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A New Distribution Using Sine Function- Its Application To Bladder Cancer Patients Data

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Abstract: A new life time distribution is proposed by the use of Sine function in terms of some life time distribution as baseline distribution. It is derived for the baseline distribution as exponential distribution and some statistical properties of the new distribution, thus obtained have been studied. The new distribution have been shown better fit to the bladder cancer patients data as compared to some well known distributions available in the statistical literature through Akaike information criteria (AIC), Bayesian information criteria (BIC), - log-likelihood and the associated Kolmogorov-Smirnov (KS) test values.

Keywords: Life Time Distribution, Reliability Analysis and Bayesian Inferences.

1 Introduction

In statistical literature, several methods are available to propose new life time distribution by the use of some existing life time distribution as baseline distribution. For example, Gupta et al. [4] proposed the cumulative distribution function (cdf) $G_1(x)$ of new distribution corresponding to the cdf $F_1(x)$ of baseline distribution as,

$$G_1(x) = (F_1(x))^{\alpha},$$

where, $\alpha > 0$ is the shape parameter of the proposed one. For $\alpha = 1$, the new distribution and the baseline distribution are the same.

Several researchers generalise some useful distributions by the idea of Gupta et al. [4]. For example, Nadarajah and Kotz [6] introduced four exponentiated type distributions that are the generalizations of the standard gamma, standard Weibull, standard Gumbel and the standard Frechet distributions and studied some mathematical properties for each distribution. Nadarajah [7] derived exponentiated standard Gumbel distribution with a hope that it would attract wider applicability in climate modeling as the standard Gumbel distribution do. Many other genralizations can be found in the statistical literatures.

Another idea of generalizing a baseline distribution is to transmute it by using the quadratic rank transmutation map (QRTM) (see, Shaw and Buckley [8]). If $G_2(x)$ be the cdf of transmuted distribution corresponding to the baseline distribution having cdf $F_2(x)$, then

$$G_2(x) = (1+\lambda)F_2(x) - \lambda\{F_2(x)\}^2,$$

where $|\lambda| \leq 1$. For $\lambda = 0$, the new distribution is same as the baseline distribution.

Recently, various generalizations has been introduced based on QRTM. For example, transmuted extreme value distribution (see, Aryal and Tsokos [9]), transmuted log-logistic distribution (see, Aryal [13]), transmuted modified Weibull distribution (see, Khan and King [15]), transmuted inverse Weibull distribution (see, Khan, King and Hudson

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[19]) and many more.

In the present study, we propose a method to get new life distribution by the use of any baseline life distribution. If $f(x)$ and $F(x)$ be the pdf and cdf of some baseline life distribution, then the cdf $G(x)$ of new life distrbution is proposed by,

$$G(x) = \sin\left(\frac{\pi}{2}F(x)\right) \tag{1}$$

Further, if $g(x)$ be the pdf and $h(x)$ be the hazard rate function corresponding to the cdf $G(x)$, then,

$$g(x) = \frac{\pi}{2}\, f(x)\, \cos\left(\frac{\pi}{2}F(x)\right) \tag{2}$$

and

$$h(x) = \frac{\pi}{2}\, f(x)\, \tan\left(\frac{\pi}{4} + \frac{\pi}{4}\, F(x)\right) \tag{3}$$

respectively.

We will call the transformation (1) and (2) as SS transformation for frequently used purpose in the present article or elsewhere.

The rest of the paper is organized as follows: In section 2, we propose a new distribtion, as obtain by SS transformation (2) by considering Exp(θ)-distribution as the baseline distribution and studied some of its statistical characteristics; like moment generating function (MGF), moments, median and mode. Further, in section 3, we have shown the applicability of the new distribution obtained in the section 2, to the bladder cancer patients data in terms of assesing its fitting in comparison to some available distributions. In section 4, we have derived MLE and Bayes estimators of the parameter θ of the distribution, thus obtained under GELF and SELF. Finally, comparison and conclusion has been shown in the sections 5 and 6 respectively .

