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Small-Sample Accelerated Life Test Method based on the Inverse Power Law Model

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Abstract: To obtain the characteristic life of a chain tool magazine and automatic tool changer under normal working condition, a small-sample accelerated life test method was developed. The inverse power law model and small-sample virtual augmented theory was applied while taking vibration as the accelerated stress. The accelerated life test of the automatic tool changer was carried out under different stress levels. The results showed that the "Mean Cycles between Failures" of exchanging tool under normal stress was about 28283 times for the chain tool magazine and automatic tool changer. The proposed small-sample accelerated life test method, which based on the inverse power law model, could also be used in other relevant large-scale and complex products.

Keywords: Tool Magazine and Automatic Tool Changer, Accelerated Life Test, Small Sample, Inverse Power Law Model

1 Introduction

The investment of a tool magazine and automatic tool changer comprises about 30%-50% of the entire machine, and the failure rate ranks first according to a recent application report. Therefore, it is important to predict its characteristic life under normal working condition.

The accelerated life test is one of the most powerful tools which have been widely used to evaluate life characteristics within shorter time and less cost. Some available life-stress models, such as numerical simulation method [1], Arrhenius, Eyring, and inverse power law model, are most commonly used for the accelerated life test: Nelson [2] and Bai [3] studied the accelerated life test with the inverse power law model about 30 years ago; Shin [4] developed the accelerated life test method using the inverse power law model for small DC motors; Rajkumar [5] used the life-stress relationship based on the inverse power law-Weibull model to predict reliability under the normal usage level; Nogueira [6] proposed an accelerated life model that involves temperature, humidity and current parameters to evaluate the reliability of LED; Zhang [7] studied the constant-step-stress accelerated life test under Weibull distribution case; Lee [8] discussed the parameter estimation method of Step-Stress Accelerated Life Testing model by utilizing Generalized Linear Model techniques; while Tang [9] focused on the general k-step step-stress accelerated life tests for two-parameter Weibull distributions; Besides, Allegri [10] provided an acceptable estimation of the relative fatigue damage accumulation based on the inverse power law model.

Most of the works above mainly aimed at those products that could easily reach large-sample size due to their simple structure and low cost, such as capacitor, insulating fluid, LED, bearing, etc. However, these works were very limited-used in those products which could only have small-sample size. Wen [11] took the in-pipe robot as an example and did a small-sample accelerated life experimental analysis. As one of typical large-scale and complex products, the tool magazine and automatic tool changer requires a very short development cycle and a long life-expectancy. However, Complex structure and exorbitant cost make it impossible to carry out a large-sample size test, even there is only single or a few samples in some circumstances. Therefore, it is necessary to find a suitable small-sample accelerated life test method for predicting the characteristic life of such large-scale and complex products. Moreover, for some products, measuring life by cycles is more appropriate than time [12]. Considering the major task of the tool magazine and automatic tool changer is exchanging tool automatically, the exchanging tool cycles would be taken as the counting unit in this study.

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This paper is organized as follows. The Weibull model based on probability weighted moments (PWM) method was first introduced, and an accelerated life test method with the inverse power law model and small-sample virtual augmented theory was then discussed. Finally, the proposed method was applied to a certain type of chain tool magazine and automatic tool changer, and the parameter of "Mean Cycles between Failures" under the normal working condition was predicted.

2 Weibull model based on probability weighted moments method

As a wide applicability distribution model, the Weibull distribution can well describe each stage of the bathtub curve, and therefore often be used to indicate the failure distribution of mechanical products. This paper would study the characteristic life of tool magazine and automatic tool changer based on the Weibull distribution.

The failure density function under the Weibull distribution is

$$f(\tau)=\frac{m}{\eta}\left(\frac{\tau-\gamma}{\eta}\right)^{m-1}e^{-\left(\frac{\tau-\gamma}{\eta}\right)^{m}} \tag{1}$$

And the failure distribution function is

$$F(\tau)=1-e^{-\left(\frac{\tau-\gamma}{\eta}\right)^{m}}. \tag{2}$$

Where m is the shape parameter, η is the scale parameter, and γ is the location parameter. When $m>1$, the failure rate increases with time, which indicates that the product has been in the accidental failure stage; and when $m<1$, the failure rate decreases with time, which indicates that the product is in the early failure stage.

