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Magnetic Field Effect on Three Plane Waves Propagation at Interface between Solid-Liquid Media Placed under Initial Stress in the Context of GL Model

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Abstract: In this work, the effects of magnetic field and initial stress on plane waves propagation is investigated. The problem of reflection and transmission of thermoelastic wave at a solid-liquid interface in presence of initial stress and magnetic field has been investigated. In the context of Green-Lindsay theory of generalized, the problem has been solved. The boundary conditions applied at the interface are (i) displacement continuity, (ii) Vanishing the tangential displacement, (iii) Continuity of normal force per unit initial area, (iv) Tangential force per unit initial area must vanish, and (v) Continuity of temperature. The appropriate expressions to find the amplitudes ratios for the three incidence waves (P-, SV, and T-wave) have been obtained. The reflection and transmitted coefficients for the incident waves are computed numerically, considering the initial stress and magnetic field effect and presented graphically.

Keywords: Initial stress, magnetic field, reflection, transmission, thermoelasticity, relaxation time, , solid-liquid.

1 Introduction

The dynamical problem of surface waves propagation in a homogeneous and non-homogeneous elastic media are of considerable importance in earthquake, engineering, and seismology on account of the occurrence of non-homogeneities in the earth's crust, as the earth is made up of different layers. During the last five decades, wide spread attention has been given to thermoelasticity theories which consider finite speed for the propagation of thermal signal. Initial stresses develop in the medium due to various reasons, such as the difference of temperature, process of quenching shot pinning and cold working, slow process of creep, differential external forces, and gravity variations. The Earth is under high initial stress and therefore, it is of great interest to study the effect of these stresses on the propagation of elastic waves. Alot of systematic studies have been made on the propagation of elastic waves. [1] showed that the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. [2] reported a new theory based on a modified Fourier's law of heat conduction with one relaxation time and subsequently a more rigorous theory of thermoelasticity was formulated by [3] introducing two relaxation times. These non-classical theories are often regarded as the generalized dynamic theory of thermoelasticity. Various problems have been investigated and discussed in the light of these two theories and the studies reveal some interesting phenomena. Problem on wave propagation phenomena in coupled or generalized thermoelasticity is discussed by [4]. [5] discussed the reflection of SV-wave in a generalized thermoelastic medium. [6] investigated the reflection of generalized magneto-thermo-viscoelastic waves at the boundary of a semi-infinite solid considering that the free surface of the solid be adjacent to vacuum and the solid is subjected to a constant temperature and magnetic field. [7] investigated the reflection and refraction of thermoelastic waves at an interface of two semi-infinite media in welded contact, in the context of generalized thermoelasticity with two relaxation times. A survey article of various representative theories in the range of generalized thermoelasticity is prepared by [8]. [9] investigated solution of the field equations governing small motions of a micropolar viscoelastic solid half-space with stretch to study the reflection and

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transmission at the interface between a liquid and a micropolar viscoelastic solid with stretch. Problem on reflection and refraction in coupled or generalized thermoelasticity have been a topic of research for various authors as [10]. [11] discussed the effects of applied magnetic field on reflection and refraction of shear waves in two semi-infinite elastic media having viscoelasticity of general linear type and the values of reflection and transmission coefficients are derived for two specific orientations of the magnetic field. The generalized magneto-thermoelasticity model with two relaxation times in an isotropic elastic medium under the effect of reference temperature on the modulus of elasticity is pointed out by [12]. Estimation on magnetic field effect in an elastic solid half-space under thermoelastic diffusion is discussed and the expressions for the reflection coefficients for the four reflected waves are obtained by [13]. The impact of magnetic field, initial pressure, and hydrostatic initial stress on reflection of P and SV waves considering a Green Lindsay theory is discussed by [14]. [15] studied P waves propagation in an isotropic homogeneous solid half-space under the influence of magnetic field, thermal relaxation time and rotation with voids. [16] illustrated reflection of P and SV waves from stress-free surface elastic half-space under influence of magnetic field and hydrostatic initial stress without energy dissipation in the context of the Green and Naghdi theory of type III. [17] studied relaxation times and magnetic field sense effects on the reflection of thermoelastic waves phenomena from isothermal and insulated boundaries of a half space. [18] estimated Maxwell's stresses effect on reflection and transmission of plane waves between two thermo-elastic media in the context of GN Model. [19] studied the problem of reflection and refraction of thermo-elastic wave under normal initial stress at a solid-solid interface under perfect boundary condition. [20] pointed out the re?ection and refraction at an interface between two dissimilar thermally conducting viscous liquid half-spaces. [21] studied the radial deformation and the corresponding stresses in a homogeneous annular fin for an isotropic material. Recently, [22] investigated rotational and voids effects on the reflection of P waves from stress-free surface of an elastic half-space under magnetic field, initial stress and without energy dissipation. Reflection and refraction of P-, SV- and thermal waves, at an initially stressed solid-liquid interface in generalized thermoelasticity has been discussed by Singh and Chakraborty [23]. [24] investigated the calculation of bulk acoustic wave propagation velocities in trigonal piezoelectric smart materials. [25] investigated SV-waves incidence at interface between solid-liquid media under magnetic field, initial stress and two thermal relaxation times. [26] pointed out Green Lindsay model on reflection and refraction of p- and SV-waves at interface between solid-liquid media presence in magnetic field and initial stress. [27] investigated the problem of reflection and refraction of thermoelastic waves at a magnetized solid-liquid interface in presence of initial stress in the context of CT (Classical theory).

In this paper, the plane waves propagation is investigated under influence of magnetic field and initial stress. The problem of reflection and transmission of thermoelastic wave at a solid-liquid interface in presence of initial stress and magnetic field considering GL theory of generalized has been solved. The boundary conditions at the interface are applied to solve the problem. The appropriate expressions to find the amplitudes ratios for the three incidence (p-, SV-, and T-waves) have been obtained to calculate the reflection and transmitted coefficients and computed numerically, considering the initial stress and magnetic field effect and displayed graphically.

2 Formulation of the problem

We consider a plane interface between solid half-space homogeneous an isotropic elastic and liquid haf-space are with a primary temperature T_0 and magnetic field acts on z-direction. Both media placed under different initial stress. A plane waves are incident in medium M at the plane interface which reflected to p-wave (dilatational wave), SV-wave (rotational wave) and thermal wave (dilatational wave). Rest of the wave continues to travel in the other medium M' after refraction, as p-wave and one thermal wave as shown in (Fig. 1). We assume a Cartesian coordinate system oxyz with origin "o" on the plane y = 0. Since we consider a two-dimensional problem, we restrict our analysis to plane strain parallel to oxy-plane. Hence all the field variables depend only on x, y and time t. For easy reference we follow a convention: All quantities in medium M' except initial stress that acts only in a solid media are represented unprimed whereas corresponding quantities in medium M' are represented as primed. The initial stress components in medium M are shown in Fig. 2. where, the initial stress affects on the medium M only.

3 Basic equations

1.The dynamical equations of motion the rotating frame of reference for a plane strain under initial stress in absence of heat source, given by [1], taking into account the presence of Lorenz's force are

$$\begin{aligned}\frac{\partial S_{11}}{\partial x}+\frac{\partial S_{12}}{\partial y}-P\frac{\partial \bar{\omega}}{\partial y}+F_x=\rho\frac{\partial^2 u}{\partial t^2},\\ \frac{\partial S_{21}}{\partial x}+\frac{\partial S_{22}}{\partial y}-P\frac{\partial \bar{\omega}}{\partial x}+F_y=\rho\frac{\partial^2 v}{\partial t^2}.\end{aligned} \quad (1)$$

Where, $\bar{\omega}=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$.

F_x and F_y are components of the magnetic field in x and y directions, respectively.

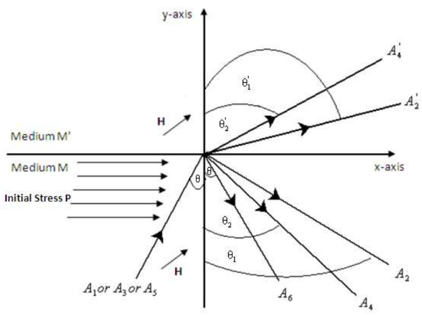

Fig. 1: Geometry of the problem.