2 SS transformation of Exp(θ)-distribution

Let the baseline distribution is Exp(θ)-distribution with pdf,

$$f(x) = \theta\, e^{-\theta x} \quad ; \quad x > 0 \tag{4}$$

and the corresponding cdf is given by,

$$F(x) = 1 - e^{-\theta x} \tag{5}$$

Here, $\theta > 0$ is the rate parameter or inverse scale parameter of Exp(θ)-distribution.

Let $g(x)$ be the pdf of the new distribution; obtained by SS transformation (2), corresponding to the baseline pdf (4), then

$$g(x) = \frac{\pi}{2}\, \theta\, e^{-\theta x}\, \sin\left(\frac{\pi}{2}\, e^{-\theta x}\right) \quad ; \quad x > 0 \tag{6}$$

For simplicity in terms of use, we name/call the distribution having pdf (6) as SS transformation of Exp (θ)-distribution and we will write it as SS$_E(\theta)$-distribution.

The cdf and hazard rate function of SS$_E(\theta)$-distribution are given by,

$$G(x) = \cos\left(\frac{\pi}{2}\, e^{-\theta x}\right) \tag{7}$$

and

$$h(x) = \frac{\pi}{2}\, \theta\, e^{-\theta x}\, \cot\left(\frac{\pi}{4}\, e^{-\theta x}\right) \tag{8}$$

respectively.

The plots of pdf and hazard rate function, for different values of θ are shown in Figures 1 and 2 respectively.

2.1 Moment Generating Function of $SS_E(\theta)$-distribution

The moment generating function of $\mathrm{SS}_E(\theta)$-distribution having pdf (6) is obtained as follows,

$$M_X(t) = \theta \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k+2}}{(2k+1)!} \left\{ \frac{1}{(2k+2)\theta - t} \right\} \tag{9}$$

provided $t < 2\theta$.

2.2 Raw Moments of $SS_E(\theta)$-distribution

The r^{th} moment about origin (i.e. raw moment) of $\mathrm{SS}_E(\theta)$-distribution is obtained as follows,

$$\begin{aligned} \mu_r^{'} &= \left[\frac{\partial^r M_X(t)}{\partial t^r} \right]_{t=0} \\ &= \frac{r!}{\theta^r} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k+2}}{(2k+1)!\,(2k+2)^{r+1}} \end{aligned} \tag{10}$$

2.3 Median of $SS_E(\theta)$-distribution

The median of $\mathrm{SS}_E(\theta)$-distribution is the solution of the following equation for M,

$$G(M) = \frac{1}{2}$$

and the same is obtained as follows,

$$M = -\frac{1}{\theta} \ln\left(\frac{2}{3}\right) \tag{11}$$

2.4 Mode of $SS_E(\theta)$-distribution

Differentiating (6) partially w. r. to x on both sides, we get

$$g^{'}(x) = -\frac{\pi}{2}\, \theta^2 \, e^{-\theta x} \left\{ \sin\left(\frac{\pi}{2}\, e^{-\theta x}\right) + \frac{\pi}{2}\, e^{-\theta x} \cos\left(\frac{\pi}{2}\, e^{-\theta x}\right) \right\} \tag{12}$$

Clearly

$$g^{'}(x) < 0 \quad \forall \quad x,\ \ \theta$$

and this shows that $g(x)$ is a decreasing function of $x > 0$ ($\forall\ \theta$) and hence its mode is $x = 0$.

