$$\begin{aligned}&x^{(5)}+\tfrac{1}{1+(x''')^2}x^{(4)}+x'''-\{3+\exp(-x^2-x^2(t-r)-...-u^2-u^2(t-r))\}x''\\&+x'(t)-x(t-r)-4arctgx(t-r)=0.\end{aligned} \tag{9}$$

We write Eq. (9) in system form as follows

$$\begin{aligned}&x'=y, y'=z, z'=w, w'=u,\\&u'=-\tfrac{u}{1+w^2}-w+\{3+\exp(-x^2-...-u^2(t-r)\}z\\&-y+x+4arctgx\\&-\int\limits_{t-r}^{t} y(s)ds-4\int\limits_{t-r}^{t}\tfrac{1}{1+x^2(s)}y(s)ds.\end{aligned}$$

It follows that Eq. (9) is special case of Eq. (6) and

$$f_5(w)=\frac{1}{1+w^2}\geq 0,$$

$$f_4(z)=1,$$

$$f_3(x,...,u(t-r))=-3-\exp\{-x^2-...-u^2(t-r)\}\leq -3=a_3,$$

$$f_2(y)=y, f_2(0)=0,$$

$$f_1(x)=x+4arctgx, -\frac{\pi}{2}<x<\frac{\pi}{2}, f_1'(x)=1+\frac{4}{1+x^2}$$

and

$$\bar{a}_5=5\geq 1+\frac{4}{1+x^2}\geq 1=a_5.$$

In view of the above estimates, we conclude that all the assumptions of Theorem 2 hold. Hence, we conclude that the zero solution, $x=0$, of Eq. (9) is unstable provided that $r<\frac{2}{5}$.

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