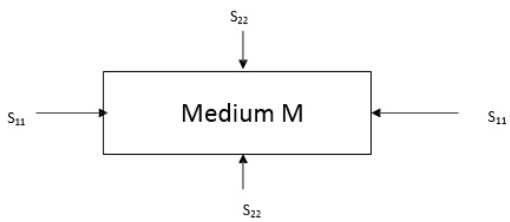

Fig. 2: Components of initial stress on the solid medium.

2.The stress-strain relations with incremental isotropy are given by *Biot* [1]:

$$S_{11} = (\lambda + 2\mu + P)e_{xx} + (\lambda + P)e_{yy} - \gamma(T + \tau_1 \frac{\partial T}{\partial t})$$
$$S_{22} = \lambda e_{xx} + (\lambda + 2\mu)e_{yy} - \gamma(T + \tau_1 \frac{\partial T}{\partial t}) \quad (2)$$
$$S_{12} = 2\mu e_{xy}$$

3.The incremental strain- components are given by *Biot* [1]

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial y}, \quad e_{xy} = \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}). \quad (3)$$

4.The modified heat conduction equation is:

$$K\nabla^2 T = \rho C_e(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}) + T_0\gamma[\frac{\partial}{\partial t}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + \tau_0\delta_{ij}\frac{\partial^2}{\partial t^2}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})] \quad (4)$$

where, C_e is specific heat per unit mass, e_{ij} is strain components, K is thermal conductivity, P is initial stress, s_{11}, s_{22}, s_{12} is incremental stress components, λ and μ are Lame's constants, T_0 is natural temperature of the medium, δ_{ij} is Krnecker delta, T is absolute temperature of the medium, τ_0 and τ_1 are thermal relaxation times, α_t is coefficient of linear thermal expansion, u_i is components of the displacement vector, $\overline{\omega}$ is magnitude of local rotation.

5.Taking into account the absence of displacement current, the linearized Maxwell equations governing the electromagnetic fields for a slowly moving solid medium having perfect electrical conductivity are

$$curl \overrightarrow{h} = \overrightarrow{J}, \qquad curl \overrightarrow{E} = -\mu_e \frac{\partial \overrightarrow{h}}{\partial t}, \quad (5)$$
$$div \overrightarrow{h} = 0, \qquad div \overrightarrow{E} = 0$$

where

$$\overrightarrow{h} = curl(\overrightarrow{u} \times \overrightarrow{H}), \qquad \overrightarrow{F} = \overrightarrow{J} \times \overrightarrow{B}$$
$$\overrightarrow{H} = \overrightarrow{H_0} + \overrightarrow{h}(x,y,t), \qquad \overrightarrow{H_0} = (0, 0H).$$

Using equation (5), we obtain:

$$F_x = \mu_e H^2(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y})$$
$$F_y = \mu_e H^2(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2}) \quad (6)$$

where, $\overrightarrow{B}$ is magnetic induction vector, $\overrightarrow{E}$ is electric intensity vector, $\overrightarrow{F}$ is Lorenz's body forces vector, $\overrightarrow{h}$ is perturbed magnetic field vector, $\overrightarrow{H}$ is magnetic field vector, $\overrightarrow{H_0}$ is primary constant magnetic field vector, $\overrightarrow{J}$ is electric current density vector, μ_e is magnetic permeability.
Again Maxwell's stress equation can be given in the form as:

$$\tau_{ij} = \mu_e[H_i h_j + H_j h_i - H_k h_k \delta_{ij}] \quad (7a)$$

Where τ_{ij} is Maxwell's stress tensor, which reduces to:

$$\tau_{11} = \tau_{22} = \mu_e H^2(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}), \qquad \tau_{12} = 0. \quad (7b)$$

4 Solution of the problem

Substituting Eqs. (2), (3) and (7) into (1), we get:

$$\left(\lambda + 2\mu + P + \mu_e H^2\right) \frac{\partial^2 u}{\partial x^2} + \left(\lambda + \frac{P}{2} + \mu + \mu_e H^2\right) \frac{\partial^2 v}{\partial x \partial y} + \left(\mu + \frac{P}{2}\right) \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2} + \gamma\left(\frac{\partial T}{\partial x} + \tau_1 \frac{\partial^2 T}{\partial x \partial t}\right), \quad (8)$$

$$\left(\mu - \frac{P}{2}\right) \frac{\partial^2 v}{\partial x^2} + \left(\lambda + \frac{P}{2} + \mu + \mu_e H^2\right) \frac{\partial^2 u}{\partial x \partial y} + \left(2\mu + \lambda + \mu_e H^2\right) \frac{\partial^2 v}{\partial y^2} = \rho\left(\frac{\partial^2 v}{\partial t^2}\right) + \gamma\left(\frac{\partial T}{\partial y} + \tau_1 \frac{\partial^2 T}{\partial y \partial t}\right). \quad (9)$$

To separate the dilatational and rotational components of strain, we introduce displacement scalar and vector potentials Φ and Ψ defined by the following relations:

$$u=\frac{\partial\Phi}{\partial x}-\frac{\partial\Psi}{\partial y},\qquad v=\frac{\partial\Phi}{\partial y}+\frac{\partial\Psi}{\partial x} \tag{10}$$

where, $\overrightarrow{\Psi}=(0,0,-\Psi)$.

Substituting Eq. (10) into Eq.(8), we obtain:

$$\nabla^2\Phi=\frac{\rho}{(\lambda+2\mu+P+\mu_eH^2)}(\frac{\partial^2\Phi}{\partial t^2})+\frac{\gamma}{(\lambda+2\mu+P+\mu_eH^2)}(T+\tau_1\frac{\partial T}{\partial t}). \tag{11}$$

$$\nabla^2\Psi=\frac{\rho}{(\mu+\frac{P}{2})}[\frac{\partial^2\Psi}{\partial t^2}] \tag{12}$$

Also, from Eqs. (9) and (10), we get:

$$\nabla^2\Phi=\frac{\rho}{(\lambda+2\mu+\mu_eH^2)}(\frac{\partial^2\Phi}{\partial t^2})+\frac{\gamma}{(\lambda+2\mu+\mu_eH^2)}(T+\tau_1\frac{\partial T}{\partial t}). \tag{13}$$

$$\nabla^2\Psi=\frac{\rho}{(\mu-\frac{P}{2})}[\frac{\partial^2\Psi}{\partial t^2}] \tag{14}$$

Where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ (Laplace operator). Using Eq.(10), the temperature Eq. (4) tends to the following form:

$$K\nabla^2T=\rho C_e(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2T}{\partial t^2})+T_0\gamma\frac{\partial}{\partial t}(1+t_0\delta_{ij}\frac{\partial}{\partial t})\nabla^2\Phi \tag{15}$$

5 Solution using GL model.

In Green-Lindsay theory: $\tau_1\geq\tau_0>0$ and $\delta_{ji}=0$, Eqs. (11) and (14) can be rewritten as:

$$\nabla^2\Phi=\frac{1}{C_1^2(1+R_H)}\frac{\partial^2\Phi}{\partial t^2}+\frac{\gamma}{\rho C_1^2(1+R_H)}(T+\tau_1\frac{\partial T}{\partial t}), \tag{16}$$

$$\nabla^2\Psi=\frac{1}{C_2^2}[\frac{\partial^2\Psi}{\partial t^2}] \tag{17}$$

where $R_H=\frac{C_A^2}{C_1^2}$, $C_1^2=\frac{\lambda+2\mu+P}{\rho}$, $C_2^2=\frac{\mu-\frac{P}{2}}{\rho}$, $C_A^2=\frac{\mu_eH^2}{\rho}$.
Here, R_H,C_1,C_2 represent the magnetic sense (Alfven speed), velocities of isothermal dilatational and rotational waves respectively, in medium M.
Using GL theory, Eq. (15) can be written as:

$$K\nabla^2T=\rho C_e(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2T}{\partial t^2})+T_0\gamma\frac{\partial}{\partial t}(\nabla^2\Psi). \tag{18}$$

Eliminating T from Eqs. (16) and (18), we obtain a fourth order differential equation in terms of Ψ as:

$$T=(1+\tau_1\frac{\partial}{\partial t})^{-1}[\frac{\rho C_1^2(1+R_H)}{\gamma}\nabla^2\Phi-\frac{\rho}{\gamma}(\frac{\partial^2\Phi}{\partial t^2})], \tag{19}$$

then

$$C_3^2(1+R_H)\nabla^4\Phi-[(1+R_H+\varepsilon_T)\frac{\partial}{\partial t}+((1+R_H)\tau_0+\varepsilon_t\tau_1+\frac{C_3^2}{C_1^2})\frac{\partial^2}{\partial t^2}]\nabla^2\Phi+\frac{1}{C_1^2}(1+\tau_0\frac{\partial}{\partial t})\frac{\partial^3\Phi}{\partial t^3}=0 \tag{20}$$

where, $C_2^3=\frac{K}{\rho C_e}$, $\varepsilon_T=\frac{T_0\gamma^2}{\rho^2C_eC_1^2}$ is thermoelastic coupling constant of the solid medium M.

