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An Analytical Solution for Effect of Rotation and Magnetic Field on the Composite Infinite Cylinder in Non-Homogeneity Viscoelastic Media.

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Abstract: The aim of this paper is to study the effect of rotation and magnetic field on the composite infinite cylinder in non-homogenity viscoelastic media. The one-dimensional equation of elastodynamic problem is solved in terms of radial displacement. A harmonic function techniques are used. The components of stress and displacement have been obtained. Numerical results are presented and illustrated graphically to illustrate the effect of rotation, magnetic field, viscosity, non-homogeneity. Comparison was made with the results obtained in the presence and absence of magnetic field ,rotation and viscosity . The results indicate that the effect of rotation, magnetic field, viscosity and non-homogeneity are very pronounced.

Keywords: Composite infinite cylinder, Rotation, Magnetic field, Non-homogenity, Viscosity.

1 Introduction

Interactions between strain and electromagnetic fields are largely being undertaken due to its various applications in many branches of science and technology. Development of magnetoelasticity also induces us to study various problems of geophysics, seismology and related topics. Bazer [3] made the survey of linear and non-linear wave motion in a perfect magnetoelastic medium. Without going into the details of the research work published so far in the fields of magnetoelasticity, magneto-thermo-elasticity, magneto-thermo viscoelasticity we mention some recent papers. Acharya and Sengupta [1] studied the magneto-thermoelastic surface waves in initially stressed conducting media. Chaudhary et al. [2] investigated the reflection/Transmission of plane SH wave through a senf-reinforced elastic layer between two half-spaces. Plane SH-wave response from elastic slab interposed between two different self-reinforced elastic solids studied by Chaudhary et al. [3]. discussed the surface waves in magnetoelastic initially stressed conducting media been illustrated by De and Sengupta [4]. Othman and Song [5] studied the effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation. Magneto-thermoelastic wave propagation at the interface between two micropolar viscoelastic media discussed by Song et al. [6]. Tianhu et al. [7] studied a two-dimensional generalized thermal shock problem for a half-space in electromagneto-thermoelasticity. Verma et al. [8] investigated the magneto-elastic transverse surface waves in self-reinforced elastic solids. Effect of the rotation on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity studied by Abd-Alla et al. [9]. Bayones [10] studied the influence of diffusion on generalized magneto- thermo-viscoelastic problem of a homogenous isotropic material. Viscoelastic materials are those for which the relationship between stress and strain depends on time. All materials exhibit some viscoelastic response. In common metals such as steel, aluminum , copper etc. Abd-Alla and Abo-Dahab [11] investigated the time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Roychoudhuri and Banerjee [12] investigated the magneto-thermoelastic interactions in an infinite viscoelastic cylinder of

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temperature rate dependent material subjected to a periodic loading. Spherically symmetric thermo-viscoelastic waves in a viscoelastic medium with a spherical cavity discussed by Banerjee et al. [13]. The problem of magneto-thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to a periodic loading is discussed by Abd-Alla et al.[14]. Kaliski [15] studied the absorption of magneto-viscoelastic surface waves in a real conductor in a magnetic field. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in refs.[19]-[38]. In this paper, we investigated the effect of rotation, magnetic field and viscosity on the composite infinite cylinder in non-homogeneity media. A harmonic function techniques are used. The components of stress and displacement have been obtained. Some special cases are studied , first case in non-homogeneity viscoelastic media with neglect of rotation and second case in non-homogeneity elastic media with effect of rotation and third case in non-homogeneity viscoelastic media with effect of rotation and neglect magnetic field. Finally, the effects of rotation, magnetic field and viscosity are calculated numerically and illustrated graphically.