3 Estimation of the parameter θ of $\mathrm{SS}_E(\theta)$-distribution

3.1 Maximum Likelihood Estimator

Let n identical items are put on a life testing experiment and suppose $\underline{\mathbf{X}} = (X_1, X_2, \ldots, X_n)$ be their independent lives such that each X_i ($\forall\ i = 1[1]n$) follow $\mathrm{SS}_E(\theta)$-distribution having pdf (6). Then the likelihood function for $\underline{\mathbf{X}}$ is given by,

$$L(\underline{\mathbf{X}}|\theta) = \prod_{i=1}^{n} g(x_i) \tag{13}$$

Putting the value of g at x_i from (6) in (13), we get

$$L(\underline{\mathbf{X}}|\theta) = \prod_{i=1}^{n} \left\{ \frac{\pi}{2}\ \theta\ e^{-\theta x_i}\ \sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right) \right\}$$
$$= \left(\frac{\pi}{2}\right)^n \theta^n\ e^{-\theta \sum\limits_{i=1}^{n} x_i} \prod_{i=1}^{n} \sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right) \tag{14}$$

The log- likelihood function for $\underline{\mathbf{X}}$ is obtained as,

$$l = \ln L(\underline{\mathbf{X}}|\theta)$$
$$= K + n\ln\theta - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \ln\left\{\sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\right\} \tag{15}$$

where $K = n\ \ln\left(\frac{\pi}{2}\right)$ is a constant.

Hence, the log- likelihood equation for estimating θ is given by,

$$\frac{\partial l}{\partial \theta} = 0$$
$$\frac{n}{\theta} - \sum_{i=1}^{n} x_i - \frac{\pi}{2}\sum_{i=1}^{n}\left\{x_i\ e^{-\theta x_i}\ \cot\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\right\} = 0 \tag{16}$$

Above equation is not solvable analytically for θ. We propose Newton- Raphson method for its numerical solution.

3.2 Bayes Estimators

An important element in Bayesian estimation problem is the specification of the loss function. The choice is basically depends on the problem in hand. For more discussion on the choice of a suitable loss function, readers may refer to Singh et al. [11]. Another, important element is the choice of the appropriate prior distribution that covers all the prior knowledge regarding the parameter of interest. For the criteria of choosing an approprriate prior distribution, see Singh et al. [12].

With the above philosophical point of view, we are motivated to take the prior for θ as $G(\alpha,\beta)$-distribution with the pdf

$$\pi(\theta) = \frac{\alpha^{\beta}}{\Gamma(\beta)} e^{-\alpha\theta}\theta^{\beta-1} \quad ; \quad \theta > 0 \tag{17}$$

where $\alpha > 0$ and $\beta > 0$ are the hyper- parameters. These can be obtained, if any two independent informations on θ are available, say prior mean and prior variance are known (see, Singh et al. [12]). The mean and variance of the prior distribution (17) are $\frac{\beta}{\alpha}$ and $\frac{\beta}{\alpha^2}$ respectively. Thus, we may take $M = \frac{\beta}{\alpha}$ and $V = \frac{\beta}{\alpha^2}$, giving $\alpha = \frac{M^2}{V}$ and $\beta = \frac{M}{V}$. For any finite value of M and V to be sufficiently large, (17) behaves as like as non-informative prior.

The posterior pdf of θ given $\underline{\mathbf{X}}$ corresponding to the considered prior pdf $\pi(\theta)$ of θ is given by,

$$\psi(\theta|\underline{\mathbf{X}}) = \frac{L(\underline{\mathbf{X}}|\theta)\ \pi(\theta)}{\int\limits_0^{\infty} L(\underline{\mathbf{X}}|\theta)\ \pi(\theta)\partial\theta}$$
$$= \frac{e^{-\left(\alpha+\sum\limits_{i=1}^{n} x_i\right)\theta}\ \theta^{\beta+n-1}\ \prod\limits_{i=1}^{n}\sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)}{\int\limits_0^{\infty} e^{-\left(\alpha+\sum\limits_{i=1}^{n} x_i\right)\theta}\ \theta^{\beta+n-1}\ \prod\limits_{i=1}^{n}\sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\partial\theta} \tag{18}$$

Now, to have an idea about the shapes of the prior and corresponding posterior pdfs for different confidence levels in the guessed value of θ as its true value, we randomly generate a sample from $\mathrm{SS}_E(\theta)$-distribution for fixed values $n = 15$,

$\theta = 2$, $M = 2$, $V = 0.1$ (showing a higher confidence in the guessed value) and $V = 500$ (showing a weak confidence in the guessed value). The sample thus generated is,

$\underline{\mathbf{X}} = (0.009334163, 0.035661012, 0.041706382, 0.054252838, 0.058706749, 0.085740112, 0.094581234, 0.119499688, 0.144828571, 0.145472486, 0.148218681, 0.281091001, 0.411933061, 0.449613798, 0.933292489)$

The graphs are shown in Figures 3 and 4 respectively.

