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# Application of the Local Fractional Variational Iteration Method to Solve System of Coupled Partial Differential Equations Involving Local Fractional Operator

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**Abstract:** In this paper, the local fractional variational iteration method (LFVIM) is employed to obtain approximate analytical solution to system of linear/nonlinear coupled partial differential equations within local fractional operator. LFVIM yields solutions in convergent series forms with easily computable terms. Generally, the closed form of the exact solution or its expansion is obtained without any noise terms. Test examples demonstrate the efficiency of local fractional variational iteration method.

**Keywords:** Linear/Nonlinear coupled partial differential equations, Local fractional variational iteration method, Approximate analytical solutions, Local fractional operator

## 1 Introduction

There are many analytical and numerical methods used to solve local fractional partial differential equations such as, local fractional function decomposition method [1,2], local fractional Adomian decomposition method [2,3], local fractional series expansion method [4,5], local fractional Laplace transform method [6,7], local fractional Fourier series method [8], local fractional Laplace decomposition method [9,10], local fractional Laplace variational iteration method [11,12,13], and another methods.

The local fractional variational iteration method was applied to solve the partial differential equations arising in mathematical physics, for example, Laplace equation [2], wave and diffusion equations [14,15], Fokker Plank equation [16], heat conduction problem [17,18], damped wave and dissipative wave equations [19], Helmholtz equation [20], Poisson equation [21], and also it used to solve integro-differential equations [22]. In this paper, our aim is to present the local fractional variational iteration method, and to used it to solve the system of coupled partial differential equations within local fractional derivative operators. The structure of the paper is as follows. In Section 2, we give analysis of the local fractional variational iteration method. In Section 3, we consider some illustrative examples. Finally, in Section 4, we present our conclusions.

## 2 Local Fractional Variational Iteration Method (LFVIM)

In order to illustrate variational iteration method, we investigate systems of local fractional partial differential equations as follows:

$$L_\alpha u_i(x,t) + R_i(U) + N_i(U) = g_i(x,t), i = 1,2,\ldots,n, \quad (1)$$

with the initial conditions

$$u_i(x,0) = f_i(x), \quad (2)$$

where $U = [u_1(x,t), u_2(x,t), \ldots, u_n(x,t)]$, $L_\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$ denotes linear local fractional derivative operator of order $\alpha$, $R_i$ denote remaining linear local fractional derivative

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operators, $N_i$ denote nonlinear local fractional derivative operators and $g_i(x,t)$ is a source term of nondifferentiable functions.

According to the rule of local fractional variational iteration method [18,19] , the correction local fractional functional for (1) can be set in the form:

$$u_{i(m+1)}(t) = u_{im}(t) + {}_0I_t^{(\alpha)}\left(\frac{\lambda_i(\xi)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_{im}(\xi) + R_i(\widetilde{U}_m) + N_i(\widetilde{U}_m) - g_i(\xi)\right]\right), \quad (3)$$

where $\widetilde{U}_m = [\widetilde{u}_{1m}, \widetilde{u}_{2m}, \ldots, \widetilde{u}_{nm}]$, $\frac{\lambda_i(\xi)^\alpha}{\Gamma(1+\alpha)}$ are fractal Lagrange multipliers, and the local fractional operator be defined as

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)}\lim_{\triangle t\longrightarrow 0}\sum_{j=0}^{N-1} f(t_j)(\triangle t_j)^\alpha. \end{aligned}$$

with the partition of the interval $[a,b]$ is denoted as $(t_j,t_{j+1}), j = 0,...,N-1, t_0 = a$ and $t_N = b$ with $\triangle t_j = t_{j+1} - t_j$ and $\triangle t = \max\{\triangle t_0, \triangle t_1, \ldots\}$.