We seek the solutions for Φ, Ψ and T in the form:

$$\Phi=f(y)exp[ik(x-ct)], \tag{21a}$$
$$\Psi=g(y)exp[ik(x-ct)], \tag{21b}$$
$$T=h(y)exp[ik(x-ct)], \tag{21c}$$

where, k is wave number, ω is frequency, υ is phase speed, $c=\frac{\omega}{k}$. Since Eq. (21) is a solution of Eq. (20), it must satisfy Eq. (21).
Putting Eq. (21a) in (20), we get,

$$C_3^2(1+R_H)[-k^4f+\frac{\partial^4f}{\partial y^4}-2k^2\frac{\partial^2f}{\partial y^2}]-[(1+R_H+\varepsilon_T)+(ik^3cf(y)-ikc\frac{\partial^2f}{\partial y^2})+((1+R_H)\tau_0+\varepsilon_t\tau_1\frac{C_3^2}{C_1^2})(K^4C^2f(y)-k^2c^2\frac{\partial^2f}{\partial y^2})]+\frac{1}{C_1^2}((f(y)ik^3c^3)+\tau_0(f(y)k^4c^4)=0 \tag{22}$$

Which tends to:

$$(1+R_H)\frac{\partial^4f}{\partial y^4}+[-2k^2(1+R_H)+ikc\frac{(1+R_H+\varepsilon_T)}{C_3^2}+k^2c^2\frac{((1+R_H)\tau_0+\varepsilon_t\tau_1+\frac{C_3^2}{C_1^2})}{C_3^2}]\frac{\partial^2f}{\partial y^2}+[k^4(1+R_H)-ik^3c\frac{(1+R_H+\varepsilon_T)}{C_3^2}-k^4c^2\frac{((1+R_H)\tau_0+\varepsilon_t\tau_1+\frac{C_3^2}{C_1^2})}{C_3^2}+\frac{ik^3c^3}{C_1^2C_1^2}((1-i\tau_0kc)-\frac{C_1^2}{C^2}(1+R_H+\varepsilon_t))]f=0 \tag{23}$$

Eq. (23) being a fourth order differential equation in $f(y)$, the solution gives four values of $f(y)$ and Eq. (21a) becomes

$$\Phi = \begin{bmatrix} (A_1 exp(ikm_1 y) + (A_2 exp(ikm_1 y)) \\ +((A_3 exp(ikm_2 y) + (A_4 exp(ikm_2 y)) \end{bmatrix} exp[ik(x-ct)] \quad (24)$$

Where $m_1 = \sqrt{q^2c^2-1}, \qquad m_2 = \sqrt{p^2c^2-1}$

$$p^2, q^2 = \frac{1}{2c_1^2c_3^2}[\{c_1^2(\tau_0(1+R_H)+\varepsilon_T\tau_1) + c_3^2 + \frac{i(1+R_H+\varepsilon_T)c_1^2}{\omega}\} \pm \sqrt{N}] \quad (25)$$

$$N = [c_1^2(\tau_0(1+R_H)+\varepsilon_T\tau_1)+c_3^2 + \frac{i(1+R_H+\varepsilon_T)c_1^2}{\omega}]^2 - \frac{4i(1+R_H)(1-i\omega\tau_0)c_1^2c_3^2}{\omega} \quad (26)$$

Using Eq. (21b) in (17), we get

$$\frac{\partial^2 g}{\partial y^2} + k^2(\frac{c^2}{c_2^2}-1)g = 0 \quad (27)$$

Eq. (27) suggests that the solution yields two values of $g(y)$, and Eq. (21b) can be written as

$$\Psi = [A_5 exp(ikm_3 y) + A_6 exp(-ikm_3 y)]exp[ik(x-ct)] \quad (28)$$

Where $m_3 = \sqrt{(\frac{c^2}{c_2^2}-1)}$ The constants $A_i (i = 1,2,3,4,5,6)$ in pairs represent the amplitudes of incident and reflected thermal, $P-$ and SV-waves respectively. Substituting Eqs. (24) and (21c) in Eq. (16), we get

$$T = \frac{\rho}{\gamma\tau}\begin{bmatrix} b_1(A_1 exp(ikm_1 y) + (A_2 exp(ikm_1 y)) \\ +b_2((A_3 exp(ikm_2 y) + (A_4 exp(ikm_2 y)) \end{bmatrix} \times exp[ik(x-ct)] \quad (29)$$

Where

$$\tau = (1-i\omega\tau_1), \quad b_1 = \omega^2(1-(1+R_H)q^2c_1^2),$$
$$b_2 = \omega^2(1-(1+R_H)p^2c_1^2).$$

Setting $\mu = P = 0$ in Eqs. (1)-(4) we obtain the basic equations for a non-viscous liquid medium in presence of body forces and using them, we get displacement equations and temperature field equation, valid for the liquid medium M'. The equations are as follows:

$$(\lambda' + \mu_e' H'^2)\frac{\partial^2 u'}{\partial x^2} + (\lambda' + \mu_e' H'^2)\frac{\partial^2 v'}{\partial x \partial y} = \rho'\frac{\partial^2 u'}{\partial t^2} + \gamma'(\frac{\partial T'}{\partial x} + \tau_1'\frac{\partial^2 T'}{\partial x \partial t}), \quad (30)$$

$$(\lambda' + \mu_e' H'^2)\frac{\partial^2 u'}{\partial x \partial y} + (\lambda' + \mu_e' H'^2)\frac{\partial^2 v'}{\partial y^2} = \rho'\frac{\partial^2 v'}{\partial t^2} + \gamma'(\frac{\partial T'}{\partial y} + \tau_1'\frac{\partial^2 T'}{\partial y \partial t}), \quad (30)$$

$$k'\nabla^2 T' = \rho' C_e'(\frac{\partial T'}{\partial t} + \tau_0'\frac{\partial^2 T'}{\partial t}) + T_0'\gamma'\frac{\partial}{\partial t}(\nabla^2\Phi'). \quad (31)$$

The primes have been used to designate the corresponding quantities in the liquid medium M' as already been defined in case of solid medium M.
Taking

$$u = \frac{\partial \Phi'}{\partial x}, \quad v = \frac{\partial \Phi'}{\partial y} \quad (32)$$

we get

$$\nabla^2\Phi' = \frac{1}{C_1'^2(1+R_H')}\frac{\partial^2\Phi'}{\partial t^2} + \frac{\gamma'}{\rho' C_1'^2(1+R_H')}(1+\tau_1'\frac{\partial}{\partial t})T', \quad (33)$$

$$K'\nabla^2 T' = \rho' C_e'(\frac{\partial T'}{\partial t} + \tau_0'\frac{\partial^2 T'}{\partial t^2}) + T_0'\gamma'\frac{\partial}{\partial t}(\nabla^2\Phi') \quad (34)$$

Solving Eqs. (33) and (34) and proceeding exactly in a similar way as in solid medium M, we get the appropriate solution for Φ' and T' as

$$\Phi' = [A_2' exp(ikm_1' y) + A_4' exp(ikm_2' y)]exp[ik(x-ct)], \quad (35)$$

$$T' = \frac{\rho'}{\gamma'\tau'}[b_1' A_2' exp(ikm_1' y) + b_2' A_4' exp(ikm_2' y)]exp[ik(x-ct)], \quad (36)$$

where

$$\tau' = (1-i\omega t_1'), b_1' = \omega^2(1-(1+R_H')q'^2c_1'^2),$$
$$b_2' = \omega^2(1-(1+R_H')P'^2c_1'^2). \quad (37)$$

The constants A_2' and A_4' represent the amplitudes of refracted thermal and $p-waves$, respectively.