The loss functions we considered here are general entropy loss function (GELF) and squared error loss function (SELF), which are defined by,

$$L_G\left(\hat{\theta},\ \theta\right) = \left(\frac{\hat{\theta}}{\theta}\right)^{\delta} - \delta\ \ln\left(\frac{\hat{\theta}}{\theta}\right) - 1 \tag{19}$$

and

$$L_S\left(\hat{\theta},\ \theta\right) = \left(\hat{\theta} - \theta\right)^2 \tag{20}$$

respectively.

The Bayes estimators of θ under GELF (19) and SELF (20) are given by

$$\hat{\theta}_G = \left[E\left\{\theta^{-\delta}|\underline{\mathbf{X}}\right\}\right]^{-\frac{1}{\delta}} \tag{21}$$

and

$$\hat{\theta}_S = E\left[\theta|\underline{\mathbf{X}}\right] \tag{22}$$

respectively. It is easy to see that when $\delta = -1$, the Bayes estimator (21) under GELF reduces to the Bayes estimator (22) under SELF.

It is name-worthy to note here that GELF (19) was proposed by Calabria and Pulcini [3] and SELF (20) was proposed at first by Legendre [1] and Gauss [2] when he was developing the least square theory. For more applications related to GELF, readers may refer to Singh et al. [16,17,18].

Now, the Bayes estimator of the parameter θ of $SS_E(\theta)$-distribution having pdf (6) under GELF is obtained as follows,

$$\begin{aligned}\hat{\theta}_G &= \left[E\left\{\theta^{-\delta}|\underline{\mathbf{X}}\right\}\right]^{-\frac{1}{\delta}} \\ &= \left[\frac{\int_0^\infty e^{-\left(\alpha+\sum_{i=1}^n x_i\right)\theta}\ \theta^{\beta-\delta+n-1}\ \prod_{i=1}^n \sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\partial\theta}{\int_0^\infty e^{-\left(\alpha+\sum_{i=1}^n x_i\right)\theta}\ \theta^{\beta+n-1}\ \prod_{i=1}^n \sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\partial\theta}\right]^{-\frac{1}{\delta}}\end{aligned} \tag{23}$$

Further, if $\hat{\theta}_S$ denotes the Bayes estimator of θ under SELF, then it can be obtained by putting $\delta = -1$ in (23) and therefore the same is given by,

$$\hat{\theta}_S = \frac{\int_0^\infty e^{-\left(\alpha+\sum_{i=1}^n x_i\right)\theta}\ \theta^{\beta+n}\ \prod_{i=1}^n \sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\partial\theta}{\int_0^\infty e^{-\left(\alpha+\sum_{i=1}^n x_i\right)\theta}\ \theta^{\beta+n-1}\ \prod_{i=1}^n \sin\left(\frac{\pi}{2}\ e^{-\theta x_i}\right)\partial\theta} \tag{24}$$

The integral involved in Bayes estimators do not solved analytically, therefore we propose Gauss - Lagurre's quadrature method for their numerical evaluation.

