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Soret and Dufour Effects on Peristaltic Transport of MHD Fluid with Variable Viscosity

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Abstract: This article looks at the Soret and Dufour effects on the magnetohydrodynamic (MHD) peristaltic flow of variable viscosity fluid in a symmetric channel. Analysis is presented in the presence of Ohmic heating. Results for the stream function, temperature and concentration are constructed. The variations of sundry parameters are analyzed.

Keywords: Dufour Effects, Peristaltic Transport, MHD Fluid
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1 Introduction

The peristaltic flow problems in channel/tube are widely seen in several processes of engineering and physiology. Such flows appear in urine transport from kidney to bladder, swallowing of food through oesophagus, movement of chyme in gastrointestinal tract, spermatozoa transport in the ducts efferentes of male reproductive tract, ovum movement in the female fallopian tube, vasomotion of small blood vessels, water transport from ground to upper branches of tall trees etc. The industrial applications of peristalsis include blood pumps in heart lung machine, sanitary fluid transport and transport of corrosive fluids. Earliest experimental and theoretical models of peristaltic transport in viscous fluids were presented by Latham [1] and Shapiro et al [2]. Since then a vast amount of information on peristalsis under different aspects has been given by various investigators. Few recent studies on the title can be seen through the refs [3, 4, 5, 6, 7, 8, 9, 10, 16, 17, 18].

Most of the published papers regarding peristalsis in channels/tubes have been discussed for constant viscosity fluid. Very little attention is given to the situations which can shed light on the peristalsis of variable viscosity fluid. For instance [11, 12, 13, 14, 15]. To date no information is available on MHD peristaltic flows with variable viscosity and Soret and Dufour effects. Interaction of peristaltic motion with heat transfer is significant in oxygenation and hemodialysis processes. Simultaneous considerations of heat and mass transfer are available in chemical industry problems for example in reservoir engineering regarding thermal recovery process, catalytic reactors, analysis of hot springs in the sea and medicine diffusion in blood veins. Further simultaneous occurrence of heat and mass transfer affecting each other lead to the Soret and Dufour effects. The magnetohydrodynamic character of fluid has a pivotal role in solidification processes of metal and metal alloys, study of nuclear fuel debris, control of underground spreading of chemical wastes and pollution, design of MHD power generators, blood and blood pump machines, treatment of cancer tumor etc. In view of such discussion, the objective of present article is to analyze the MHD peristaltic transport of variable viscosity fluid in a symmetric channel when Soret and Dufour effects are present. Problem formulation invokes the long wavelength and low Reynolds number assumptions. The series solutions are presented and discussed very carefully.

2 Mathematical analysis

Let us investigate the magnetohydrodynamic flow of an incompressible viscous fluid in a channel with width $2a$. The $\overline{X}-$ axis is chosen along the walls of channel and $\overline{Y}-$ axis is taken normal to the $\overline{X}-$ axis. A constant magnetic

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field of strength $\mathbf{B}_0$ is exerted in the $\overline{Y}-$ direction. Induced magnetic field is not accounted because of small magnetic Reynolds number. Sinusoidal wave propagation on channel walls with constant wave speed c are represented by

$$H(\overline{X},\bar{t}) = a + b\cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\bar{t})\right). \tag{1}$$

If $(\overline{U},\overline{V})$ and $(\overline{u},\overline{v})$ are the velocity components in the laboratory $(\overline{X},\overline{Y})$ and wave $(\overline{x},\overline{y})$ frames respectively, then transformations between laboratory and wave frames are

$$\overline{x} = \overline{X} - c\bar{t},\ \ \overline{y} = \overline{Y},\ \ \overline{u} = \overline{U} - c,\ \ \overline{v} = \overline{V},\ \ \overline{p}(\overline{x},\overline{y}) = \overline{P}(\overline{X},\overline{Y},\bar{t}). \tag{2}$$

In above expressions b is the wave amplitude, λ is the wavelength, $\bar{t}$ is the time and $\overline{P}$ and $\overline{p}$ are the pressures in laboratory and wave frames respectively. Introducing the variables in the forms

$$\begin{gathered} x = \frac{\overline{x}}{\lambda},\ y = \frac{\overline{y}}{a},\ u = \frac{\overline{u}}{c},\ v = \frac{\overline{v}}{c\delta},\ \delta = \frac{a}{\lambda},\ h = \frac{H}{a},\ d = \frac{b}{a},\ p = \frac{a^2\overline{p}}{c\lambda\mu_0}, \\ \theta = \frac{T - T_0}{T_0},\ \phi = \frac{C - C_0}{C_0},\ M = \left(\frac{\sigma}{\mu_0}\right)^{1/2}\mathbf{B}_0 a,\ \upsilon = \frac{\mu_0}{\rho}, Re = \frac{\rho c a}{\mu_0}, \\ t = \frac{c\bar{t}}{\lambda}, u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x},\ Br = \mathrm{Pr}E,\ Du = \frac{DC_0K_T}{C_s\zeta\mu_0T_0},\ Sr = \frac{\rho DK_T T_0}{\mu_0 T_m C_0}, \\ Sc = \frac{\mu_0}{\rho D},\ E = \frac{c^2}{\zeta T_0},\ Pr = \frac{\mu\zeta}{K}, \text{ and } \mu(y) = \frac{\bar{\mu}(\bar{y})}{\mu_0}. \end{gathered} \tag{3}$$

the conservation laws of mass, linear momentum, energy and concentration after utilizing long wavelength and low Reynolds number assumptions yield

$$\frac{dp}{dx} = \frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - M^2\left(\frac{\partial\psi}{\partial y} + 1\right), \tag{4}$$

