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A Lagrangian Heuristic Algorithm for an Automobile Distribution Network Optimization Problem

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Abstract: The distribution network design problem for an automobile company in China is investigated. Through optimizing the location of the distribution centers and allocation of retailers to distribution centers or plants, the total profit for the automobile company is maximized. The demand is assumed to be sensitive to the lead time and a Linear Integer Programming model is proposed to formulate the problem. A Lagrangian heuristic algorithm is developed to solve the problem, in which the subgradient algorithm and a heuristic algorithm is combined. Large scale examples including up to 20 plants, 100 distribution centers and 500 retailers are used to test the algorithm. Computational results show that the solution approach can obtain near-optimal solution for the problem in short time.

Keywords: Build-to-order, Supply chain, Distribution network design, Integer programming, Lagrangian relaxation

1 Introduction

After the success of Build-to-Order (BTO) strategy in BMW, more and more companies in automobile industry begin to apply this strategy, and BTO supply chain is considered to be a strategy that can improve their competition in the future uncertain market.

When BTO strategy prevails in automobile industry, some new challenges arise. For example, 65% buyers in UK think the waiting time from order to delivery is very important for their final choice for the car and 61% of customers would like their vehicle to be delivered within 14 days or less, while the majority of US consumers would only be willing to wait up to three weeks to receive their vehicle after they submit their orders [1]. But there is a lack of research on the trade-off between lead time and the cost of logistics [2].

In recent years, automobile companies in China, such as Volkswagen China, also adopt BTO strategy for some of their products. In many situations, customers in China have to wait for 1-2 months for their cars, which influences the demand of the automobile products. Therefore, more and more automobile companies in China face the problem to optimize their distribution networks to increase the response to customers. We consider the design of the distribution network for an automobile company in China. The distribution network consists of plants, distribution centers (DCs) and retailers (demand zones). The products are assembled immediately in plant and sent to the retailers directly or through the DCs as soon as they receive the orders. We assume that each retailer's demand for one plant is served either by a single DC or directly by the plant if they are close enough, but not by both.

Gunasekaran and Ngai [2] take a comprehensive and overall review on BTO supply chain management, present a framework for developing it, and point out five deficiencies in current research on BTO supply chain, one of which is the trade-off between responsiveness and the cost of logistics. Gunasekaran and Ngai [3] further review the modeling and analysis of BTO supply chain, and present some new challenges. Holweg and Miemczyk [4] evaluate current automotive distribution logistics systems in UK and suggest that responsive delivery is actually more cost efficient than the current system given the implementation of certain mitigating measures. Holweg and Miemczyk [5] compare the implications on inbound, outbound and sea transportation logistics and develop a strategic framework for BTO automotive logistics.

BTO strategy has put pressure on logistics to reduce lead times, but there are insufficient considerations on the lead time in the distribution network design or facility location [6,7,8]. Although some traditional models can be used in BTO supply chain design, the objectives of the

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optimization and decision variables may be need to change [2]. And fewer models are developed specially for automobile BTO companies or consider the lose sale caused by lead time. Some of the few papers considering the lead time cost in distribution network design are by [9,10], in which a large scale model is presented, but the demand is considered to be known and steady.

Here we reinvestigate the demand pattern for the automobile BTO supply chain, and the demand is assumed to be sensitive to the lead time. By this assumption, the lead time can be considered form the view of its impact on demand. Therefore, the model we develop is more efficient and suitable for the automobile BTO distribution network design. We assume that the customers go to the retailers and know the waiting time form the retailers. If they can be tolerant to it they will submit the orders, otherwise they will abandon them and/or turn to other company's products. If the products are abandoned because of lead time they will be treated as lead-time-dependent lost sales. Thus, we assume the demand follows a nonincreasing function of the lead time. We optimize the tradeoff of lead time with other factors in the design of automobile BTO distribution network by introducing waiting-time-dependent demand into the model.

The paper is organized as follows. In section 2, the automobile distribution network is introduced in detail and the problem is formulated by Linear Integer Programming model. The Lagrangian heuristic algorithm is developed to solve the problem in section 3 and the computational results are discussed in section 4. We conclude the paper in section 5.