4 Bladder Cancer Patients Data

In this section, we analyze a real data set to illustrate that $SS_E(\theta)$-distribution can be a good lifetime model, comparing with many known distributions available in statistical literature. For the purpose, we have considered a real data of the remission times (in months) of a random sample of 128 bladder cancer patients. The data is extracted from Lee and Wang [5] and is as shown below:

$\underline{\mathbf{X}} = (0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06,$
$7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76,$
$26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18,$
$5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12,$
$46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71,$
$7.93, 1.46, 18.10, 11.79, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 13.31, 4.51, 6.54, 8.53,$
$12.03, 20.28, 2.02, 3.36, 12.07, 6.76, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69)$

Khan et al. [19] showed the applicability of transmuted inverse Weibull distribution (TIWD) on this data by the fitting criteria in terms of Akaike information criteria (AIC), Bayesian information criteria (BIC), mean square error (MSE) and the associated Kolmogorov-Smirnov (KS) test values. They compared some life time distributions namely transmuted inverse Rayleigh distribution (TIRD), transmuted inverted exponential distribution (TIED) and inverse Weibull distribution (IWD) in terms of their AIC, BIC, MSE and KS test values and found that the TIWD has the lowest AIC, BIC, MSE and KS test value, indicating that TIWD provides a better fit than the other three lifetime distributions to the bladder cancer patients data.

We have computed MLE of the parameter θ of $SS_E(\theta)$-distribution having pdf (6) for the above data set and found it as 0.05925657. The AIC, BIC and KS test value for $SS_E(\theta)$-distribution are calculated and we get their values as in Table 1. We have extracted the values of AIC, BIC, -log–likelihood (-LL) and KS test values for TIWD, TIED, IWD and TIRD for the above considered data from Khan et al. [19] and present their values in the following comparative Table 1.

Table 1: AIC, BIC, -LL and KS test values for $SS_E(\theta)$-distribution, TIWD, TIED, IWD and TIRD

Distributions	AIC	BIC	-LL	KS test value
$SS_E(\theta)$-distribution	832.6	835.5	415.3	0.067
TIWD	879.4	879.7	438.5	0.119
TIED	889.6	889.8	442.8	0.155
IWD	892.0	892.2	444.0	0.131
TIRD	1424.4	1424.6	710.2	0.676

The plots of empirical cdf F_n and fitted cdf $G(x)$ of $SS_E(\theta)$-distribution having pdf (6) for above data of the remission times of a random sample of 128 bladder cancer patients are shown in Figure 5.

From Table 1, it is observed that $SS_E(\theta)$-distribution having pdf (6) has the lowest AIC, BIC, -LL and KS test value in comparision to those of TIWD, TIED, IWD and TIRD; indicating that $SS_E(\theta)$-distribution provides a better fit than the other four lifetime distributions namely TIWD, TIED, IWD and TIRD.

5 Comparison of the estimators

In this section, we compared the considered estimators i.e. $\hat{\theta}_M$, $\hat{\theta}_S$, $\hat{\theta}_G$ of the parameter θ of pdf (6) in terms of simulated risks (average loss over sample space) under GELF. It is clear that the expressions for the risks cannot be obtained in nice closed form. So, the risks of the estimators are estimated on the basis of Monte Carlo simulation study of 5000 samples from pdf (6). It may be noted that the risks of the estimators will be a function of number of items put on test n, parameter θ of the model, the hyper parameters α and β of the prior distribution and the GELF parameter δ. In order to consider the variation of these values, we obtained the simulated risks for $n = 15$, $\theta = 2$, $M = 1, 2, 3$, $V = 0.1, 0.5, 1, 2, 5, 10, 100, 500$ and $\delta = \pm 3$.

Table 2 shows the risks of the estimators of θ when guessed value of θ ($M = 1$) is less than its true value ($\theta = 2$) and we observed that when over estimation is more serious than under estimation, the estimator $\hat{\theta}_G$ performs better (in the sense of having smallest risk) in comparison to $\hat{\theta}_S$ and $\hat{\theta}_M$ for lower confidence in the guessed value and for high confidence in guessed value, $\hat{\theta}_S$ performs better than $\hat{\theta}_G$ and $\hat{\theta}_M$. But in the reverse situation, the only change is noted that for high confidence in the guessed value, $\hat{\theta}_M$ performs better than $\hat{\theta}_G$ and $\hat{\theta}_S$.