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - M^2\frac{\partial\psi^2}{\partial y^2} = 0, \tag{5}$$

$$0 = \frac{\partial^2\theta}{\partial y^2} + Br\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + BrM^2\left(\frac{\partial\psi}{\partial y} + 1\right)^2 + \mathrm{Pr}Du\frac{\partial^2\phi}{\partial y^2}, \tag{6}$$

$$0 = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} + Sr\frac{\partial^2\theta}{\partial y^2}, \tag{7}$$

where p is the pressure, C the concentration field, T the temperature field, σ the electric conductivity, D the mass diffusivity, K_T the thermal diffusion ratio, ζ the specific heat, C_s the concentration susceptibility, K the thermal conductivity, $\mu(y)$ the variable viscosity, μ_0 the absolute viscosity, T_m the fluid mean temperature, υ the kinematic viscosity, M the Hartman number, Re the Reynolds number, Br the Brinkman number, Du the Dufour parameter, Sr the Soret parameter, Sc the Schmidt number, E the Eckret number, Pr the Prandtl number, δ the wave number, ψ the stream function, C_0 and T_0 the concentration and temperature at the boundary, θ the dimensionless temperature and ϕ the concentration. The boundary conditions are

$$\begin{aligned} &\psi = 0,\quad \frac{\partial^2\psi}{\partial y^2} = 0,\ \frac{\partial\theta}{\partial y} = 0,\ \frac{\partial\phi}{\partial y} = 0,\quad at\ \ y = 0, \\ &\psi = F,\quad \frac{\partial\psi}{\partial y} = -1,\ \theta = 0,\ \phi = 0,\quad at\ \ y = h, \end{aligned} \tag{8}$$

$$h(x) = 1 + d\cos(2\pi x),\ F = \int_0^h \frac{\partial\psi}{\partial y}dy, \tag{9}$$

where $h(x)$ is the dimensionless wall shape and F is the dimensionless flow rate in the wave frame.

Pressure rise per wavelength Δp_λ is

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx}dx. \tag{10}$$

The dimensionless expression of space dependent viscosity is [11]

$$\mu(y) = e^{-\alpha y} = 1 - \alpha y \qquad \alpha \ll 1,$$

where "α" is the viscosity parameter.

We look for solutions in the series form represented below

$$\begin{aligned} \psi &= \psi_0 + \alpha\psi_1 + \ldots. \\ F &= F_0 + \alpha F_1 + \ldots. \\ p &= p_0 + \alpha p_1 + \ldots. \end{aligned}$$

Employing the procedure of perturbation method and retaining the results up to order $O(\alpha)$ we have

$$\psi = A_1 + \frac{(F + h)\,\alpha A_2}{8\left(-hM\cosh(hM) + \sinh(hM)\right)^2}, \tag{11}$$

$$\begin{aligned} \theta = &-\frac{BrM^2B_1}{8\left(-1 + \mathrm{Pr}ScSrDu\right)\left(hM\cosh(hM) - \sinh(hM)\right)^2} \\ &-\frac{BrM\alpha\left[B_2 + B_3 + B_4 - B_5 + B_6 - B_7 + B_8\right]}{32\left(-1 + \mathrm{Pr}ScSrDu\right)\left(hM\cosh(hM) - \sinh(hM)\right)^3}, \end{aligned} \tag{12}$$

$$\begin{aligned} \phi = &-\frac{BrM^2ScSr\left[C_1 - C_2 + C_3\right]}{8\left(-1 + \mathrm{Pr}ScSrDu\right)\left(hM\cosh(hM) - \sinh(hM)\right)^2} \\ &+\frac{BrMScSr\alpha\left[C_4 + C_5 + 2\left(C_6 + C_7 + \frac{1}{4}M\left(C_8 + C_9 - C_{10}\right) - C_{11} - C_{12} + C_{13}\right)\right]}{64\left(-1 + \mathrm{Pr}ScSrDu\right)\left(hM\cosh(hM) - \sinh(hM)\right)^3}, \end{aligned} \tag{13}$$

where the involved $\mathrm{A}_i(i = 1,2)$, $\mathrm{B}_j(j = 1-8)$ and $\mathrm{C}_k(k = 1-13)$ are presented in the Appendix. The heat transfer coefficient is a vital quantity to be analyzed in the problems of heat transfer as it incorporates the geometry of the problems in to the analysis of heat transfer. The heat transfer coefficient for this problem is defined as follows:

$$Z = h_x\theta_y.$$

3 Discussion

This section describes the impacts of pertinent parameters on the temperature and concentration. Here 3-D graphs are explicitly plotted in order to analyze the quantities of interest in a more detailed manner. These types of graphs allow the readers to analyze the physical quantities continuously within a given domain which is not perhaps visible in the 2-D plots. Also these plots explain the changing behavior of physical quantities in a better way. We recall that theme of this study is to point out the influences of Soret and Dufour. Therefore, the results of various parameters associated with velocity are not included. In order to achieve the desired objective, we present the plots in such a way that the left panels (all the ”a” parts of the Figs.) are for constant viscosity (i.e. for $\alpha = 0.0$) and the right ones (all the ”b” parts of the Figs.) are for variable viscosity (i.e. for $\alpha = 0.4$). It is observed in all the graphs related to temperature that the rise in temperature for variable viscosity fluid is relatively slower when compared with fluid having constant viscosity. Here Figs. 1-5 are drawn for temperature whereas the Figs. 6-10 show variation of concentration. Temperature increases with an increase in M (Fig. 1). Further an increase in temperature is abrupt in view of Ohmic heating. The variations of Du, Sr, Sc and Br on the temperature are displayed in the Figs. 2-5. These Figs. indicate that there is an increase in temperature by increasing Du, