Applied Mathematics & Information Sciences
An International Journal

A New Method of Information Aggregation for Multi-type Evaluation Subjects-Involving Teachers' Performance Evaluation Based on TDW Operator

Faming Zhang

School of Economics &Management, Nanchang University, Nanchang 330031,China
Email: zfm1214@163.com

Received: 23 Mar. 2012; Revised 17 Jul. 2012; Accepted 22 Aug. 2012

Abstract: Traditional university teachers' performance evaluation rarely considers quantitative evaluation, and usually involves only one subject. In order to fill the inadequacies, this paper put forwards a new method to aggregate the evaluation information of university teachers. The new method involves many different parts of the evaluation. First, a new information aggregation method——TDW operator method is presented; then TOPSIS is employed to handle evaluation results of different types of evaluation subjects in order to avoid the difficulty of information aggregation preference of different types; third, method to determine density weighted vector according to the relevant degree of different information is introduced. Finally, an example is given to illustrate the effectiveness.

Keywords: Multiple types of evaluation objects; Performance evaluation of high education teacher; Evaluation; Aggregation; TDW operator.

1 Introduction

The core competitiveness of a university comes from the overall quality of its teachers and it's very important to evaluate the performance of university teachers [1]. University teachers' performance evaluation is to evaluate the work of teachers in a certain period of time. Reasonable and scientific evaluation is very important to encourage teachers to perform their duties earnestly, and improve the quality of teaching and scientific research. It has the function of guiding, appraising, improving, encouraging and managing [2].

Educational management scientist Ernest Boyer (1990) and R.Eugene Rice (2002) published separately far-reaching reports on performance evaluation of American teachers. The two reports set off "Boyer Reform" which reconsidered the academic activities of university teachers and adjusted the reward system of teachers [3, 4]. As a continuation, Carnegie Foundation proposed six standards to evaluate teachers' performance in another report. They are definite goal, adequate preparation, proper method, remarkable result, effective popularization and reflective self-assessment [5]. The report also discussed the relationship between teaching and researching in university teachers' performance evaluation. It concluded that university teachers did not pay due attention to undergraduate teaching; on the contrary, they focused on abstruse research and thus get astrayed from the centric mission of modern university. Thus it proposed to reform the perfor mance evaluation system so as to motivate teachers to teach and get involved in academic activities more actively. Up to now, hundreds of American universities adopt the new difinition of academic activity proposed by Boyer and Rice in their evaluation of teachers' performance. The reward system becomes more balanced, teachers become more satisfied with their work, and they get more involved in various university activities [6].

Linda Darling, Hammond and other scholars proposed four basic goals for teachers' performance evaluation in the 1980s, including teacher's profes sional development, personnel decision, school development and judgement of school status [7].Wright (1984) did research on the relationship between self-assessment of teachers and students' evaluation of teachers [8]. Eiszter (2002) focused on the effectiveness and reliability of students' evaluation of teachers [9].

Reviewing the research in China and abroad and considering the practice of performance evaluation of university teachers in China, we sill find follo wing deficiencies. First, evaluation subject is single. Traditionally, teachers' performance is evaluated by school authorities—leaders of different departm ents. Second, qualitative evaluation is preferred while quantitative evaluation is ignored. Third, quantitative evaluation method, if adopted, is single. To make up these deficiencies, it is necessary to involve multiple types of evaluation subjects in teachers' performance evaluation. Multiple types of evaluation subjects here include teaching objects (students), colleagues, and members of teaching management department, members of teaching supervision department, leaders and the evaluation objects themselves.

A major problem in multi-type evaluation subjects-involved teachers' performance evaluation is how to aggregate effectively the evaluation infor mation of different types of evaluation subjects. That is the main task of this paper—to solve the performance evaluation information aggregation problem of different types of evaluation subjects with nontraditional TDW operator (two-dimension al density weighted operator) aggregation method.