Further, Table 3 shows the risks of the estimators of θ when guessed value of θ ($M = 2$) is same as its true value ($\theta = 2$) and it is observed that the estimator $\hat{\theta}_G$ performs better than the other estimators for moderate and lower confidence in the guessed value, while for higher confidence in the guessed value, $\hat{\theta}_S$ performs better for whatever may be the situation is serious.

Finally, Table 4 shows the risks of the estimators of θ when guessed value of θ ($M = 3$) is greater than its true value ($\theta = 2$) and it is observed that when over estimation is more serious than under estimation, the estimator $\hat{\theta}_G$ performs better in comparison to $\hat{\theta}_S$ and $\hat{\theta}_M$ for lower confidence in the guessed value and for high confidence in guessed value, $\hat{\theta}_S$ performs better. But in the reverse situation, the estimator $\hat{\theta}_M$ performs well for higher confidence, $\hat{\theta}_S$;performs batter for moderate confidence and for lower confidence, the estimator $\hat{\theta}_G$ performs better.

Table 2: Risks of the estimators of θ under GELF for fixed $n = 15$, $\theta = 2$, $M = 1$ and $\delta = \pm 3$

V	$\delta = -3$			$\delta = +3$		
	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$
0.1	0.2738009	0.8279515	0.5602645	0.378193	0.3246863	0.5540848
0.5	0.2743367	0.2698518	0.1989104	0.3770018	0.1684724	0.2507738
1	0.2718506	0.2561712	0.2097545	0.3928168	0.2311955	0.2512197
5	0.2718565	0.267155	0.2412518	0.3756553	0.3288731	0.278374
10	0.2696276	0.2665932	0.246442	0.3686863	0.3419576	0.278791
100	0.2738389	0.2751602	0.2541608	0.3761437	0.3673616	0.2906435
500	0.2805843	0.2833976	0.2628969	0.3793893	0.373038	0.2910866

Table 3: Risks of the estimators of θ under GELF for fixed $n = 15$, $\theta = 2$, $M = 2$ and $\delta = \pm 3$

V	$\delta = -3$			$\delta = +3$		
	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$
0.1	0.2779026	0.04231212	0.04093879	0.3857821	0.03682427	0.03842067
0.5	0.2724498	0.1058654	0.1103778	0.3884136	0.1308066	0.1080364
1	0.273504	0.1628869	0.1618809	0.3793466	0.1975366	0.1708151
5	0.2735559	0.2464482	0.231951	0.3710224	0.3160623	0.2569307
10	0.2779773	0.2647012	0.2482753	0.3689269	0.3368521	0.2714024
100	0.2711806	0.2700362	0.2521126	0.3748748	0.3653429	0.2896318
500	0.2764858	0.2788361	0.2566196	0.3686769	0.3618619	0.2879823

Table 4: Risks of the estimators of θ under GELF for fixed $n = 15$, $\theta = 2$, $M = 3$ and $\delta = \pm 3$

V	$\delta = -3$			$\delta = +3$		
	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_G)$
0.1	0.2715807	0.4357377	0.4365846	0.3884268	0.9126745	0.9036269
0.5	0.266041	0.2199453	0.2558782	0.3975871	0.43974	0.2773379
1	0.2777014	0.1767826	0.2118104	0.3691888	0.337568	0.1935476
5	0.2757049	0.2222811	0.22444	0.3687834	0.3365857	0.2431336
10	0.27652	0.2474642	0.2411496	0.3751386	0.353347	0.2676574
100	0.2737185	0.2712784	0.2533212	0.3783093	0.370335	0.2929393
500	0.276816	0.2764202	0.2568241	0.3842771	0.3776275	0.2966648