2 Model Formulation

In the BTO supply chain each plant produces a different product and can satisfy all the customer demand. The products are first delivered to the DCs from the plants and then delivered to the retailers, and they can be also sent to the retailer directly from the plant if they are close enough. The products form the same plant can either be delivered form plants or one DC, but not from both or from more than one DC.

2.1 Parameters and variables

The following notations are used in our subsequent models: Indices:

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$i = 1,2,\ldots,I$:index of plants

$j = 1,2,\ldots,J$:index of potential distribution centers

$k = 1,2,\ldots,K$:index of potential distribution centers

Parameters:

r_{ik}=unit sale revenue of product from plant i at retailer k

di_{ik}= the ideal demand of retailer k from plant i when there is no lead time

de_{ik}= the expect demand of retailer k from plant i if delivered directly

de_{ijk}= the expect demand of retailer k from plant i if delivered through DC j

tp_i= the dwell time at plant i

td_j= the dwell time at DC j

tpd_{ij}= the transportation time from plant i to DC j

tdr_{jk} the transportation time from DC j to retailer k

tpr_{ik}= the transportation time from plant i to retailer k

cpd_{ij}= the cost of transporting per product from plant i to DC j

cdr_{jk}= the cost of transporting per product from DC j to retailer k

cpr_{ik}= the cost of transporting per product directly from plant i to retailer k

f_j= the setup cost of DC j

The dwell time at plant, tp_i, is the overall time spent on the plant i from receiving the order until sending out the products, which includes the order processing time, the assembly time, the time spent on administrative procedures, the upload time and some other times. The dwell time at DC, td_j, is the overall time spent on the DC j from receiving the products until sending out them, which includes the unloading time, the sorting time, the upload time and the time spent on administrative procedures.

We define the following decision variables:

U_{ijk} 1 if the demands of retailer k are delivered from plant i through DC j, otherwise 0;

V_{ik} 1 if the demands of retailer k are delivered from plant i directly, otherwise 0;

W_j 1 if DC j is setup, otherwise 0.

2.2 Lead-time-dependent demand

Here we make the assumption that the demand are dependent on the lead time following a nonlinear decreasing function is mainly based on the following practice. Elias [11] investigates the behavior of new vehicle buyer and finds that 65 buyers think the waiting time

from order to delivery is very important for their final car choice. And the investigation results for the customers' expecting length of waiting time from order to delivery as follows (see Fig 1).

Thus, it is necessary and reasonable to consider the lead-time-dependent demand in the supply chain design. We assume that the excepted demand follows a negative exponential function of the lead time, that is,

$$de = di * g(t) \tag{1}$$

where t is the order-to-delivery lead time and $g(t) = exp(-\lambda t)$

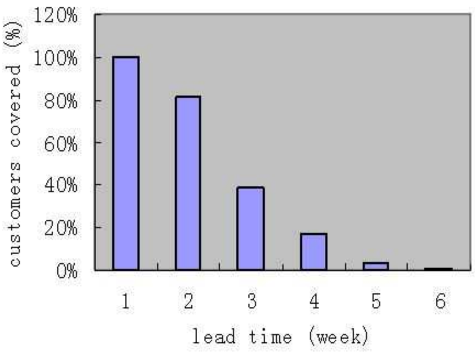

Fig. 1: Demand varying with lead time

Note that,

$t_{ijk} = tp_i + tpd_{ij} + td_j + tdr_{jk}$ =the transportation time from plant i to retailer k through DC j.

$t_{ik} = tp_i + tpr_{jk}$ =the transportation time from plant i to retailer k directly.

2.3 Formulation of optimization model

In fact our distribution network design problem is a production-distribution integrating optimization problem which considers the tradeoff between lead time and distribution cost to maximize the total profit, and it can also be viewed as a special case of two-level facility location problem.