2 Formulation of the Problem

In this paper, we derive the analytical formulation of the problem in cylindrical coordinates (r,θ,z) with the z- axis coinciding with the axis of the cylinder. We consider the strains axis symmetrical about the z-axis. We have only the radial displacement $u=u(r,t)$, the circumferential displacement $u_\theta=0$ and the longitudinal displacement $u_z=0$, which are independent of θ and z. The elastic medium is rotating uniformly with an angular velocity $\underline{\Omega}=\Omega\underline{n}$ where n is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame has two additional term centripetal acceleration, $\overrightarrow{\Omega}\times(\overrightarrow{\Omega}\times\overrightarrow{u})$ is the centripetal acceleration due to time varying motion only and $2\overrightarrow{\Omega}\times\dot{\overrightarrow{u}}$ is the Coriolis acceleration, and $\overrightarrow{\Omega}=(0,\Omega,0)$. Let us consider the medium is a perfect electric conductor and the linearized Maxwell equations governing the electromagnetic field , in the absence of the displacement current (SI) in the form as in Kraus [16] and Roychoudhuri and Mukhopadhyay [17]:

$$\begin{aligned}
&\mathbf{j}=curl\,\overrightarrow{h},\\
&-\mu_e\frac{\partial\overrightarrow{h}}{\partial t}=curl\,\overrightarrow{E},\\
&div\,\overrightarrow{h}=0,\\
&div\,\overrightarrow{E}=0\\
&\overrightarrow{E}=-\mu_e\left(\frac{\partial\overrightarrow{u}}{\partial t}\times\overrightarrow{H}\right)
\end{aligned}\tag{1}$$

where $\overrightarrow{h}$ is the perturbed magnetic field over the primary magnetic field , $\overrightarrow{E}$ is the electric intensity , $\overrightarrow{J}$ is the electric current density , μ_e is the magnetic permeability, $\overrightarrow{H}$ is the constant primary magnetic field and $\overrightarrow{u}$ is the displacement vector . Applying an initial magnetic field vector $\overrightarrow{H}(0,0,H_Z)$ in cylinder coordinate (r,θ,z) to Eq.(1) we have:

$$\overrightarrow{J}=\left(0,\frac{-\partial h_z}{\partial r},0\right),h_z=-H_Z\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\tag{2}$$

The non-vanishing displacement component is the radial one $u_r=u(r,t)$, is

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),e_{jj}=\Delta,\ i,j=1,2,3.\tag{3}$$

Then, the strain tensor has the following components

$$e_{rr}=\frac{\partial u}{\partial r},e_{\theta\theta}=e_{ZZ}=\frac{u}{r}\tag{4}$$

The cubical dilatation is thus given by:

$$e=\frac{\partial u}{\partial r}+\frac{2u}{r}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2u\right)\tag{5}$$

The stresses - displacement relations are given by:

$$\begin{aligned}
&\sigma_{rr}=\frac{2\mu\tau_m}{1-2\upsilon}[\,(1-\upsilon)\frac{\partial u}{\partial r}+\upsilon\frac{u}{r}\,]\\
&\sigma_{\theta\theta}=\frac{2\mu\tau_m}{1-2\upsilon}[\,(1-\upsilon)\frac{u}{r}+\upsilon\frac{\partial u}{\partial r}\,]\\
&\sigma_{zz}=\frac{2\mu\upsilon\tau_m}{1-2\upsilon}[\,\frac{\partial u}{\partial r}+\frac{u}{r}\,]\\
&\tau_{rz}=\tau_{r\theta}=\tau_{\theta z}=0.
\end{aligned}\tag{6}$$

where υ is the Poisson's ratio of the material and μ is the rigidity modulus. τ_0 is the mechanical relaxation time due to the viscosity , $\tau_m=(1+\tau_0\frac{\partial}{\partial t})$. The dynamical equation of the motion in the absence of body force in the direction of r is given by:

$$\frac{\partial\tau_{rr}}{\partial r}+\frac{1}{r}(\tau_{rr}-\tau_{\theta\theta})+f_r=\rho\left[\ddot{u}_i+(\Omega\times\Omega\times u)_i\right]\tag{7}$$

where ρ is the density of the cylinder material and $\mathbf{F}$ is defined as Lorentz's force and f is the radial component of Lorentz's force , Kraus [16], which may be written as:

$$\mathbf{F}=\mu_e(\overline{J}\times\overline{H})\equiv(\mu_eH_z^2\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right),0,0)\equiv f_r\mathbf{e}_r.\tag{8}$$

Where μ_e is the magnetic permeability. We characterize the rigidity modulus μ and the magnetic permeability μ_e and the density ρ of the non-homogeneous material in the form:

$$\mu=\mu_0r^{2m}and\rho=\rho_0r^{2m},\mu_e=\mu_{e0}r^{2m}\tag{9}$$

where μ_0 and ρ,μ_{e0} , are the values of μ and μ_e , in the homogeneous case respectively, and m is a rational number.