Making the local fractional variation of (3), we have

$$\delta^\alpha u_{i(m+1)}(t) = \delta^\alpha u_{im}(t) + {}_0I_t^{(\alpha)} \delta^\alpha\left(\frac{\lambda_i(\xi)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_{im}(\xi) + R_i(\widetilde{U}_m) + N_i(\widetilde{U}_m) - g_i(\xi)\right]\right), \quad (4)$$

The extremum condition of $u_{n+1}(x,t)$ is given by

$$\delta^\alpha u_{i(m+1)}(x,t) = 0. \quad (5)$$

From (4) and (5), we have the following stationary conditions

$$1 + \frac{\lambda_i(\xi)^\alpha}{\Gamma(1+\alpha} |_{\xi=t} = 0, \left[\frac{\lambda_i(\xi)^\alpha}{\Gamma(1+\alpha}\right]^{(\alpha)} |_{\xi=t} = 0. \quad (6)$$

This in turn gives

$$\frac{\lambda_i(\xi)^\alpha}{\Gamma(1+\alpha} = -1. \quad (7)$$

so that iteration is expressed as

$$u_{i(m+1)}(t) = u_{im}(t) + {}_0I_t^{(\alpha)}\left[L_\alpha u_{im}(\xi) + R_i(U_m) + N_i(U_m) - g_i(\xi)\right], \quad (8)$$

Finally, we obtain the solution of (1) as follows:

$$u_i(x,t) = \lim_{m\to\infty} u_{im}(x,t) \quad (9)$$

## 3 Applications

To illustrate local fractional variational iteration method for system of local fractional coupled partial differential equations we take three examples in this section.

*Example 1.*Let us consider the system of linear coupled partial differential equations involving local fractional operator:

$$\begin{aligned} &\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + \frac{\partial^\alpha v(x,t)}{\partial x^\alpha} - u(x,t) - v(x,t) = 0, \\ &\frac{\partial^\alpha v(x,t)}{\partial t^\alpha} + \frac{\partial^\alpha u(x,t)}{\partial x^\alpha} - v(x,t) - u(x,t) = 0, \end{aligned} \quad (10)$$

with initial conditions

$$\begin{aligned} u(x,0) &= \sinh_\alpha(x^\alpha), \\ v(x,0) &= \cosh_\alpha(x^\alpha). \end{aligned} \quad (11)$$

According to local fractional variational iteration method, formula (8) for (10) can be expressed in the following form:

$$\begin{aligned} u_{m+1}(x,t) &= u_m - {}_0I_t^{(\alpha)}\left[\frac{\partial^\alpha u_m}{\partial t^\alpha} + \frac{\partial^\alpha v_m}{\partial x^\alpha} - u_m - v_m\right], \\ v_{m+1}(x,t) &= v_m - {}_0I_t^{(\alpha)}\left[\frac{\partial^\alpha v_m}{\partial t^\alpha} + \frac{\partial^\alpha u_m}{\partial x^\alpha} - v_m - u_m\right]. \end{aligned} \quad (12)$$

Suppose that an initial approximation has the following form which satisfies the initial condition:

$$\begin{aligned} u_0(x,t) &= \sinh_\alpha(x^\alpha), \\ v_0(x,t) &= \cosh_\alpha(x^\alpha). \end{aligned} \quad (13)$$

Now by iteration formula (12) , we obtain the following approximations:

$$\begin{aligned} u_1(x,t) &= u_0(x,t) - {}_0I_t^{(\alpha)}\left[\frac{\partial^\alpha u_0}{\partial t^\alpha} + \frac{\partial^\alpha v_0}{\partial x^\alpha} - u_0 - v_0\right] \\ v_1(x,t) &= v_0(x,t) - {}_0I_t^{(\alpha)}\left[\frac{\partial^\alpha v_0}{\partial t^\alpha} + \frac{\partial^\alpha u_0}{\partial x^\alpha} - v_0 - u_0\right] \\ &= \sinh_\alpha(x^\alpha) + \frac{1}{\Gamma(1+\alpha)}\int_0^t [\cosh_\alpha(x^\alpha)]\,(d\tau)^\alpha \\ &= \cosh_\alpha(x^\alpha) + \frac{1}{\Gamma(1+\alpha)}\int_0^t [\sinh_\alpha(x^\alpha)]\,(d\tau)^\alpha \\ &= \sinh_\alpha(x^\alpha) + \frac{t^\alpha}{\Gamma(1+\alpha)}\cosh_\alpha(x^\alpha), \\ &= \cosh_\alpha(x^\alpha) + \frac{t^\alpha}{\Gamma(1+\alpha)}\sinh_\alpha(x^\alpha), \end{aligned}$$