2 A new information aggregation method based on TDW operator

To solve multi-type evaluation subjects-involved teachers' performance evaluation problem, let $\boldsymbol{O}=\{o_1,o_2,\cdots,o_n\}$ be the evaluation objects (evaluated teachers) set, $\boldsymbol{S}=\{s_1,s_2,\cdots,s_5\}$ be the evaluation subjects set which include five types, namely students, colleagues, members of teaching management department, members of teaching supervision department and leaders. In each type, there are certain numbers of evaluation subjects. Each evaluation subject has different perspective viewing the same evaluation object. Thus let $a_{ij}(j=1,2,\cdots,m)$ be the evaluation subject $s_j(j= 1,2,\cdots,m)$'s evaluation value of the evaluation object $o_i(i=1,2,\cdots,n)$. The evaluation value is expressed in scores. Then the corresponding evaluation value vector would be $\boldsymbol{a}_j(j=1,2,\cdots,m)$ and set the evaluation value matrix as A (without loss of generality, let $n\geq 3$, and $m\geq 3$). Obviously, A is a two-dimensional data matrix.

$$\boldsymbol{A}=[a_{ij}]_{n\times m}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

2.1 TDW operator

Definition 1 In the two-dimensional data set $A=(\boldsymbol{a}_1,\boldsymbol{a}_2,\cdots,\boldsymbol{a}_m)$, set $TDWA$: $\mathbf{R}^n\rightarrow\mathbf{R}$, if

$$TDWA_{\xi}(\boldsymbol{a}_1,\boldsymbol{a}_2,\cdots,\boldsymbol{a}_m)=\boldsymbol{y}=\sum_{r=1}^{q}\xi_r\cdot\boldsymbol{y}(A_r) \qquad (1)$$

Then function $TDWA$ is called two-dimensional density weighted arithmetical average operator, or $TDWA$ operator. In formula (1), $\boldsymbol{y}=(y_1,y_2,\cdots,y_n)^{\mathrm{T}}$; $\boldsymbol{y}(A_r)=\boldsymbol{y}_r=(y_{1r},y_{2r},\cdots,y_{nr})^{\mathrm{T}}$, A_r denotes the evalua-tion information of the $r(r=1,2,\cdots,q;q=5)^{\text{th}}$ type of evaluation subjects, such as the evaluation information of students, the evaluation information of colleagues, etc., and is a two-dimensional data set. $\boldsymbol{y}_r$ denotes the comprehensive evaluation value of the r[th] type of evaluation subjects. $A_r=\{\boldsymbol{a}_j^{(r)}|r=1,2\cdots,5;j=1,2,\cdots,n_r\}$, $\sum_{r=1}^{5}n_r=m$, $\boldsymbol{a}_j^{(r)}$ is an element in A_r and is a one-dimensional column vector; $\xi=(\xi_1,\xi_2,\cdots,\xi_5)$ is a density weighted vector, and $\xi_r\in[0,1],r\in Q,\sum\xi_r=1$.

Definition 2 In two-dimensional data set $A=(\boldsymbol{a}_1,\boldsymbol{a}_2,\cdots,\boldsymbol{a}_m)$, set $TDWGA$: $\mathbf{R}^n\rightarrow\mathbf{R}$, if

$$TDWGA_{\xi}(\boldsymbol{a}_1,\boldsymbol{a}_2,\cdots,\boldsymbol{a}_m)=\boldsymbol{y}=\sum_{r=1}^{q}\boldsymbol{y}(A_r)^{\xi_r} \qquad (2)$$

Then we call function $TDWGA$ two-dimensio nal density weighted geometrical average operator, or $TDWGA$ operator. In formula (2), both $\boldsymbol{y}$ and $\boldsymbol{y}(A_r)$ are one-dimensional vectors, and $\boldsymbol{y}=(y_1,y_2,\cdots,y_n)^{\mathrm{T}}$, $\boldsymbol{y}(A_r)=\boldsymbol{y}_r=(y_{1r},y_{2r},\cdots,y_{nr})^{\mathrm{T}}$.Other elements in the formula have the same denotation as in definition 1.