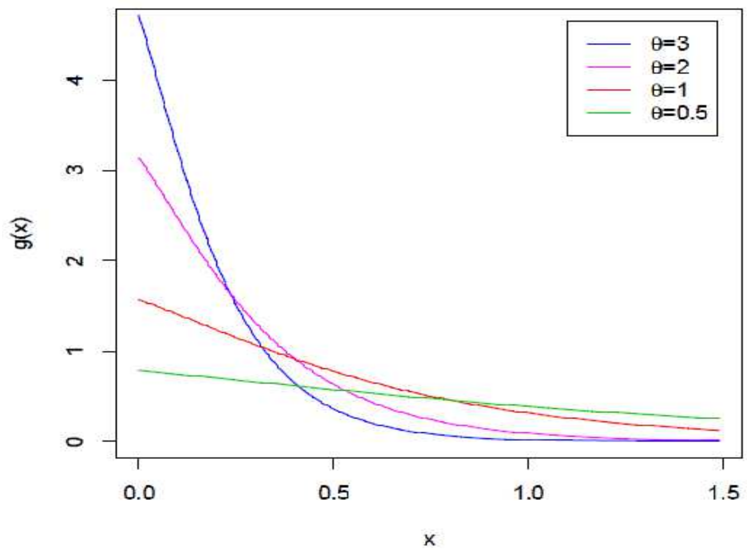

Fig. 1: Plots of probability density function of $SS_E(\theta)$-distribution for different values of θ

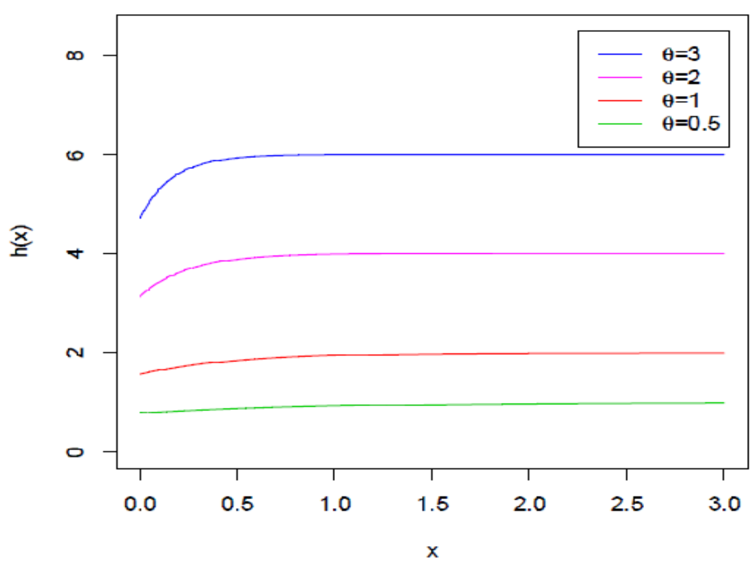

Fig. 2: Plots of hazard rate function of $SS_E(\theta)$-distribution for different values of θ

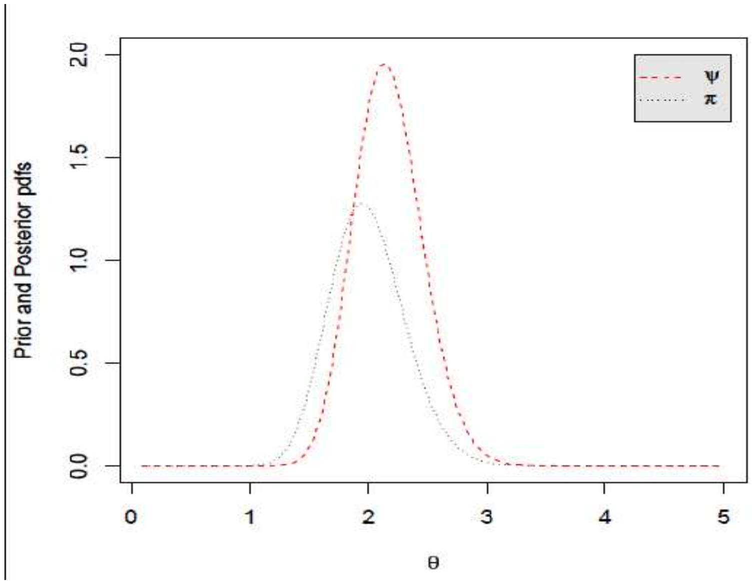

Fig. 3: Prior and Posterior pdfs of θ for a randomly generated sample $\underline{\mathbf{X}}$ from $SS_E(\theta)$-distribution for fixed $n = 15$, $\theta = 2$, $M = 2$ and **V=0.1**