The PWM method is a widely used parameter estimation method. After transforming a test sample set $(X_1,X_2,\cdots,X_n)$ into the sample order statistics $(x_1,x_2,\cdots,x_n)$, the estimated value of PWM is [13]

$$M_{1,0,k}=\frac{1}{n}\sum_{i=1}^{n}x_i\left(1-\frac{i-0.35}{n}\right)^{k},k=0,1,3 \tag{3}$$

Therefore, these three parameters of the Weibull distribution expressed by k-order PWM are respectively

$$\gamma=\frac{4(M_{1,0,3}M_{1,0,0}-M_{1,0,1}^{2})}{4M_{1,0,3}+M_{1,0,0}-4M_{1,0,1}} \tag{4}$$

$$m=\frac{\ln 2}{\ln\frac{M_{1,0,0}-2M_{1,0,1}}{2(M_{1,0,1}-2M_{1,0,3})}} \tag{5}$$

$$\eta=\frac{M_{1,0,0}-\gamma}{\Gamma\left(\frac{\ln\left(\frac{M_{1,0,0}-2M_{1,0,1}}{M_{1,0,1}-2M_{1,0,3}}\right)}{\ln 2}\right)} \tag{6}$$

Finally, the failure density function and the failure distribution function of products can be obtained using the formulas above.

3 Small-sample accelerated life test method

If the sample size of test objects is very small or even only single, such as the tool magazine and automatic tool changer, the accelerated life test method based on the small-sample virtual augmented theory should be used in order to obtain its "Mean Cycles between Failures" under the normal condition in a short time. Detailed steps are as follows:

1) Choose an appropriate accelerated stress type and censoring way of accelerated life test;

2) Choose an appropriate acceleration model and determine different accelerated stress levels, then carry out accelerated life test under those different stress levels;

3) Augment the test results based on the small-sample virtual augmented theory, so as to make the set of random numbers follow the Weibull distribution and has same distribution characteristics with the test results;

4) Calculate the characteristic life of the tool magazine and automatic tool changer under different stress levels;

5) Predict the characteristic life of the automatic tool changer under the normal working condition.

3.1 Accelerated life test method

The tool magazine and automatic tool changer is one of high reliability products. In order to effectively reduce test time and cost, the type-I censoring and type-II censoring ways are always chosen to carry out the accelerated life test in a laboratory. Due to the occasionality and uncontrollability of failure during the operating process of tool magazine and automatic tool changer, the way of type-I censoring is recommended. Besides, considering the operability of small-sample test, the step-stress accelerated life test is applied. It means that in order to obtain the "Mean Cycles between Failures" of automatic tool changer under the normal stress level S_0, a group of stress levels $S_1,S_2,\cdots,S_k$ would be applied on the automatic tool changer in sequence with $S_0<S_1<S_2<\cdots<S_k$, and the test end time is T_i under the stress level S_i, while stopping test until the predetermined total test time T.

Because of the complex structure and exorbitant price of tool magazine and automatic tool changer, a small-sample accelerated life test method should be used. In this paper, through generating a set of random numbers, the small-sample test data would be augmented to a large-sample based on the small-sample virtual augmentation theory [14], and make sure that these two sample sets have similar distribution characteristics and same mean and variance.

According to the formula (2), the reliability function can be obtained:

$$R(\tau) = 1 - F(\tau) = e^{-\left(\frac{\tau-\gamma}{\eta}\right)^m}. \quad (7)$$

After performing the Weibull linear transformation,

$$y = m[x - \ln(\eta)], \quad (8)$$

where $y = \ln[-\ln[R(\tau)]], x = \ln(\tau - \gamma)$

Assuming a group of failure samples: $\tau_1, \tau_2, \cdots, \tau_n$, with corresponding reliability: $R_1, R_2, \cdots, R_n$, then the array columns $(\tau_1, R_1), (\tau_2, R_2), \cdots, (\tau_n, R_n)$ can be set up and be converted into the sample order statistics $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ by the Weibull transformation. When plotting these points on the Weibull Probability Paper (WPP), it indicates a straight line if the products life follows the Weibull distribution. Therefore, when the sample set of tool magazine and automatic tool changer is augmented, its order statistics distribution should be a straight line on the WPP considering that its life distribution should follow the Weibull distribution model.

3.2 Acceleration model

Accelerated test conditions are typically set up by testing products at higher stress levels than normal, and some parameters are always chose as the accelerated stress in the inverse power law model, such as temperature, pressure, load, vibration, voltage, current, power, cycling rate, etc. (or some combination of them). When exchanging tools, vibration will occur on the tool magazine and automatic tool changer, and different tool weight loads will lead to different degrees of vibration. The Root Mean Square (RMS) of vibration will increase with increased load. Since vibration is one of major factors making mechanical parts fatigue and wear, it will be chosen as the acceleration stress in the inverse power law model in this paper. Finally, the characteristic life of the automatic tool changer can be predicted by applying different tool weight load levels to obtain corresponding vibration characteristics.