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$$u_2(x,t) = u_1(x,t) - {}_0I_t^{(\alpha)}\left[\frac{\partial^\alpha u_1}{\partial t^\alpha}+\frac{\partial^\alpha v_1}{\partial x^\alpha}-u_1-v_1\right]$$

$$\begin{aligned}
v_2(x,t) &= v_1(x,t) - {}_1I_t^{(\alpha)}\left[\frac{\partial^\alpha v_1}{\partial t^\alpha}+\frac{\partial^\alpha u_1}{\partial x^\alpha}-v_1-u_1\right]\\
&= \sinh_\alpha(x^\alpha)+\frac{t^\alpha}{\Gamma(1+\alpha)}\cosh_\alpha(x^\alpha)\\
&\quad+\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\tau^\alpha}{\Gamma(1+\alpha)}\cosh_\alpha(x^\alpha)\right](d\tau)^\alpha\\
&= \cosh_\alpha(x^\alpha)+\frac{t^\alpha}{\Gamma(1+\alpha)}\sinh_\alpha(x^\alpha)\\
&\quad+\frac{1}{\Gamma(1+\alpha)}\int_0^t\left[\frac{\tau^\alpha}{\Gamma(1+\alpha)}\sinh_\alpha(x^\alpha)\right](d\tau)^\alpha\\
&= \sinh_\alpha(x^\alpha)+\frac{t^\alpha}{\Gamma(1+\alpha)}\cosh_\alpha(x^\alpha)\\
&\quad+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\sinh_\alpha(x^\alpha),\\
&= \cosh_\alpha(x^\alpha)+\frac{t^\alpha}{\Gamma(1+\alpha)}\sinh_\alpha(x^\alpha)\\
&\quad+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\cosh_\alpha(x^\alpha),
\end{aligned}$$

and so on for other components. The series solutions are therefore given by

$$\begin{aligned}
u(x,t) &= \sinh_\alpha(x^\alpha)\left[1+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^{4\alpha}}{\Gamma(1+4\alpha)}+\cdots\right]\\
&\quad+\cosh_\alpha(x^\alpha)\left[\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots\right],\\
v(x,t) &= \cosh_\alpha(x^\alpha)\left[1+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^{4\alpha}}{\Gamma(1+4\alpha)}+\cdots\right]\\
&\quad+\sinh_\alpha(x^\alpha)\left[\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots\right],
\end{aligned}$$

and finally in its closed form gives

$$\begin{aligned}
u(x,t) &= \sinh_\alpha(x^\alpha+t^\alpha),\\
v(x,t) &= \cosh_\alpha(x^\alpha+t^\alpha).
\end{aligned}\tag{14}$$

*Example 2.*Consider the following system of coupled Burger's equations with local fractional derivative [23]:

$$\begin{aligned}
&\frac{\partial^\alpha u}{\partial t^\alpha}+\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}}-2u\frac{\partial^\alpha u}{\partial x^\alpha}-\frac{\partial^\alpha[uv]}{\partial x^\alpha}=0,\\
&\frac{\partial^\alpha v}{\partial t^\alpha}+\frac{\partial^{2\alpha} v}{\partial x^{2\alpha}}-2v\frac{\partial^\alpha v}{\partial x^\alpha}-\frac{\partial^\alpha[uv]}{\partial x^\alpha}=0,
\end{aligned}\tag{15}$$

subject to the initial conditions

$$\begin{aligned}
u(x,0) &= \cos_\alpha(x^\alpha),\\
v(x,0) &= \cos_\alpha(x^\alpha).
\end{aligned}\tag{16}$$

According to local fractional variational iteration method, formula (8) for (15) can be expressed in the following form:

$$\begin{aligned}
&u_{m+1}(x,t) = u_m(x,t)-\frac{1}{\Gamma(1+\alpha)}\times\\
&\int_0^t\left[\frac{\partial^\alpha u_m}{\partial\tau^\alpha}+\frac{\partial^{2\alpha}u_m}{\partial x^{2\alpha}}-2u_m\frac{\partial^\alpha u_m}{\partial x^\alpha}-\frac{\partial^\alpha[u_mv_m]}{\partial x^\alpha}\right](d\tau)^\alpha,\\
&v_{m+1}(x,t) = v_m(x,t)-\frac{1}{\Gamma(1+\alpha)}\times\\
&\int_0^t\left[\frac{\partial^\alpha v_m}{\partial\tau^\alpha}+\frac{\partial^{2\alpha}v_m}{\partial x^{2\alpha}}-2v_m\frac{\partial^\alpha v_m}{\partial x^\alpha}-\frac{\partial^\alpha[u_mv_m]}{\partial x^\alpha}\right](d\tau)^\alpha.
\end{aligned}\tag{17}$$

Suppose that an initial approximation has the following form which satisfies the initial condition:

$$\begin{aligned}
u_0(x,t) &= \cos_\alpha(x^\alpha),\\
v_0(x,t) &= \cos_\alpha(x^\alpha).
\end{aligned}\tag{18}$$

Now by iteration formula (17), we obtain the following approximations

$$\begin{aligned}
&u_1(x,t) = u_0(x,t)-\frac{1}{\Gamma(1+\alpha)}\times\\
&\int_0^t\left[\frac{\partial^\alpha u_0}{\partial\tau^\alpha}+\frac{\partial^{2\alpha}u_0}{\partial x^{2\alpha}}-2u_0\frac{\partial^\alpha u_0}{\partial x^\alpha}-\frac{\partial^\alpha[u_0v_0]}{\partial x^\alpha}\right](d\tau)^\alpha\\
&v_1(x,t) = v_0(x,t)-\frac{1}{\Gamma(1+\alpha)}\times\\
&\int_0^t\left[\frac{\partial^\alpha v_0}{\partial\tau^\alpha}+\frac{\partial^{2\alpha}v_0}{\partial x^{2\alpha}}-2v_0\frac{\partial^\alpha v_0}{\partial x^\alpha}-\frac{\partial^\alpha[u_0v_0]}{\partial x^\alpha}\right](d\tau)^\alpha\\
&= \cos_\alpha(x^\alpha)+\frac{1}{\Gamma(1+\alpha)}\int_0^t[\cos_\alpha(x^\alpha)](d\tau)^\alpha\\
&= \cos_\alpha(x^\alpha)+\frac{1}{\Gamma(1+\alpha)}\int_0^t[\cos_\alpha(x^\alpha)](d\tau)^\alpha\\
&= \cos_\alpha(x^\alpha)\left[1+\frac{t^\alpha}{\Gamma(1+\alpha)}\right],\\
&= \cos_\alpha(x^\alpha)\left[1+\frac{t^\alpha}{\Gamma(1+\alpha)}\right],
\end{aligned}$$

$$\begin{aligned}
u_2(x,t) &= u_1(x,t)-\frac{1}{\Gamma(1+\alpha)}\times\\
&\int_0^t\left[\frac{\partial^\alpha u_1}{\partial\tau^\alpha}+\frac{\partial^{2\alpha}u_1}{\partial x^{2\alpha}}-2u_1\frac{\partial^\alpha u_1}{\partial x^\alpha}-\frac{\partial^\alpha[u_1v_1]}{\partial x^\alpha}\right]\\
v_2(x,t) &= v_1(x,t)-\frac{1}{\Gamma(1+\alpha)}\times\\
&\int_0^t\left[\frac{\partial^\alpha v_1}{\partial\tau^\alpha}+\frac{\partial^{2\alpha}v_1}{\partial x^{2\alpha}}-2v_1\frac{\partial^\alpha v_1}{\partial x^\alpha}-\frac{\partial^\alpha[u_1v_1]}{\partial x^\alpha}\right]
\end{aligned}$$