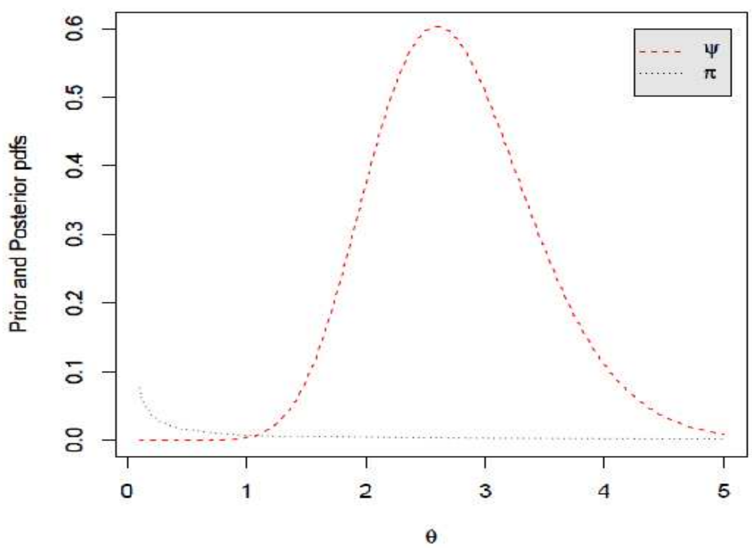

Fig. 4: Prior and Posterior pdfs of θ for a randomly generated sample $\underline{\mathbf{X}}$ from $SS_E(\theta)$-distribution for fixed $n = 15$, $\theta = 2$, $M = 2$ and **V=500**

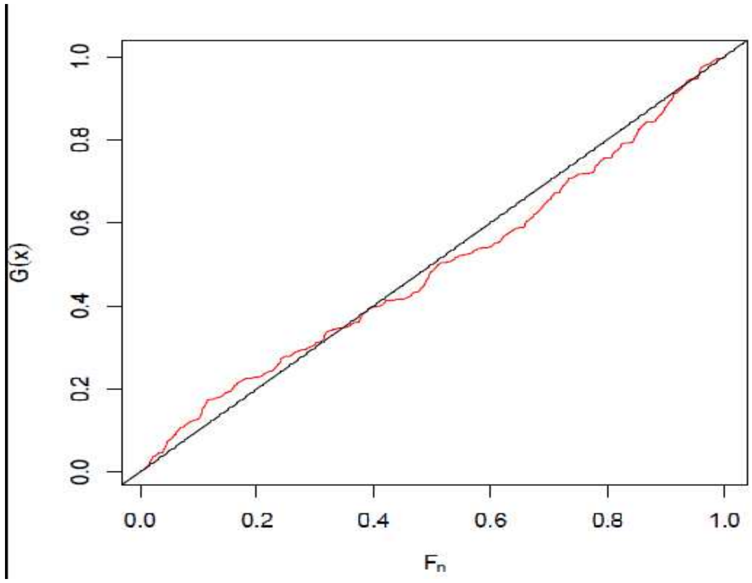

Fig. 5: Plots of empirical cdf F_n and fitted cdf $G(x)$ of $\mathrm{SS}_E(\theta)$-distribution for remission times of 128 bladder cancer patients data

6 Conclusion

From the above simulation study, it is clear that the Bayes estimators of the parameter θ of $\mathrm{SS}_E(\theta)$-distribution having pdf (6) may be recommended for their use as per confidence level in the guessed value of θ as discussed in the previous section. Further from real data analysis, it is clear that SS transformation (1) is full proof and by its use, the distribution, thus found may be appropriate for real life applications.

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