$TDWA$ operator and $TDWGA$ operator are respectively arithmetical and geometrical, and they are collectively named as two-dimensional density weighted (*TDW*) operator[10].

2.2 Information aggregation of different types of evaluation subjects based on TOPSIS

In order to use *TDW* to aggregate the information of different types of evaluation subjects and obtain the final comprehensive evaluation result y, first the comprehensive evaluation result of each type of evaluation subjects $\boldsymbol{y}_r$ should be obtained. $\boldsymbol{y}_r$ is essentially an aggregated group evaluation value. As group preference aggregation is a difficult issue, to avoid the aggregation problem of group preference, this paper uses TOPSIS [11] to deal with the evaluation results of different types of evaluation subjects. TOPSIS calculates the difference between the target value and the ideal value of the evaluation object, and orders the differences to make final decisions. The positive ideal value and negative ideal value of the TOPSIS are virtual evaluation scores of evaluation objects, and they work as a reference point for evaluating and making decision. By selecting in the normalized matrix A the largest and smallest element of each column in corresponding type of evaluation subjects, the positive ideal value vector $u_j^{r+}(j=1,2,\cdots,n_r)$ and negative ideal value vector $u_j^{r-}(j=1,2,\cdots,n_r)$ can be obtained. Considering individual influence, Euclid Norm method is employed as the measurement of difference to obtain the difference parameter $d_i^{r+}, d_i^{r-}(i=1,2,\cdots,n_r)$ between the evaluation value of the evaluation object $o_i(i=1,2, \cdots,n)$ and the positive ideal value vector $u_j^{r+}(j=1,2,\cdots,n_r)$ and negative ideal value vector $u_j^{r-}(j=1,2,\cdots,n_r)$.

$$d_i^{r+}=(\sum_{j=1}^{n_r}\varepsilon_j^r(a_{ij}^r-u_j^{r-})^2)^{1/2} \ ; \qquad d_i^{r-}=(\sum_{j=1}^{n_r}\varepsilon_j^t(a_{ij}^t-u_j^{r-})^2)^{1/2} \tag{3}$$

In formula (3), $\varepsilon_j^r(r=1,2,\cdots,5; j=1,2,\cdots,n_r)$ denotes the influence coefficient of evaluation subject s_j in the r[th] type of evaluation subjects. When there isn't much difference in the influence, set $\varepsilon_1^r=\varepsilon_1^r=\cdots,\varepsilon_{n_r}^r=n_r/m$.

After obtaining the value of u_j^{r+} and u_j^{r-}, define the relative adjacency coefficient between the evaluation value and ideal value as

$$y_{ir}=d_i^{r-}/(d_i^{r+}+d_i^{r-}) \quad 0\le c_i^r\le 1 \tag{4}$$

In formula (4), y_{ir} is the comprehensive evaluation value of evaluation object o_i ($i=1,2,\cdots,n$) in the r[th] type of evaluation subjects. In an ideal evaluation scheme, $y_{ir}=1$; in a negative ideal evaluation scheme, $y_{ir}=0$; in general situation, $0<y_{ir}<1$, and the closer y_{ir} is to 1, the better the evaluation object meets the standards of evaluation subjects.

Based on the above calculating and selecting, the comprehensive evaluation value $\boldsymbol{y}_r=(y_{1r},y_{2r},\cdots,y_{nr})^{\mathrm{T}}$ of the r[th] evaluation subjects can be obtained.

2.3 Determination of density weighted vector based on similarity of evaluation subjects

Another key factor in using *TDW* operator is to determine the density weighted vector ξ_r . The function of density weighted vector is to aggregate the comprehensive evaluation value $\boldsymbol{y}_r$ of different types of evaluation subjects while considering the information density. However, different from traditional density operator which determines the value of ξ_r based on the number of evaluation subjects in each type, in university teachers' performance evaluation, the type of evaluation subjects is determined before information aggregation, thus there is no need to cluster individual evaluation subject and the value of ξ_r can not be determined by the number of evaluation subjects in each type. As density weighted vector ξ_r is related to information density, in this paper information correlation intensity is used to determine information density of different types of evaluation subjects.