3 Solution of the Problem

Substituting from(6), (7) and (8) in Eq.(7), we get

$$\tau_m \frac{\partial^2 u}{\partial r^2} + \frac{\tau_m}{r}(2m+1)\frac{\partial u}{\partial r} + \frac{\nu \tau_m (2m+1)}{(1-\nu)}\frac{u}{r^2} + \mu_{e0} H_z^2 \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = \frac{\rho_0(1-2\nu)}{2\mu_0(1-\upsilon)}\left[\frac{\partial^2 u_r}{\partial t^2} - \Omega^2 u\right] \quad (10)$$

By taking the harmonic vibrations, we simplify the function u is follows:

$$u(r,t) = U(r)e^{-i\omega t}, i = \sqrt{-1} \quad (11)$$

Using the harmonic vibrations (11) in Eq.(10), we have

$$\begin{aligned} &\frac{d^2U}{dr^2} + \frac{1}{r}\frac{[(1-i\omega\tau_0)(2m+1)+\mu_0 H_Z^2]}{[(1-i\omega\tau_0)+\mu_{e0}H_Z^2]}\frac{dU}{dr} \\ &+\frac{1}{r^2}\left[\frac{\nu(1-i\omega\tau_0)(2m+1)-\mu_{e0}{H_Z}^2(1-\upsilon)}{(1-\nu)[(1-i\omega\tau_0)+\mu_{e0}H_Z^2]}\right]U \\ &= \left[\frac{-\rho_0(1-2\nu)(\omega^2+\Omega^2)}{2\mu_0(1-\nu)[(1-i\omega\tau_0)+\mu_{e0}H_Z^2]}\right]U \end{aligned} \quad (12)$$

Let:

$$U(r) = r^{-m}\varphi(r) \quad (13)$$

Which can be written as

$$\frac{d^2\varphi(r)}{dr^2} + \frac{\eta_1}{r}\frac{d\varphi(r)}{dr} + \left[\frac{\eta_2}{r^2} + \alpha^2\right]\varphi(r) = 0 \quad (14)$$

where

$$\eta_1 = \frac{(1-i\omega\tau_0)}{(1-i\omega\tau_0+\mu_{e0}H_z^2)}$$
$$,\eta_2 = \frac{m^2(\mu_{e0}{H_Z}^2-(1-i\omega\tau_0))+\nu(2m+1)(1-i\omega\tau_0)-\mu_{e0}{H_z}^2(1-\upsilon)}{(1-\nu)\left[(1-i\omega\tau_0)+\mu_{e0}H_z^2\right]}$$
$$\alpha^2 = \frac{\rho_0(1-2\nu)(\omega^2+\Omega^2)}{2\mu_0(1-\nu)[(1-i\omega\tau_0)+\mu_{e0}H_z^2]}$$

Eq.(14) is called Bessel's equation and which its general solution is known in the form

$$\varphi(r) = r^{\frac{(1-\eta_1)}{2}}[C_1 J_n(\alpha r) + C_2 Y_n(\alpha r)] \quad (15)$$

Where,

$$n = \frac{1}{2}\sqrt{1-2\eta_1+\eta_1^2-4\eta_2}$$

where C1 and C2 are arbitrary constants , $J_n(\alpha r)$ and $Y_n(\alpha r)$ denote Bessel's functions of the first and second kind of order n , respectively.