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$$
\begin{aligned}
&= \cos_\alpha(x^\alpha) + \frac{t^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(x^\alpha) \\
&\quad + \frac{1}{\Gamma(1+\alpha)}\int_0^t \left[\frac{\tau^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(x^\alpha)\right](d\tau)^\alpha \\
&= \cos_\alpha(x^\alpha) + \frac{t^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(x^\alpha) \\
&\quad + \frac{1}{\Gamma(1+\alpha)}\int_0^t \left[\frac{\tau^\alpha}{\Gamma(1+\alpha)}\cos_\alpha(x^\alpha)\right](d\tau)^\alpha \\
&= \cos_\alpha(x^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)}\right], \\
&= \cos_\alpha(x^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)}\right], \\
&\vdots
\end{aligned}
$$

$$
\begin{aligned}
u_m(x,t) &= \cos_\alpha(x^\alpha)\sum_{k=0}^{m}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}, \\
u_m(x,t) &= \cos_\alpha(x^\alpha)\sum_{k=0}^{m}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)},
\end{aligned}
$$

Therefore, the series solutions can be written in the form:

$$
\begin{aligned}
u(x,t) &= \cos_\alpha(x^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \cdots\right], \\
u(x,t) &= \cos_\alpha(x^\alpha)\left[1 + \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \cdots\right],
\end{aligned}
$$

and finally in its closed form gives

$$
\begin{aligned}
u(x,t) &= E_\alpha(t^\alpha)\cos_\alpha(x^\alpha), \\
u(x,t) &= E_\alpha(t^\alpha)\cos_\alpha(x^\alpha),
\end{aligned} \tag{19}
$$

*Example 3.*Consider the system of nonlinear coupled partial differential equations with local fractional operators:

$$
\begin{aligned}
&\frac{\partial^\alpha u}{\partial t^\alpha} - \frac{\partial^\alpha v}{\partial x^\alpha}\frac{\partial^\alpha w}{\partial y^\alpha} = 1, \\
&\frac{\partial^\alpha v}{\partial t^\alpha} - \frac{\partial^\alpha w}{\partial x^\alpha}\frac{\partial^\alpha u}{\partial y^\alpha} = 5, \\
&\frac{\partial^\alpha w}{\partial t^\alpha} + \frac{\partial^\alpha u}{\partial x^\alpha}\frac{\partial^\alpha v}{\partial y^\alpha} = 5,
\end{aligned} \tag{20}
$$

with the initial conditions

$$
\begin{aligned}
u(x,y,0) &= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)}, \\
v(x,y,0) &= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)}, \\
w(x,y,0) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)},
\end{aligned} \tag{21}
$$

According to local fractional variational iteration method, formula (8) for (20) can be expressed in the following form:

$$
\begin{aligned}
&u_{m+1}(x,t) = u_m(x,t) - \frac{1}{\Gamma(1+\alpha)}\times \\
&\int_0^t \left[\frac{\partial^\alpha u_m}{\partial \tau^\alpha} - \frac{\partial^\alpha v_m}{\partial x^\alpha}\frac{\partial^\alpha w_m}{\partial y^\alpha} - 1\right](d\tau)^\alpha, \\
&v_{m+1}(x,t) = v_m(x,t) - \frac{1}{\Gamma(1+\alpha)}\times \\
&\int_0^t \left[\frac{\partial^\alpha v_m}{\partial \tau^\alpha} - \frac{\partial^\alpha w_m}{\partial x^\alpha}\frac{\partial^\alpha u_m}{\partial y^\alpha} - 5\right](d\tau)^\alpha. \\
&w_{m+1}(x,t) = w_m(x,t) - \frac{1}{\Gamma(1+\alpha)}\times \\
&\int_0^t \left[\frac{\partial^\alpha w_m}{\partial \tau^\alpha} + \frac{\partial^\alpha u_m}{\partial x^\alpha}\frac{\partial^\alpha v_m}{\partial y^\alpha} - 5\right](d\tau)^\alpha.
\end{aligned} \tag{22}
$$