In one type, if evaluation subject s_i and $s_j(i,j=1,2,\cdots,m;i\neq j)$ have the same evaluation results (rank) towards evaluation object $\boldsymbol{O}$, then the two subjects have the largest evaluation information correlation degree; on the contrary, if evaluation subject s_i and s_j have completely different evaluation results (rank) towards evaluation object $\boldsymbol{O}$, then the two have the smallest evaluation information correlation degree. Spearman rank correlation coefficient is used as the measurement of correlation degree of individual evaluation result. The calculation formula is as follows:

$$\eta_{ij}=1-\frac{6\sum_{k=1}^{n}d_k^2}{n(n^2-1)} \tag{5}$$

In formula (5), η_{ij} denotes the information correlation coefficient between s_i and s_j, $d_k(k=1,2,\cdots,n)$ denotes the rank difference of s_i and s_j towards the i[th] evaluation object. The larger η_{ij} is, the more correlated the two subjects are, vice versa. Meanwhile, the overall similarity of one type can be calculated based on the information correlativity of different evaluation subjects.

Definition 3 The average correlation degree of different evaluation subjects reflects the overall similarity of one type. Set

$$\lambda_r=\frac{1}{(n_r-1)n_r}\sum_{i=1}^{n_r}\sum_{j=1}^{n_r}\eta_{ij} \tag{6}$$

Then λ_r is called the similarity coefficient of r[th] $(r=1,2,\cdots,5)$ type of evaluation subjects. λ_r reflects the evaluation information correlation degree in one type. The larger λ_r is, the higher the evaluation information correlation degree in one type. As information correlation degree reflects the information density, then the value density weighted vector $\boldsymbol{\xi}$ can be determined by similarity coefficient vector $\boldsymbol{\lambda}$, namely

$$\xi_r=(1+\lambda_r)\Big/\sum_{r=1}^{5}(1+\lambda_r) \tag{7}$$

In formula (7), ξ_r denotes the density weighted value of the r[th] $(r=1,2,\cdots,5)$ type of evaluation subjects, and the corresponding density weighted vector is $\boldsymbol{\xi}=(\xi_1,\xi_2,\ \cdots,\xi_5)^{\mathrm{T}}$.

In using *TDW* to aggregate the evaluation information of different types of evaluation subjects, we can revise the density weighted vector based on the actual need to reflect the preference degree of information density. That means in aggregating, if information with high density is emphasized, then set density weighted vector as $\boldsymbol{\xi}=(\xi_1,\xi_2,\cdots,\xi_5)^{\mathrm{T}}$; on the contrary, if information with low density is emphasized, then set density weighted vector as $\boldsymbol{\xi}'=1-(\xi_1,\xi_2,\cdots,\xi_5)^{\mathrm{T}}$.

2.4 Steps of aggregation

The steps to aggregate the evaluation inform ation for multi-type of evaluation subjects-involved university teachers' performance evaluation are as follows:

Step 1: Determine the positive ideal point vector u_j^{r+} and negative ideal point vector u_j^{r-} of the r[th] type A_r, then use formula (3), and (4) to calculate the relative adjacency coefficient $y_{ir}(i=1,2,\cdots,n;r=1,2,\cdots,5)$ and comprehensive evaluation value vector of a type $\boldsymbol{y}_r=(y_{1r},y_{2r},\ \cdots,y_{nr})^{\mathrm{T}}$;

Step 2: Use formula (5) to calculate the information correlation coefficient $\eta_{ij}(i,j=1,2,\cdots,n_r;i\neq j)$ among different subjects in type s_r;