Substituting from Eq.(15) and Eq.(13) into Eq.(11), the complete solution of Eq.(7) is written in the form

$$u(r,t) = r^{-m+\frac{(1-\eta_1)}{2}}[C_1 J_n(\alpha r) + C_2 Y_n(\alpha r)]e^{-i\omega t}. \quad (15)$$

Substituting from Eq.(13) and Eq.(6) into Eq.(4), we have

$$\begin{aligned} \sigma_{rr} = &\frac{2(1-i\omega\tau_0)\mu_0 r^{-m+\frac{1-\eta_1}{2}}}{1-2\nu} * \\ &\{C_1(\tfrac{1}{r}[\,(1-\nu)\left(\tfrac{1-\eta_1-2m}{2}+n\right)+\nu\,]J_n(\alpha r) \\ &-\alpha(1-\nu)J_{n+1}(\alpha r)) \\ &+C_2(\tfrac{1}{r}\left[(1-\nu)\left(\tfrac{1-\eta_1-2m}{2}+n\right)+\nu\right]Y_n(\alpha r) \\ &-\alpha(1-\nu)Y_{n+1}(\alpha r))\}e^{-i\omega t} \end{aligned} \quad (16)$$

$$\begin{aligned} \sigma_{\theta\theta} = &\frac{2(1-i\omega\tau_0)\mu_0 r^{-m+\frac{1-\eta_1}{2}}}{1-2\nu} * \\ &\{C_1(\tfrac{1}{r}[(1-\nu)+\nu\left(n+\tfrac{1-\eta_1-2m}{2}\right)]J_n(\alpha r) \\ &-\alpha\nu J_{n+1}(\alpha r)) \\ &+C_2(\tfrac{1}{r}\left[(1-\nu)+\nu\left(n+\tfrac{1-\eta_1-2m}{2}\right)\right]Y_n(\alpha r) \\ &-\alpha\nu Y_{n+1}(\alpha r))\}e^{-i\omega t} \end{aligned} \quad (17)$$

$$\begin{aligned} \sigma_{zz} = &\frac{2(1-i\omega\tau_0)\mu_0 \nu r^{-m+\frac{1-\eta_1}{2}}}{1-2\nu} * \\ &\left\{C_1\left(\tfrac{1}{r}\left(1+n+\tfrac{1-\eta_1-2m}{2}\right)J_n(\alpha r)-\alpha J_{n+1}(\alpha r)\right)\right. \\ &\left.+C_2\left(\tfrac{1}{r}\left(1+n+\tfrac{1-\eta_1-2m}{2}\right)Y_n(\alpha r)-\alpha Y_{n+1}(\alpha r)\right)\right\} * \\ &e^{-i\omega t} \end{aligned} \quad (18)$$

4 Boundary Conditions

We consider the dynamical problem of a composite cylinder of radius $r = b$ containing a rigid core of radius $r = a$. The mixed boundary conditions are:

$$\begin{aligned} u(r,t) &= 0 \qquad on \qquad r = a, \\ \sigma_{rr} &= -pe^{-i\omega t} \qquad on \qquad r = b. \end{aligned} \quad (19)$$

From Eqs.(16), (17) and (20), the constants C1 and C2 are written in the form

$$C_1 = \frac{-p(1-2\nu)Y_n(\alpha a)}{2\mu_0(1-i\omega\tau_0)b^{-m+\frac{1-\eta_1}{2}}\left[\frac{1}{b}\ell_1(J_n(\alpha b)Y_n(\alpha a)+Y_n(\alpha b)J_n(\alpha a))-\ell_2(J_{n+1}(\alpha b)Y_n(\alpha a)+Y_{n+1}(\alpha b)J_n(\alpha a))\right]} \quad (20)$$

$$C_2 = \frac{-p(1-2\nu)J_n(\alpha a)}{2\mu_0(1-i\omega\tau_0)b^{-m+\frac{1-\eta_1}{2}}\left[\frac{1}{b}\ell_1(J_n(\alpha b)Y_n(\alpha a)+Y_n(\alpha b)J_n(\alpha a))-\ell_2(J_{n+1}(\alpha b)Y_n(\alpha a)+Y_{n+1}(\alpha b)J_n(\alpha a))\right]} \quad (21)$$

where,

$$\ell_1 = (1-\nu)\left(\frac{1-\eta_1-2m}{2}+n\right)+\nu\,,\ \ell_2 = \alpha(1-\nu).$$

The displacement and stresses can be calculated by substituting from (21) and (22) in (16)-(19). Now we discuss the above by using Maple program is clear up from figs(1-7).