6 Boundary conditions

1. Normal displacement is continuous at the interface, $i.e. v = v'$. This leads to

$$\frac{\partial\Phi}{\partial y} + \frac{\partial\Psi}{\partial x} = \frac{\partial\Phi'}{\partial x}. \quad (38)$$

Using Eqs. (24), (28) and (35) in the above continuity relation, we get,

$$m_1A_1 - m_1A_2 + m_2A_3 - m_2A_4 + A_5 + A_6 - m_1'A_2' - m_2'A_4' = 0 \quad (39)$$

2.Tangential displacement must vanish at the interface *i.e.* $u = 0$.
This leads to $\frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y} = 0$.
Using Eqs. (24) and (28) in the above boundary condition, we get

$$A_1 + A_2 + A_3 + A_4 - m_3 A_5 + m_3 A_6 = 0 \quad (40)$$

3.Normal force per unit initial area must be continuous at the interface *i.e.* $\bigtriangledown f_y = \bigtriangledown f'_y$
This leads tp $s_{22} + \tau_{22} = s'_{22} + \tau'_{22}$ Using Eqs. (2), (3) and (7) for medium M and their corresponding equations for medium M' we get, with the help of Eqs. (10) and (32),

$$(\lambda + \mu_e H^2)(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}) + 2\mu(\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial x \partial y}) - \gamma(T + \tau_1 \frac{\partial T}{\partial t}) = (\lambda' + \mu'_e H'^2)(\frac{\partial^2 \Phi'}{\partial x^2} + \frac{\partial^2 \Phi'}{\partial y^2}) - \gamma'(T' + \tau_1 \frac{\partial T'}{\partial t}) \quad (41)$$

Substituting Eqs. (24), (28), (29), (35) and (36) in the above equation, we get,

$$[-(2+\beta) + c^2(\frac{1}{c_2^2} - \beta q^2)](A_1 + A_2) + [-(2+\beta) + c^2(\frac{1}{c_2^2} - \beta p^2)](A_3 + A_4) + (2+\beta) m_3 (A_5 - A_6) - \rho^*(1 + m_3^2)(A'_2 + A'_4) = 0 \quad (42)$$

where, $\rho^* = \frac{\rho'}{\rho}$ *and* $\beta = \frac{P}{\rho c_2^2}$

4.Tangential force per unit initial area must vanish at the interface *i.e.* $\bigtriangledown f_x = 0$
This leads to $s_{12} + Pe_{xy} + \tau_{12} = 0$
Using Eqs. (2), (3), (7), (10), (24) and (28) in the above equation, we get,

$$m_1(A_1 - A_2) + m_2(A_3 - A_4) - \frac{1}{2}(m_3^2 - 1)(A_5 - A_6) = 0 \quad (43)$$

5.Temperature must be continuous at the interface i.e.,$T = T'$. Using Eqs. (29) and (36) and simplifying, we get

$$(1 - (1 + R_H) q^2 c_1^2)(A_1 + A_2) + (1 - (1 + R_H) p^2 c_1^2)(A_3 + A_4) - \frac{\rho^*}{\gamma^* \tau^*}[(1 - (1 + R'_H q'^2 c_1'^2) A'_2) + (1 - (1 + R'_H) p'^2 c_1'^2) A'_4] = 0 \quad (44)$$

where, $\gamma^* = \frac{\gamma'}{\gamma}$ and $\tau^* = \frac{\tau'}{\tau}$.

7 Equations for the reflection and refraction coefficients

To consider the reflection and refraction of a thermoelastic plane wave which is incident at the solid-liquid interface at $y = 0$ making an angle θ with the y-axis, we have three different cases.

Case I: For p-wave incidence, we put $c = p^{-1} coses\theta$ and $A_1 = A_5 = 0$.
Case II: For thermal wave incidence, we put $c = q^{-1} coses\theta$ and $A_3 = A_5 = 0$.
Case III: For SV-wave incidence, we put $c = c_2 coses\theta$ and $A_1 = A_3 = 0$.

From Eqs. (39), (40) and (42)-(44) we get a system of five non-homogeneous equations for a thermoelastic plane wave incident,

$$\sum_{i=1}^{5} a_{ij} Z_j = y_i, \quad (j = 1, 2, ...5) \quad (45)$$

where

$$a_{11} = -m_1, a_{12} = -m_2, a_{13} = 1, a_{14} = -m_1, a_{15} = -m_2,$$
$$a_{22} = a_{21} = 1, a_{23} = m_3, a_{24} = a_{25} = 0,$$
$$a_{31} = [-(2+\beta) + C^2(\frac{1}{C_2^2} - \beta q^2)],$$
$$a_{32} = [-(2+\beta) + C^2(\frac{1}{C_2^2} - \beta q^2)],$$
$$a_{33} = -(2+\beta) m_3, a_{34} = a_{35} = -\rho^*(1 + m_3^2), a_{41} = -m_1,$$
$$a_{42} = -m_2, a_{43} = -0.5(m_3^2 - 1), a_{44} = a_{45} = 0,$$
$$a_{51} = (1 - (1 + R_H) q^2 c_1^2), a_{52} = (1 - (1 + R_H) p^2 c_1^2),$$
$$a_{53} = 0, a_{54} = -\frac{\rho^*}{\omega^* \tau^*}(1 - (1 + R'_H) q'^2 c_1'^2),$$
$$a_{55} = -\frac{\rho^*}{\omega^* \tau^*}(1 - (1 + R'_H) p'^2 c_1'^2)$$

where, $Z_j (j = 1, 2...5)$are the ratios of amplitudes of reflected thermal, p-, SV-waves and refracted thermal, p-waves to that of incident wave respectively.
For the three particular cases, we get,

(I) For incident p-wave:

$$y_1 = a_{12}, \quad y_2 = -a_{22}, \quad y_3 = -a32, \quad y_4 = a_{42}, \quad y_5 = -a_{52},$$
$$Z_1 = \frac{A_2}{A_3}, \quad Z_2 = \frac{A_4}{A_3}, \quad Z_3 = \frac{A_6}{A_3}, \quad Z_4 = \frac{A'_2}{A_3}, \quad Z_5 = \frac{A'_4}{A_3}$$

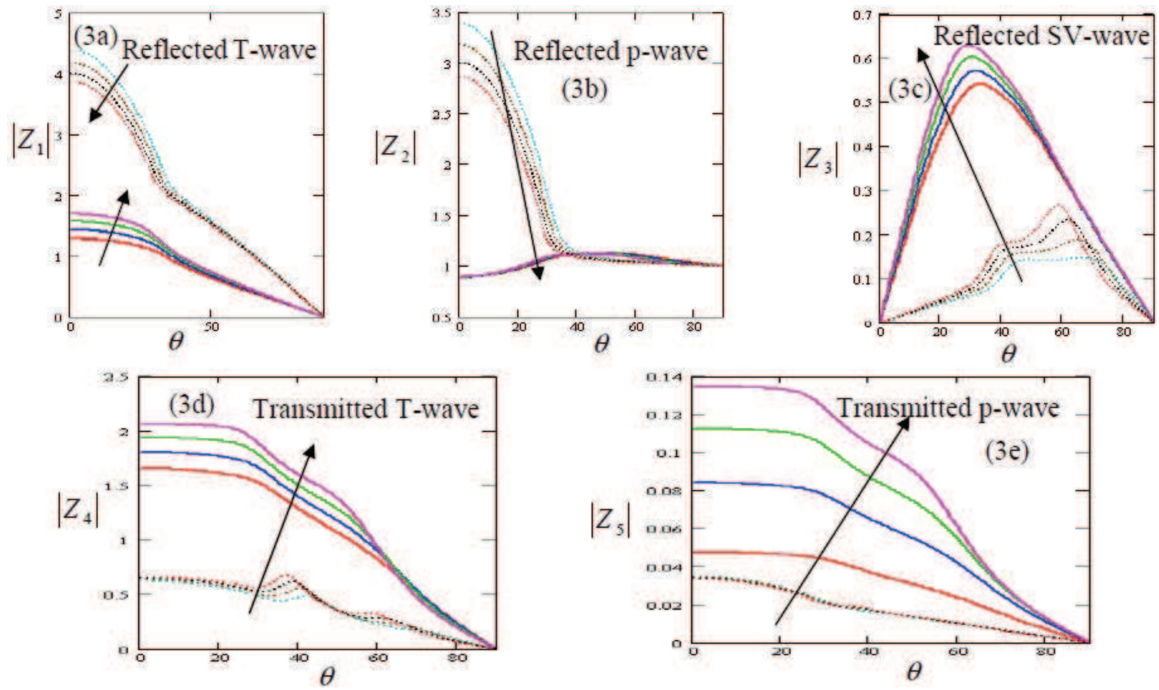

Fig. 3: Variation of the amplitudes $z_i(1,2,...5)$ with the angle of incidence of p-wave for variation of magnetic field: $H = 0.1, 0.2, 0.3, 0.4, P = 1.1(10)^{11}$-, $P = 0$.....