Max

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(r_{ik} - cpd_{ij} - cdr_{jk})di_{ik}g(t_{ijk})U_{ijk} - \sum_{j=1}^{J} f_j W_j + \sum_{i=1}^{I}\sum_{k=1}^{K}(r_{ik} - cpr_{ik})di_{ik}g(t_{ik})V_{ik} \tag{2}$$

s.t.

$$\sum_{j=1}^{J} U_{ijk} + V_{ik} = 1 \forall i,k \tag{3}$$

$$U_{ijk} \le W_j \forall i,j,k \tag{4}$$

$$U_{ijk}, V_{ik}, W_j \in \{0,1\} \forall i,j,k \tag{5}$$

The objective function is to maximize the overall profit. Constraint (3) ensures that the demand of retailer k from plant i is satisfied by shipment through one DC or by shipment directly from the plant, but not by both. Constraint (4) ensures that the products can only be delivered through the DC which has been opened. Constraint (5) is the integer restriction.

3 Solution Approach

In this section, we develop a Lagrangian heuristic algorithm to solve the distribution network design problems. The Lagrangian relaxation approach has been widely used for solving NP-hard Mixed Integer Programming problems, the analysis for the convergence efficiency can be found in [12]. In order to solve our problem, we design some heuristics to combine with the traditional Lagrangian approach. First, the Lagrangian relaxation problem is obtained by relaxing the constraints set (3) into the objective function and is decomposed into independent subproblems which are easy to solve. Then the upper bound of the problem is achieved by the subgradient algorithm. At last a heuristic algorithm is used to find a good feasible solution from the relaxed upper bound.

3.1 Lagrangian relaxation

To introduce conveniently, we note that

$a_{ijk} = (r_{ik} - cpd_{ij} - cdr_{jk} + cl_{ik})di_{ik}g(t_{ijk})$

$b_{ik} = (r_{ik} - cpr_{ik} + cl_{ik})di_{ik}g(t_{ik})$

Relax the constraint set (3), then the relaxed problem can be described as follows:

Max

$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} a_{ijk}U_{ijk} - \sum_{j=1}^{J} f_j W_j + \sum_{i=1}^{I}\sum_{k=1}^{K} b_{ik}V_{ik} + \sum_{i=1}^{I}\sum_{k=1}^{K} \mu_{ik}(1 - \sum_{j=1}^{J} U_{ijk} - V_{ik})$

Subject to constraints (4) and (5).

Then the relaxed problem is decomposed into independent subproblems which can easily solved by observing.

(1) Plant i subproblem

Max $\sum_{k=1}^{K}(b_{ik} - \mu_{ik})V_{ik}$

subject to $V_{ik} \in \{0,1\} \forall i,k$.

The optimal solution to this subproblem can be easily obtained directly by observing: if $(b_{ik} - \mu_{ik}) > 0$, $Y_{ik} = 1$; otherwise, set Y_{ik} to zero.

(2)DC j subproblem

Max $\sum_{i=1}^{I}\sum_{k=1}^{K}(a_{ijk} - \mu_{ik})U_{ijk} - f_j W_j + \sum_{i=1}^{I}\sum_{k=1}^{K} \mu_{ik}$

subject to (4) and $U_{ijk}, W_j \in \{0,1\} \forall i,j,k$.

The solution algorithm of DC j subproblem is present as follows:

Step0 (Initialization): $W_j = 0; U_{ijk} = 0$

Step1 Iteration

For $i = 1$ to I

For $k = 1$ to K

if $a_{ijk} - \mu_{ik} \ge 0$

then $\mu_{ijk} = 1$

Step2 Judge

If $\sum_{i=1}^{I}\sum_{k=1}^{K}(a_{ijk} - \mu_{ik})U_{ijk} - f_j \ge 0$

then $W_j = 1$

else set all $U_{ijk} = 0$.