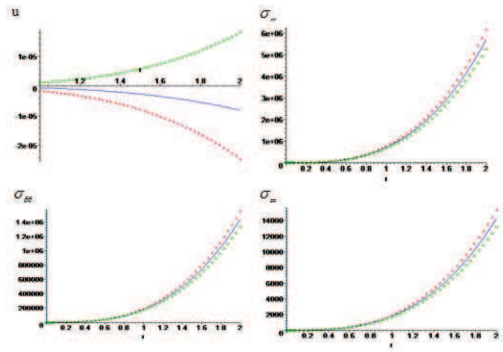

Fig. 1: Effect of τ_0 on displacement and stresses displacement with radius r, $\tau_0 = 0.1$”*Green*”, $\tau_0 = 0.3$”*Blue*”, $\tau_0 = 0.5$”*Red*”.

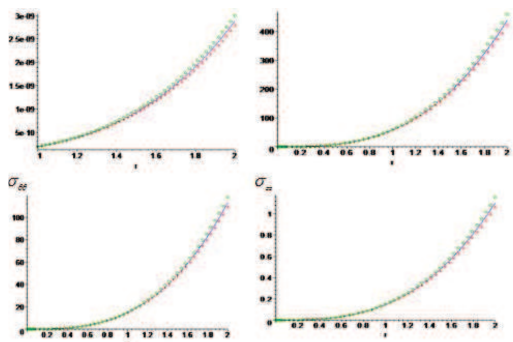

Fig. 2: Effect of Ω on displacement and stresses displacement with radius r, $\Omega = 0.3$”*Green*”, $\Omega = 0.5$”*Blue*”, $\Omega = 0.7$”*Red*”.

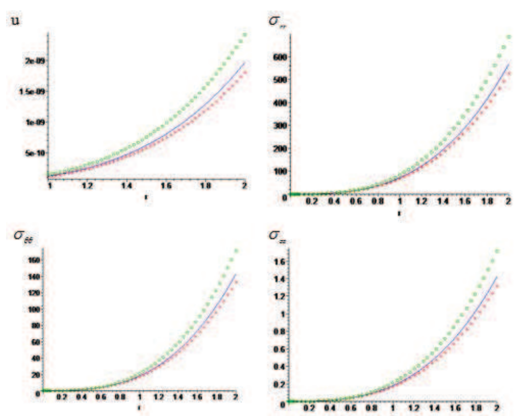

Fig. 3: Effect of H_φ on displacement and stresses displacement with radius r, $H_\varphi = 0.3$”*Green*”, $H_\varphi = 0.5$”*Blue*”, $H_\varphi = 0.7$”*Red*”.

5 Particular Case

5.1 The composite infinite cylinder in non-homogenity viscoelastic media with effect of magnetic field and neglect rotation, in this case $\Omega = 0$.

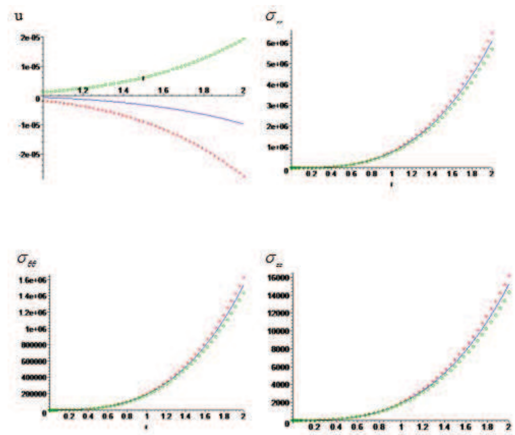

Fig. 4: Effect of τ_0 on displacement and stresses displacement with radius r, $\tau_0 = 0.1$”*Green*”, $\tau_0 = 0.3$”*Blue*”, $\tau_0 = 0.5$”*Red*”.