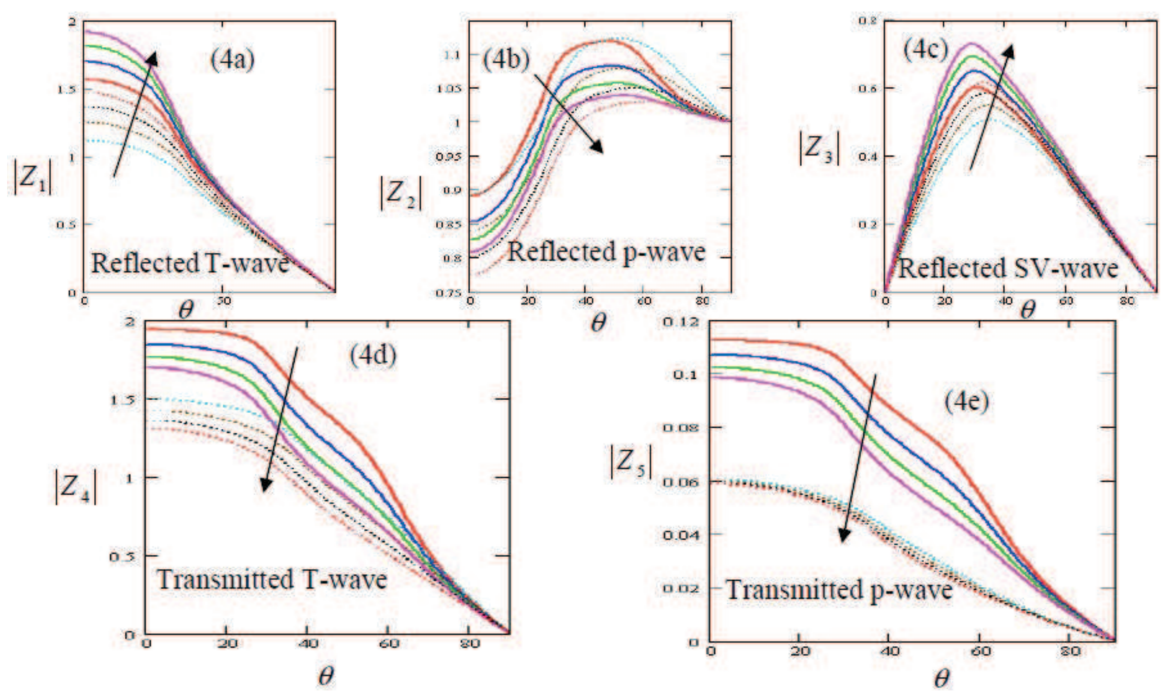

Fig. 4: Variation of the amplitudes $z_i(1,2,...5)$ with the angle of incidence of p-wave for variation of initial stress: $P = (1.1, 1.2, 1.3, 1.4)(10)^1 1, H = 0.3$-, $H = 0$.....

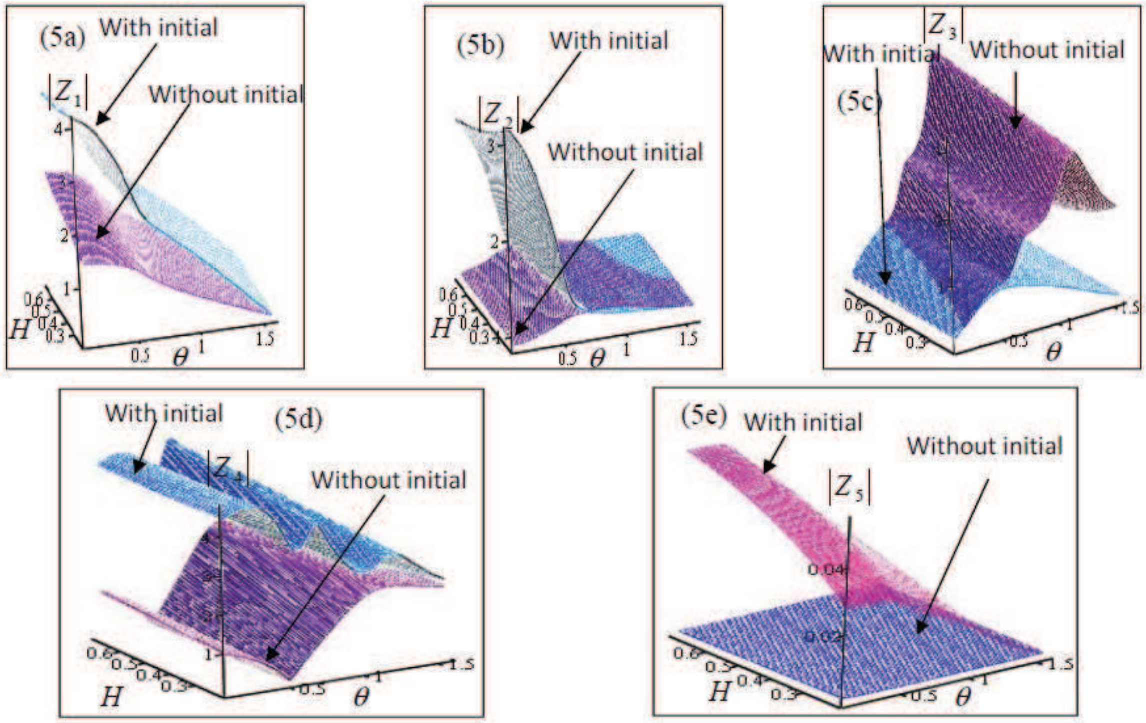

Fig. 5: Variation of the amplitudes $z_i(1,2,...5)$ with restpect to (θ, H) of p-wave with and without variation of initial stress.

(II) For incident thermal wave:

$$y_1 = a_{11}, \quad y_2 = -a_{21}, \quad y_3 = -a31, \quad y_4 = a_{41}, \quad y_5 = -a_{51},$$

$$Z_1 = \frac{A_2}{A_1}, \quad Z_2 = \frac{A_4}{A_5}, \quad Z_3 = \frac{A_6}{A_5}, \quad Z_4 = \frac{A'_2}{A_5}, \quad Z_5 = \frac{A'_4}{A_5}$$

(III) For incident SV-wave:

$$y_1 = a_{13}, \quad y_2 = -a_{23}, \quad y_3 = -a33, \quad y_4 = a_{43}, \quad y_5 = -a_{53},$$

$$Z_1 = \frac{A_2}{A_5}, \quad Z_2 = \frac{A_4}{A_5}, \quad Z_3 = \frac{A_6}{A_5}, \quad Z_4 = \frac{A'_2}{A_5}, \quad Z_5 = \frac{A'_4}{A_5}$$

8 Numerical results and discussion.

For a view to illustrate the numerical analysis of the expressions of the reflection and refraction coefficients, we have used the data for crust as solid medium following Choi and Gurnis [28] and water as liquid medium.