Suppose that an initial approximation has the following form which satisfies the initial condition:

$$
\begin{aligned}
u_0(x,y,t) &= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)}, \\
v_0(x,y,t) &= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)}, \\
w_0(x,y,t) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)}.
\end{aligned} \tag{23}
$$

Now by iteration formula (22), we obtain the following approximations

$$
\begin{aligned}
&u_1(x,t) = u_0(x,t) - \frac{1}{\Gamma(1+\alpha)}\times \\
&\int_0^t \left[\frac{\partial^\alpha u_0}{\partial \tau^\alpha} - \frac{\partial^\alpha v_0}{\partial x^\alpha}\frac{\partial^\alpha w_0}{\partial y^\alpha} - 1\right](d\tau)^\alpha, \\
&v_1(x,t) = v_0(x,t) - \frac{1}{\Gamma(1+\alpha)}\times \\
&\int_0^t \left[\frac{\partial^\alpha v_0}{\partial \tau^\alpha} - \frac{\partial^\alpha w_0}{\partial x^\alpha}\frac{\partial^\alpha u_0}{\partial y^\alpha} - 5\right](d\tau)^\alpha. \\
&w_1(x,t) = w_0(x,t) - \frac{1}{\Gamma(1+\alpha)}\times \\
&\int_0^t \left[\frac{\partial^\alpha w_0}{\partial \tau^\alpha} + \frac{\partial^\alpha u_0}{\partial x^\alpha}\frac{\partial^\alpha v_0}{\partial y^\alpha} - 5\right](d\tau)^\alpha. \\
&= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{1}{\Gamma(1+\alpha)}\int_0^t 3(d\tau)^\alpha \\
&= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{1}{\Gamma(1+\alpha)}\int_0^t 3(d\tau)^\alpha \\
&= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{1}{\Gamma(1+\alpha)}\int_0^t 3(d\tau)^\alpha \\
&= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
&= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
&= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)},
\end{aligned}
$$

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$$
\begin{aligned}
u_2(x,t) &= u_1(x,t) - \frac{1}{\Gamma(1+\alpha)} \times \\
&\int_0^t \left[ \frac{\partial^\alpha u_1}{\partial \tau^\alpha} - \frac{\partial^\alpha v_1}{\partial x^\alpha}\frac{\partial^\alpha w_1}{\partial y^\alpha} - 1 \right] (d\tau)^\alpha, \\
v_2(x,t) &= v_1(x,t) - \frac{1}{\Gamma(1+\alpha)} \times \\
&\int_0^t \left[ \frac{\partial^\alpha v_1}{\partial \tau^\alpha} - \frac{\partial^\alpha w_1}{\partial x^\alpha}\frac{\partial^\alpha u_1}{\partial y^\alpha} - 5 \right] (d\tau)^\alpha. \\
w_2(x,t) &= w_1(x,t) - \frac{1}{\Gamma(1+\alpha)} \times \\
&\int_0^t \left[ \frac{\partial^\alpha w_1}{\partial \tau^\alpha} + \frac{\partial^\alpha u_1}{\partial x^\alpha}\frac{\partial^\alpha v_1}{\partial y^\alpha} - 5 \right] (d\tau)^\alpha. \\
&= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
&= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
&= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
&\vdots \\
u_m(x,y,t) &= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
v_m(x,y,t) &= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
w_m(x,y,t) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)},
\end{aligned}
$$

Therefore, the series solutions can be written in the form

$$
\begin{aligned}
u(x,y,t) &= \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \\
v(x,y,t) &= \frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)}, \qquad (24)\\
w(x,y,t) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2y^\alpha}{\Gamma(1+\alpha)} + \frac{3t^\alpha}{\Gamma(1+\alpha)},
\end{aligned}
$$

## 4 Conclusions

The local fractional variational iteration method is a powerful method which is able to handle linear/nonlinear local fractional differential equations. The method has been applied to system of local fractional coupled partial differential equations in order to find its approximate analytical solutions. The results show that the applied method is suitable and inexpensive for obtaining the approximate solutions.


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