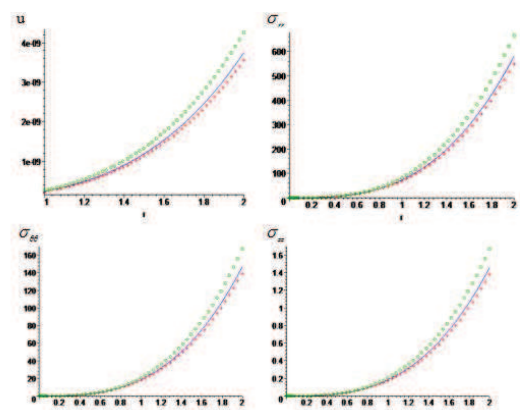

Fig. 5: Effect of H_φ on displacement and stresses displacement with radius r, $H_\varphi = 0.3$”*Green*”, $H_\varphi = 0.5$”*Blue*”, $H_\varphi = 0.7$”*Red*”.

5.2 The composite infinite cylinder in non-homogeneity of elastic media with effect of rotation and of magnetic field, in this case $\tau_0 = 0$

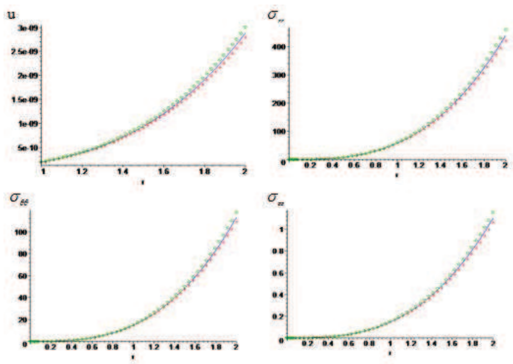

Fig. 6: Effect of Ω on displacement and stresses displacement with radius r, $\Omega = 0.3$"*Green*", $\Omega = 0.5$"*Blue*", $\Omega = 0.7$"*Red*".

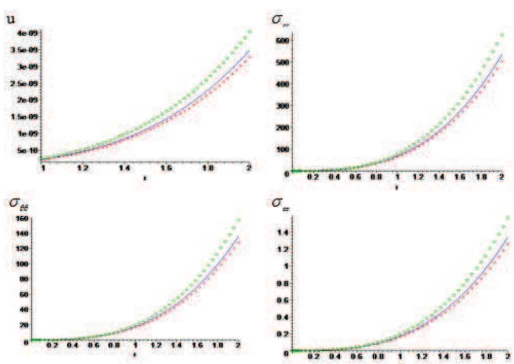

Fig. 7: Effect of H_φ on displacement and stresses displacement with radius r, $H_\varphi = 0.3$"*Green*", $H_\varphi = 0.5$"*Blue*", $H_\varphi = 0.7$"*Red*".

5.3 The composite infinite cylinder in non-homogeneity viscoelastic media with effect of rotation and neglect magnetic field, in this case H_φ

6 Numerical Results and Discussion

The numerical results are obtained taking into account evaluation an Copper material, the physical data for such material are given by [18].

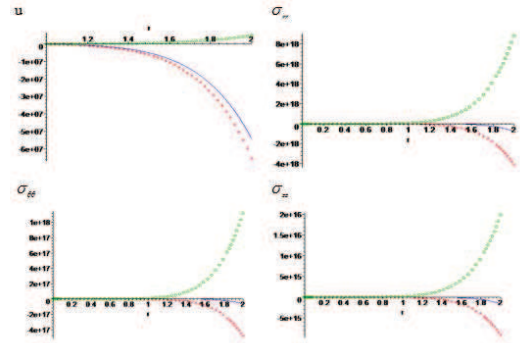

Fig. 8: Effect of τ_0 on displacement and stresses displacement with radius r, $\tau_0 = 0.1$"*Green*", $\tau_0 = 0.3$"*Blue*", $\tau_0 = 0.5$"*Red*".