For solid medium (M crust)

$$\lambda = \mu = 3\times 10^9 N.M^{-2}, \quad \alpha = 1.0667\times 10^-5K^-1,$$
$$C_e = 1100J.Kg^{-1}.K^{-1}, \quad \rho = 2900Kg.M^{-3},$$
$$k = 3W.M^{-1}.K^{-1}$$

For liquid medium (M' water)

$$\lambda' = \mu' = 20.4\times 10^9 N.M^{-2}, \quad \alpha' = 69\times 10^-6K^-1,$$
$$C'_e = 4187J.Kg^{-1}.K^{-1}, \quad \rho' = 1000Kg.M^{-3},$$
$$k' = 0.6W.M^{-1}.K^{-1}$$

Taking into consideration $\tau_0 = \frac{3k}{\rho C_e C_1^2}, \tau'_0 = \frac{3k'}{\rho' C'_e C_1'^2}$ while τ_1, τ'_1 have been taken to be of the same order (about 1.5 times) of $\tau_0, \tau'_0, \omega = 7.5\times 10^{13}, T_0 = 3000k$. [29].
Figs. 3-5, 6-8 and 9-11 show the amplitudes ratios variation with the angle of incident p-wave, Twave and SV-wave, respectively.

Figs. 3 and 5 display the variation of the amplitudes ratios $Z_i = (1,2,...,5)$ with the angle of incidence of p-wave for variation of magnetic field with and without initial stress. It is appear that the amplitudes of the reflected T-wave, refracted T- and p-waves start from their maximum values and decreases to tend zero at $\theta = 90^o$, amplitude ratio of reflected p-wave tends to the unity, on the other hand, the reflection coefficient for the reflected SV-wave equal zero at $\theta = 0^o, 90^o$, increases to arrive to

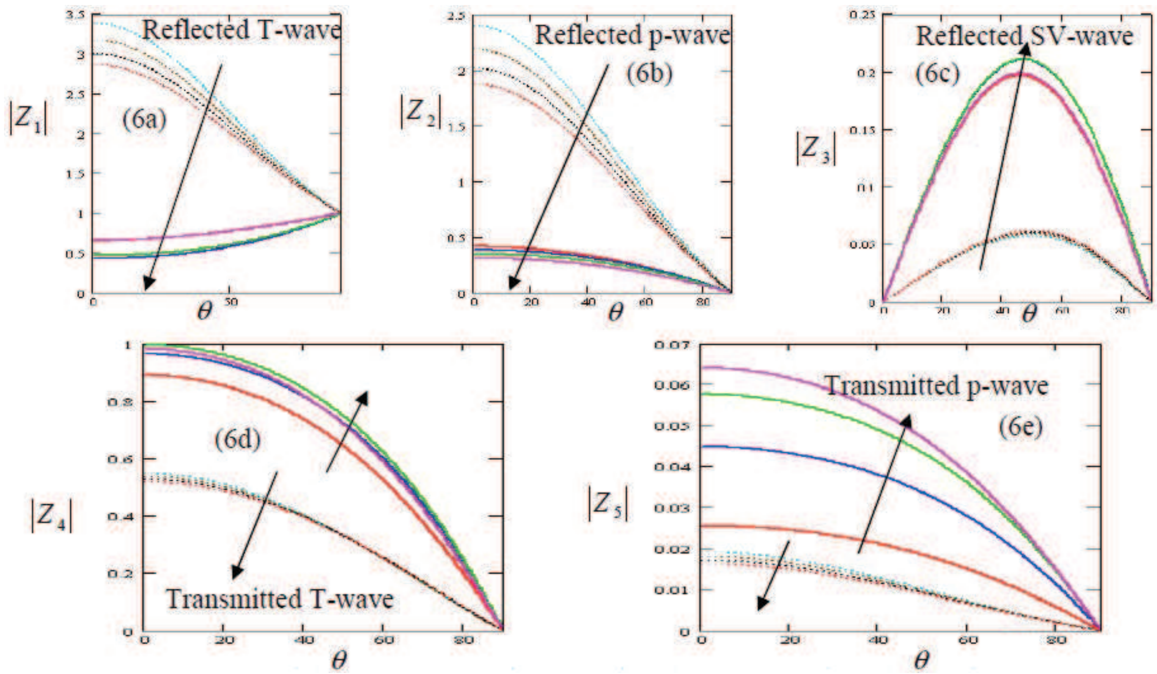

Fig. 6: Variation of the amplitudes $z_i(1,2,...5)$ with the angle of incidence of thermal-wave for variation of magnetic field: $H = 0.1, 0.2, 0.3, 0.4, P = 1.1(10)^{11}$-, $P = 0$.....

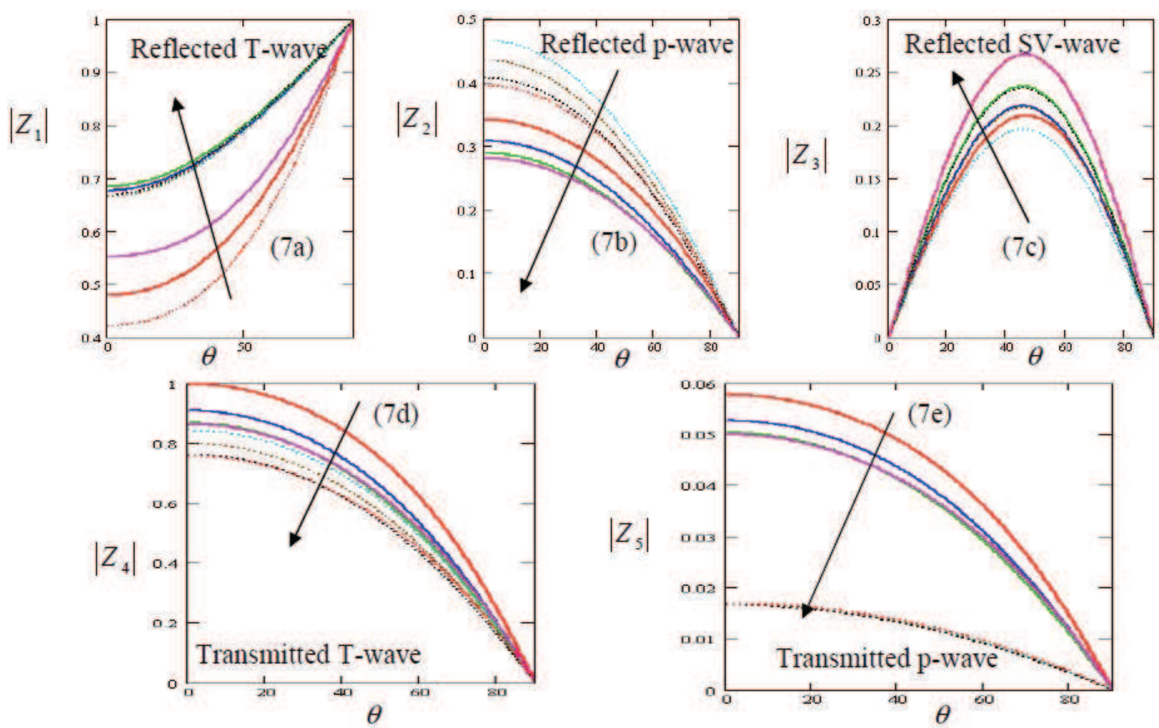

Fig. 7: Variation of the amplitudes $z_i(1,2,...5)$ with the angle of incidence of thermal-wave for variation of initial stress: $P = (1.1, 1.2, 1.3, 1.4)(10)^1 1, H = 0.3$-, $H = 0$.....