3.2 Obtaining an upper bound by subgradient algorithm

Here we use subgradient algorithm to obtain an upper bound for the problem. The main steps of the algorithm are as follows:

Step0 Initialization

Initialize $U_{ijk} = 0, W_j = 0$ and $V_{ik} = 1 (\forall i, j, k)$ to get an initial feasible solution and a lower bound LB,

let LR be the objective value of the Lagrangian dual problem, and initialize $LR = +\infty$

set $\mu_{ik} = b_{ik} + 1$, and $\alpha = 2$

Step1 Solve subproblems

Given μ_{ik}, solve plant i subproblems and DCj subproblems and get a new bound LB

Step2 Update lower bound

If $LB < LR$, update $LB = LR$

Step3 Accelerate convergence

If there is no improvement on LR after N iterations,

set $\alpha = \alpha 2$ and $LB\_update = LB + (LR - LB)n$

If $LB\_update < LR$

$LB = LB\_update$

where N and n are defined by user for different scale test data, and $N = 10, n = 30$ in our case.

Step4 Calculate new step size

Let$norm = \sum_i \sum_k (1 - \sum_j U_{ijk} - V_{ik})^2$

If $norm > 0$

$stepsize = \alpha(LR - LB)norm$

else $stepsize = stepsize2$

Step5 Update multipliers

$\mu_{ik} = max\{0, \mu_{ik} - stepsize(1 - \sum_j U_{ijk} - V_{ik})\}$

Step6 Update multipliers

If iteration times $> Ni$ or the max gap of μ_{ik} between two coterminous iteration, STOP,

else GOTO step 1

where Ni and the gap is defined by user according requirement, in our problem $Ni = 200$ and the gap is set to 0.01.

3.3 Obtaining an lower bound by greedy heuristics

Here we use a heuristics algorithm to obtain a feasible solution to the problem from the bound given by the subgradient algorithm. The main processes of the feasible solution algorithm are present below:

Step0 Find an infeasible constraint and initialize

Find an infeasible pair (i, k) which $\sum_{j=1}^{J} U_{ijk} + V_{ik} \neq 1$,

And initialize $U_{ijk} = 0 (\forall j), V_{ik} = 0$

Step1 Find a location to assign

Find a best DCj among current open DCs which $B_j = max\{a_{ijk} || W_j = 1$

If $B_j > b_{ik}$, assign retailer k to DCJ and $U_{iJk} = 1$

Else assign retailer k to the plant i and $V_{ik} = 1$

Step2 Stop criteria

If there are no infeasible pairs, STOP

Else GOTO step 0.

Table 1: Computational performances of the solution approach for the examples

Problem size	Gap(upperbound-lowerbound)/lowerbound			CPU time(second)		
	Max	*Min*	*Average*	*Max*	*Min*	*Average*
Small	0.08%	0.01%	0.03%	7.852	4.045	5.342
Middle	0.11%	0.06%	0.08%	52.635	33.418	40.579
Small	0.28%	0.03%	0.10%	241.109	181.563	211.814

4 Computational Performance

The solution algorithm is tested by solving randomly generated numerical examples with different sizes. The computational experiment is conducted on the IBM T420 laptop with Windows XP (Intel $Core^{TM}$2 Duo CPU, 2GB of RAM). Fifty small examples (5 SNs, 20 DNs and 50 CNs), fifty middle examples (10 SNs, 40 DNs and 100 CNs), and fifty large examples (20 pants, 100 candidate DCs and 500 retailers) is used to test the solution algorithm. All these examples are randomly generated, and the value ranges for generating the parameters are set according to our investigation from automobile companies in China.

The computational performance of the Lagrangian heuristic approach for the problem is summarized in Fig.2 The gap presents the percentage error between the feasible lower bound obtained from the greedy heuristic and the upper bound obtained from the subgradient algorithm. From fig.2 we can see the Lagrangian heuristic works very well for our problem and can present very good solution in short time for problems with different sizes.

5 Conclusion

In this paper, we studied the automobile build-to-order distribution network design problem by revisiting its demand pattern, and the tradeoff between lead time and logistics cost is considered in the network modeling. The problem is formulated as Linear Integer Programming models and an efficient Lagrangian heuristic algorithm is developed to solve them. Randomly generated examples with different sizes from small to large are used to test the model and the solution algorithm. Computational results show that the proposed approach can obtain good solutions for all examples in short CPU time. The lead time is viewed as constant and known, while in the practice it is usual stochastic. Thus, the stochastic lead time will be studied in the future research.

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