The inverse power law model using vibration as acceleration stress can be expressed as

$$t = AG^{-B}, \quad (9)$$

where t is the characteristic life of the Weibull distribution $W(m, \eta)$; A, B are constants; G is the RMS of vibration: $Grms(g)$. The formula (9) can also be converted as: $\ln t = a + b \ln G$, where $a = \ln A, b = -B$.

When the life test of tool magazine and automatic tool changer is carried out under a certain stress level G_i, its characteristic life t_i can be obtained, then the estimate values of parameters a and b in the acceleration model can be calculated:

$$\ln \hat{t_i} = \hat{a} + \hat{b} \ln G_i \quad (10)$$

Finally the characteristic life under the normal stress level S_0 can be calculated:

$$\ln \hat{t_0} = \hat{a} + \hat{b} \ln G_0 \quad (11)$$

If failure doesnt take place under a certain stress level S_i during the test, the life can be estimated using the first repaired period:

$$t_i = \frac{T_i}{K} \quad (12)$$

where T_i is the test time under the stress level S_i and K is an empirical correction factor which is generally set to 1.5.

4 Application Example

A certain chain tool magazine and automatic tool changer was taken as the study object in this paper and shown in Fig.1. The sample size was selected one due to the constraints of funding and resources, therefore the automatic tool changer would be overhauled after each time of collecting experimental data, which would solve the problem of lacking prototype. In this study, the type-I censoring way was applied, and the small-sample step-stress accelerated life test by using different weight tools was carried out to predict product's "Mean Cycles between Failures" in a short time.

The normal weight load S_0 of the chain tool magazine and automatic tool changer was 350kg, while the maximum weight load S_{max} was 900kg, therefore, a set of load levels: S_1 was 420kg and S_2 was 600kg, were selected in this test. In order to exclude early failures of the tool changer, no-load running should be done at the beginning of test. Subsequently, the exchanging tool test would be carried out under the selected load levels respectively, while the vibration data of tool magazine system was collected and the cycle number of tool exchanging when various failures occurred would be recorded during the test. In order to obtain more failure information, the censoring number of tool exchange was adjusted according to actual condition.

In the accelerated life test, 18762 times of exchanging tool cycles were finished under the load level 420kg without occurring failure; therefore, the first repaired period of the chain tool magazine and automatic tool changer is

$$t_1 = \frac{T_1}{K} = \frac{18762}{1.5} = 12508$$

Fig. 1: Chain tool magazine and automatic tool changer

33084 times of exchanging tool cycles were finished under the load level 600kg. All failure information occurred under the load level is listed in the Table 1:

Table 1: failure information under load level 600kg

Number of current cumulative tool exchange cycles	Failure mode	Failure cause
4948	Stuck tool	Tool holders rivets loose
6213	Solenoid valve loose	Vibration and shock
12895	Drop tool	Gripper instability
15777	Stuck tool	Position inaccuracy

1) Based on the Table 1, the number of cycles between failures were 4948, 1265, 6682, 2882, and 17307 respectively, and the mean of this data set was 6616.8, while the standard deviation was 2525.13.

2) Then the sample set would be augmented virtually to 20 random numbers which followed the Weibull distribution and had same mean and standard deviation, which means that the augmented sample set would appear as a linear distribution on the WPP, while its mean was 6616.8 and standard deviation was 2525.13. The augmented sample data set of the chain tool magazine and automatic tool changer was listed as follows: 8191.8, 8636.1, 8994.6, 4109.7, 7152.6, 7218.0, 4070.1, 4741.6, 9351.9, 6284.5, 7602.1, 6839.2, 5010.9, 5202.7, 7738.0, 4216.3, 8590.9, 8054.6, 4540.2, 5945.6.