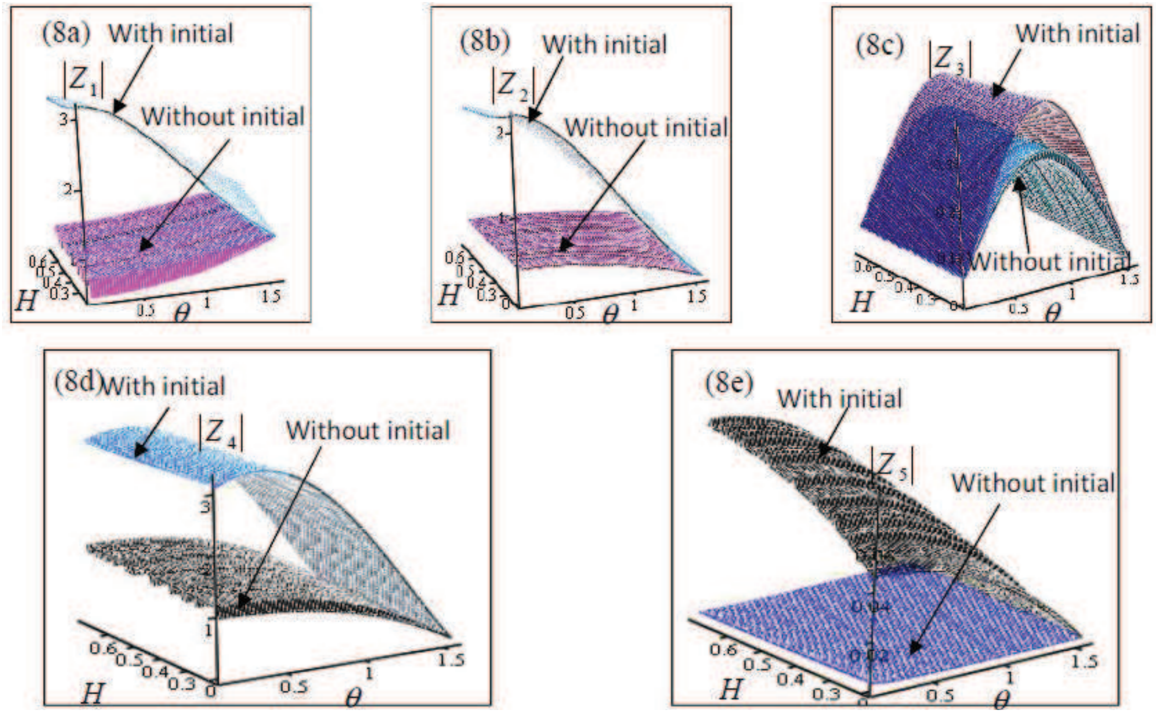

Fig. 8: Variation of the amplitudes $z_i(1,2,...5)$ with restpect to (θ,H) of thermal-wave with and without variation of initial stress.

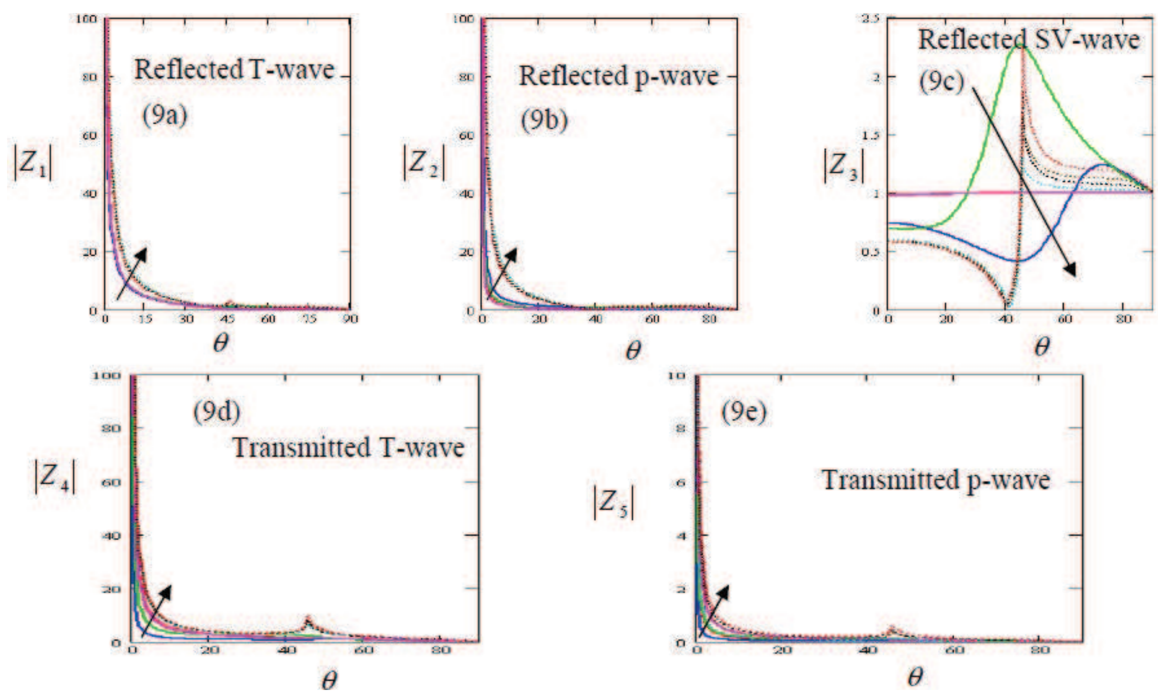

Fig. 9: Variation of the amplitudes $z_i(1,2,...5)$ with the angle of incidence of SV-wave for variation of magnetic field: $H = 0.1, 0.2, 0.3, 0.4, P = 1.1(10)^{11}$-, $P = 0$.....

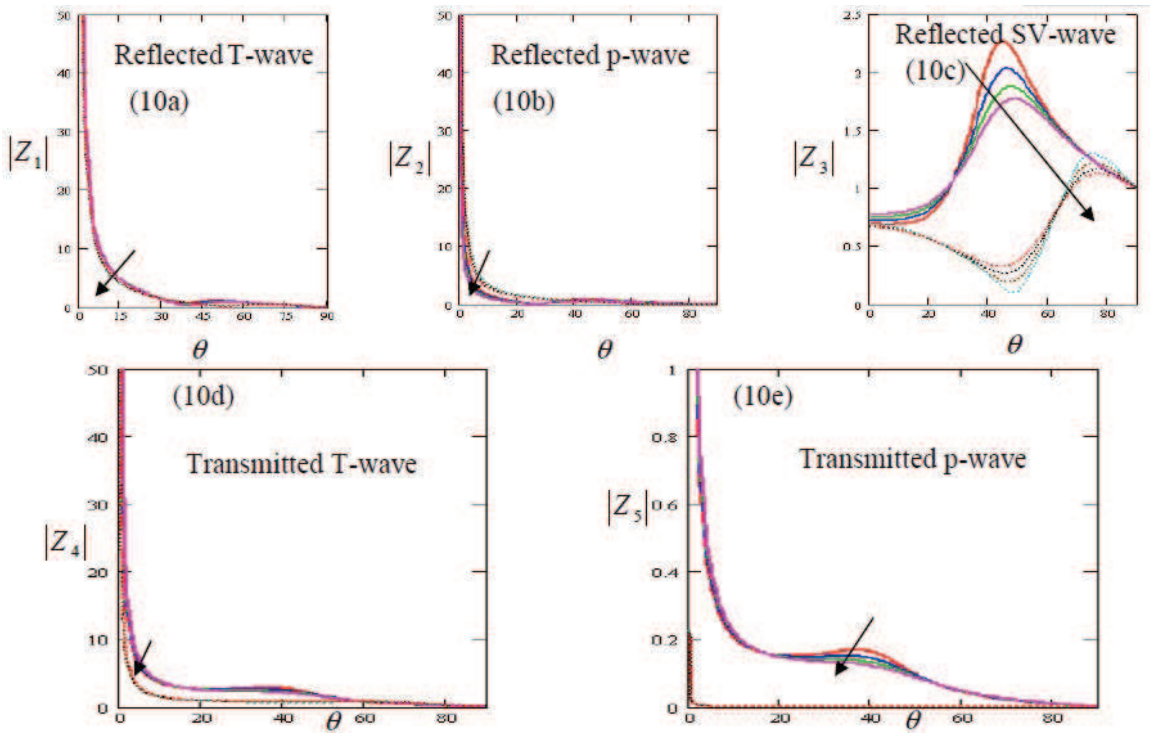

Fig. 10: Variation of the amplitudes $z_i(1,2,...5)$ with the angle of incidence of SV-wave for variation of initial stress: $P = (1.1,1.2,1.3,1.4)(10)^1 1, H = 0.3$-$, H = 0$.....

its maximum value and then decreases with the increasing of angle of incidence. Physically, we concluded that the reflected and transmitted T- and p-waves start from their maximum values and tend to zero for T-waves, refracted T- and p-waves that indicate to the interruption of them at the maximum values of the angle of incidence but reflected p-wave arrives to unity for the maximum angle of incidence, also, the reflected SV-wave starts and arrives to zero at the minimum and maximum values of θ that indicate to the creating of the reflection coefficient if $\theta = 0^o$ and interrupted at $\theta = 90^o$.