Step 3: Use formula (6) to calculate the similarity coefficient $\lambda_r(r=1,2,\cdots,5)$ of each type. Based on the similarity coefficient vector $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,\cdots,\lambda_5)^{\mathrm{T}}$, and use formula (7), calculate density weighted vector $\boldsymbol{\xi}=(\xi_1,\xi_2,\cdots,\xi_5)^{\mathrm{T}}$;

Step 4: According to aggregation preference (prefer high density information or low density information), take comprehensive evaluation value vector of a type $\boldsymbol{y}_r$ and density weighted vector $\boldsymbol{\xi}$ (or $\boldsymbol{\xi}'$) into formula (1) or formula (2), then use *TDW* to calculate the final comprehensive evaluation value of an evaluation object y_i $(i=1,2,\cdots,n)$;

Step 5: Order the evaluation objects o_i $(i=1,$

$2,\cdots,n)$ according to the value of y_i $(i=1,2,\cdots,n)$, and make related decisions.

3 Example

Suppose in a university, there is an evaluation group of 15(m=15)subjects. They are divided into five types (students, colleagues, members of teaching management department, members of teaching supervision department and leaders). The number of members in each type (A_1 to A_5) is respectively 5, 4, 2, 2, and 2. They evaluate the performance of 5 (n=5) teachers (evaluation objects). Suppose the 15 subjects have different views towards the 5 teachers, their evaluation information is shown in table 1 (expressed in scores, and full score is 10 points). The following part will focus on the information aggregation based on the method proposed above. Due to space limit, the concrete calculate process will be omitted.

Table 1 Each Subject's Evaluation Value toward Each Object

Teacher	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
o_1	3.8	1.0	6.0	9.0	8.8	9.6	0.1	4.1
o_2	0.2	2.9	3.4	5.5	3.6	3.7	3.6	9.1
o_3	9.5	0.5	7.1	8.2	9.7	4.7	3.0	7.5
o_4	5.1	3.7	9.9	0.4	2.3	0.0	9.3	1.0
o_5	7.6	6.3	1.7	4.0	5.5	7.1	5.6	1.8

Continue table 1 :

Teacher	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
o_1	8.6	1.4	2.5	0.5	0.3	3.8	1.0
o_2	4.7	4.3	3.0	9.8	8.1	0.2	2.9
o_3	3.5	7.8	0.7	2.0	0.6	9.5	0.5
o_4	2.6	7.8	6.8	8.1	7.2	5.1	3.7
o_5	9.7	6.9	5.3	8.0	8.1	7.6	6.3

The aggregation process is as follows:

1) Divide the evaluation information into five types, namely $A_1=\{\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{a}_3,\boldsymbol{a}_4,\boldsymbol{a}_5\}$, $A_2=\{\boldsymbol{a}_6,\boldsymbol{a}_7,\boldsymbol{a}_8,\boldsymbol{a}_9\}$, $A_3=\{\boldsymbol{a}_{10},\boldsymbol{a}_{11}\}$, $A_4=\{\boldsymbol{a}_{12},\boldsymbol{a}_{13}\}$, $A_5=\{\boldsymbol{a}_{14},\boldsymbol{a}_{15}\}$;

2) Use formula (3) and (4) to calculate the comprehensive evaluation value vector of each type. The results are $\boldsymbol{y}_1=(0.57,0.50,0.48,0.44,0.49)^{\mathrm{T}}$, $\boldsymbol{y}_2=(0.53,0.48,0.43,0.36,0.54)^{\mathrm{T}}$, $\boldsymbol{y}_3=(0.18,0.31,0.40,1.00,0.72)^{\mathrm{T}}$, $\boldsymbol{y}_4=(0.00,1.00,0.08,0.78,0.79)^{\mathrm{T}}$, $\boldsymbol{y}_5=(0.12,0.71,0.31,0.00,0.13)^{\mathrm{T}}$.