3) Subsequently, using the PWM method to solve the above augmented sample data set, the parameters of the Weibull model could be calculated based on the formulas (3)-(6):

$$\gamma = 838.80, \eta = 6432.20, m = 3.4821$$

Therefore, according to the formula (1), the probability density function under the Weibull distribution was listed below and shown in Fig.2, and the augmented sample set was drawn on the WPP, as Fig.3 shows:

$$f(\tau) = \frac{3.4821}{6432.2}\left(\frac{\tau - 838.80}{6432.2}\right)^{2.4821} e^{-\left(\frac{\tau - 838.80}{6432.2}\right)^{3.4821}}$$

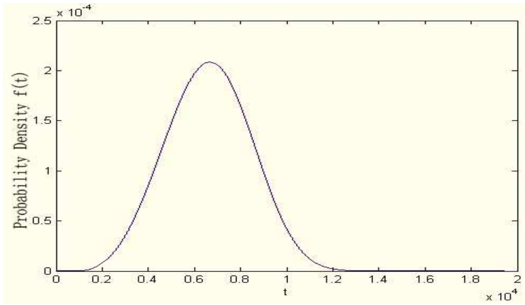

Fig. 2: failure distribution probability density function

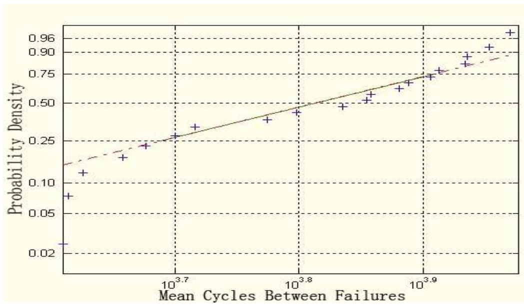

Fig. 3: the augmented sample set on the WPP

The reliability function of the chain tool magazine and automatic tool changer under the load level 600kg can be obtained based on the formula (7):

$$R(\tau) = 1 - F(\tau) = e^{-\left(\frac{\tau - \gamma}{\eta}\right)^{m}} = e^{-\left(\frac{\tau - 838.80}{6432.2}\right)^{3.4821}}$$

4) Finally, the characteristic life - "Mean cycles between failures" of the automatic tool changer under the load level 600kg can be calculated based on the formula (13) and the result was 7271:

$$t(e^{-1}) = \gamma + \eta \qquad (13)$$

The vibration signals collected during the process of exchanging tool under the three different load levels S_0:350kg, S_1:420kg and S_2:600kg were shown in Fig. 4

respectively, and the RMS of vibration: $Grms(g)$ can be calculated by formula (14).

$$\varphi_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x^2(i\Delta t)}, \quad (14)$$

where Δt was the sampling interval and N was the total number of data.

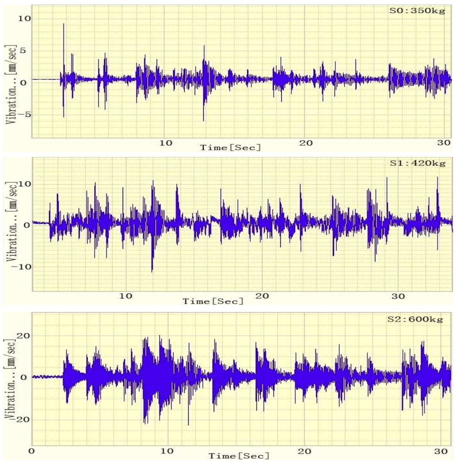

Fig. 4: Vibration signal under different load levels

The "Mean Cycles between Failures" of the chain tool magazine and automatic tool changer under the normal stress can be predicted based on the inverse power law model according to data in the Table 2.

Table 2: Data used for predicting "Mean Cycles between Failures"

Load level	S_0 : 350kg	S_1 : 420kg	S_2 : 600kg
RMS of Vibration: G	0.326	0.621	0.952
Characteristic life: t	unknown	12508	7271

According to the formula (10), the estimation value of parameters a and b can be obtained

$$\begin{cases} \ln 12508 = a + b\ln 0.621 \\ \ln 7271 = a + b\ln 0.952 \end{cases}$$

$$\hat{a} = 8.8292, \hat{b} = -1.2697$$

Using formula (11), the "Mean Cycles between Failures" of the automatic tool changer under the normal stress was

$$\hat{t_0} = 28283$$

5 Conclusion

In this paper, a small-sample accelerated life test method based on the inverse power law model was proposed and successfully applied on a chain tool magazine and automatic tool changer. In the case, vibration was taken as the accelerated stress, and small-sample data was augmented based on the sample virtual augmented theory. The "Mean Cycles between Failures" of exchanging tool under normal stress was then calculated to be about 28283 times. The result was in agreement with the actual situation. The proposed method greatly reduced test time and cost and can also be applied in other relevant large-scale and complex products.

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