With the variation of the magnetic field in the presence or absence of initial stress, it is seen that $|Z_2|$ decrease with an increasing of the magnetic field parameter but $|Z_3|,|Z_4|$ and $|Z_5|$ increase, $|Z_1|$ increase with the increasing of magnetic field in the presence of initial stress but decreases if the initial stress absence. It is shown that if the initial stress is absence, $|Z_1|,|Z_2|$ and $|Z_5|$ larger than the correspondence in the presence of initial stress, vice versa in $|Z_3|$ and $|Z_4|$.

Fig. 4 plots the amplitudes ra:os with the angle of incidence and variation of the initial stress in the presence or absence of the magnetic field. It is obvious that $|Z_1|$ and $|Z_3|$ increase with the increased values of the initial stress but $|Z_2|,|Z_4|$ and $|Z_5|$ decrease, also, we concluded that the absence of the magnetic field make small interruption on $|Z_1|,|Z_2|,|Z_3|$ and $|Z_4|$ but additional factor on $|Z_5|$.

From Figs. 6-8 that the amplitudes of the reflected p-wave, refracted T- and SV-waves start from their maximum values and decreases to tend zero at $\theta = 90^o$, amplitude ratio of reflected T-wave tends to the unity, on the other hand, the reflection coefficient for the reflected SV-wave equal zero at $\theta = 0^o, 90^o$, increases to arrive to its maximum value and then decreases with the increasing of angle of incidence. Figs. 6 and 8 display the variation of the amplitudes ratios $Z_i = (1,2,...,5)$ with the angle of incidence of T-wave for variation of magnetic field with and without initial stress. It is shown that $|Z_1|$, $|Z_2|$ and $|Z_5|$ decrease with an increasing of magnetic field in the presence and absence of initial stress, $|Z_3|$ increases but $Z_4|$increases in the presence of initial stress and decreases if the initial stress absence. Also, it is clear that $|Z_1|$, $|Z_2|$, $|Z_3|$ and $|Z_5|$ in the presence of initial stress larger than their corresponding in the absence of initial stress, that indicate to the negative effect of the initial stress on the amplitudes ratios but a positive factor on $|Z_4|$.

Fig. 7 shows the amplitudes ratios with the angle of incidence and variation of the initial stress in the presence or absence of the magnetic field. It is appear that $|Z_1|$ and $|Z_3|$ increase with the increased values of the initial stress but $|Z_2|$, $|Z_4|$ and $|Z_5|$ decrease, also, we concluded that the absence of the initial stress makes small interruption on $|Z_1|$, $|Z_3|$ and $|Z_4|$ but an additional factor on $|Z_2|$ and $|Z_5|$.

Finally, for the incidence SV-wave, Figs. 9-11 display the

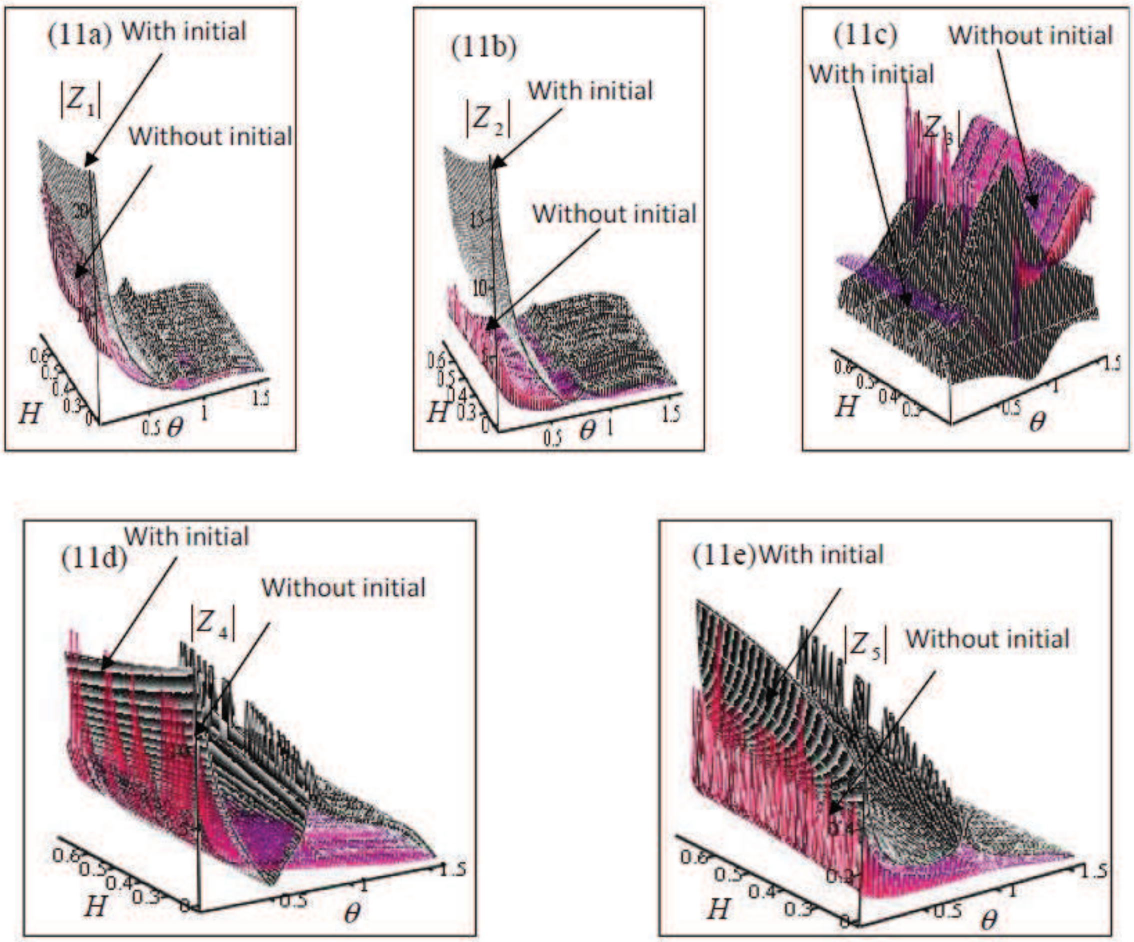

Fig. 11: Variation of the amplitudes $z_i(1,2,...5)$ with restpect to (θ,H) of SV-wave with and without variation of initial stress.

varia:on of the amplitudes ratios $Z_i = (1,2,...,5)$ with the angle of incidence of SV-wave for variation of magnetic field and initial stress. It is shown that $|Z_1|,|Z_2|,|Z_4|$ and $|Z_5|$ start from their maximum values arriving to their minimum values if $\theta = 90^o$. It is shown that the amplitudes $|Z_1|,|Z_2|,|Z_4|$ and $|Z_5|$ increase slight with an increasing of magnetic field H but $|Z_3|$ decreases, also, we concluded that if the initial stress neglected, the absolute values of the amplitudes take large values comparing with the corresponding values in the presence of initial stress. From Fig. 10, it is obvious that the absolute values of the reflection coefficients ratios except $|Z_3|$ and $|Z_5|$ decrease slight with an increasing of initial stress.

9 Concluding remarks.

We model the effect of initial stress, and magnetic field on reflection and refraction of a plane waves at a solid-liquid interface under perfect boundary conditions. The waves amplitudes ratios with initial stress and magnetic field with the angle of incidence are obtained in the framework of GL theory, discussed numerically and illustrated graphically.

The following conclusions can be made:

1. The reflected and refracted amplitudes depend on the angle of incidence, initial stress and magnetic field, the nature of this dependence is different for different reflected waves.
2. The initial stress and magnetic field play a significant role and the effect has the inverse trend for the reflected and transmitted waves.

Finally, it is observed that the reflection and refraction coefficient is strongly appear in the phenomena that has a lot of applications, especially, in Seismic waves, Earthquakes, Volcanoes, and Acoustics.

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