3) Use formula (5) and (6) to calculate the similarity coefficient of each type. The results are $\lambda_1=-0.10,\lambda_2=-0.10,\lambda_3=-0.40,\lambda_4=-0.90,\lambda_5=-0.80$.

4) Use formula (5) to calculate density weighted vector ξ . The result is $\xi=(0.13,0.13,0.20,0.28,0.27)^{\mathrm{T}}$.

5) Information aggregation. Use *TDWA* and formula (1) to aggregate information and the result is $\boldsymbol{y}=(0.211,0.652,0.303,0.522,0.532)^{\mathrm{T}}$. Thus the rank is $o_2\succ o_5\succ o_4\succ o_3\succ o_1$.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (No.71001048), Education Science " The twelfth five-year plan" planning project, Jiangxi province, China (No.11ZD040), and 2012 science and technology planning project from department of education of Jiangxi province, China (No.GJJ12129).

References

[1] Han, M.G.., Probe into the Fuzzy Comprehensive Appraisal of Teachers′ Performance Research-oriented Universities, Mathematics in practice and theory,2005, 32(15) :62-67.

[2] Liu, R.Y., Chen, S.J., Indicators and weights setting for the scientific and technical performance evaluation of teachers in colleges and universities, Science research management, 2007, 28(3):39-44.

[3] Boyer, E. L. 1990. Scholarship Reconsidered: Priorities of the Professorate. *Prineeton, NJ: Carnegie Foundation for the Advancement of Teaching.*

[4] Riee, R.E.1986.The Academic Profession in Transition: Toward a New Social Fietion, *Teaching Soeiology*, 14:12-23.

[5] Glassiek, C. E., Huber, M. T., and Maeroff, G.1. 1997. Scholarship Assessed: Evaluation of the Professoriate, *San Francisco: Jossey Bass.*

[6] O, Meara, K.2001.Scholarship Unbound: Assessing Service as Scholarship for Promotion and Tenure. *Studies in Higher Education Dissertation Series, ed. P. Altbaeh.* New York, Routledge Falmer.

[7] Dong, H. P., Thinking on Deep-seated Problems and Solutions in Higher Education Performance Evaluation, *2011 International Conference on Management and Service Science (2011)*, 2011, pp. 119-125.

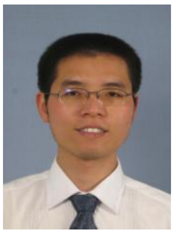
[8] Wright, P., R., Whittington &G.E. Whittenbury (1984). Student ratings of teaching effectiveness: what the

research reveals, *Journal of Accounting Education* (2):5-30.

[9] Eiszier, C.F. (2002). College students' evaluations of teaching and grade inflation. Research in Higher Education, 2002 (43): 483-501.

[10] Zhang, F.M. ,Y.J., Yi, P.T., Method of Group Evaluation Information Aggregation Based on Two-dimension Guoal Density Operator, *Journal of Systems & Management*, 2009, 18 (4): 397-401.

[11] Chen, J.Z., Xu, J.P., TOPSIS based interactive multi-attributes group decision-making method and its application, *Journal of Systems Engineering*, 2008, 23(3):276-281.



Zhang fa-ming was born in 1980 and has been carrying research on comprehensive evaluation, decision making, inofrmation management and service management, etc. since the beginning of his academic life. He is now working in Nanchang, China as a lecturer in School of Economics and Management of Nanchang University. He got his doctor degreee in 2010 in Dongbei Univesity. He has done a good job in his research area and up to now has been granted one projects National Natural Science Foundation of China. Besides, he has published more than 20 articles related to his research directions in prestigous journals in China and abroad, such as Advances in Information Sciences and Service Sciences, Applied Mechanics and Materials, Journal of Systems Engineering, Systems Engineering-Theory and Practice, Chinese Journal of Management Science, Systems Engineering and Electronics. He has been awarded a series of honors because of his outstanding research work from 2010 to the present. His email address is zfm1214@163.com.