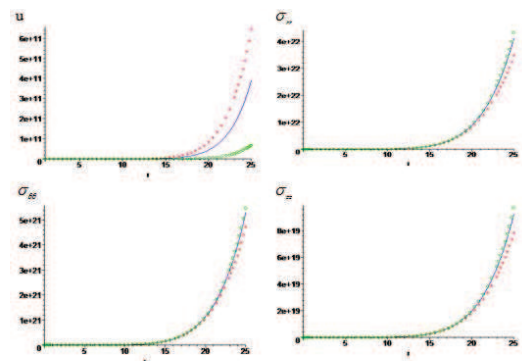

Fig. 9: Effect of Ω on displacement and stresses displacement with radius r, $\Omega = 0.3$"*Green*", $\Omega = 0.5$"*Blue*", $\Omega = 0.7$"*Red*".

$$\mu_0 = 3.86 \times 10^{10} kgm^{-1}s^{-2},$$
$$\rho_0 = 8.96 \times 10^{3} kgm^{-1}s^{-2}, \quad \omega = 1.5s^{-1}$$

we take into account the time

$m = -0.4,\ \nu = 0.2 \times 10^{-3}, a = 2 \times 10^{-3}, b = 2 \times 10^{-3}$, $\mu_e = 0.2 \times 10^{2}, H = \ldots \times 10^{-3}, p = 1.5 \times 10^{9}, t = 9 \times 10^{8'}$, $\Omega = \ldots \times 10^{2}$.

We present our results for isotropic cylinder in the forms of graphs (1-9), which it displays the components of radial displacement and stresses.

6.1 The composite infinite cylinder in non-homogeneity viscoelastic media under effects magnetic field and rotation:

From Fig .(1), that clarifies of effect of viscosity, we find that the components of displacement $u(r,t)$ decreased with increased value of τ_0 , while the components of stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} increased with increased values of τ_0. From Fig(2), that clarify of effect of rotation, we find that the components of displacement $u(r,t)$, stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of Ω. From Fig(3), that clarify of effect of magnetic field, we find that the components of displacement $u(r,t)$ decreased with increased value of τ_0 , while the components of stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} increased with increased values of H_z.

6.2 The composite infinite cylinder in non-homogeneity viscoelastic media with effect of magnetic field and neglect rotation, in this case $\Omega = 0$

From Fig (4), that clarify of effect of viscosity, we find that the components of displacement $u(r,t)$, stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of τ_0. From Fig. (5), that clarify of effect of magnetic field, we find that the components of displacement $u(r,t)$, stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of H_z.

6.3 The composite infinite cylinder in non-homogeneity of elastic media with effect of rotation and of magnetic field, in this case τ_0

From Fig. (6), that clarify of effect of rotation, we find that the components of displacement $u(r,t)$, stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of Ω. From Fig. (7), that clarify of effect of magnetic field, we find that the components of displacement $u(r,t)$, stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of H_z.

6.4 The composite infinite cylinder in non-homogeneity viscoelastic media with effect of rotation and neglect magnetic field, in this case $H_z = 0$

From Fig(8), that clarify of effect of viscosity, we find that the components of displacement $u(r,t)$, stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of τ_0. From Fig. (9), that clarify of effect of rotation, we find that the components of displacement $u(r,t)$ increasing put stresses σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} decreased with increased values of Ω respectively.

7 Conclusions

The exact solution for inhomogeneous composite cylinder subjected to magnetic field, viscosity and rotation is obtained. All material coefficients are assumed to have the same exponent-law dependence on the radial direction of the composite cylinder. The distribution of displacement and stresses are drawn and discussed in detail. The obtained solution is valid for magnetic field, viscosity and rotation on the composite cylinder.The results show that the inhomogeneous exponent, magnetic field, the viscosity and rotation have effect on the radial displacement and stresses. By selecting a proper value of m and suitable rotation, viscosity and magnetic field, it is possible for engineers to design such cylinder that can meet some special requirements. The results indicate that the effect of magnetic field, viscosity, rotation and non-homogeneity on the radial displacement and stresses are very pronounced. It is shown that the results obtained from the present method are in good agreement with those obtained using the previously methods for zero magnetic field.

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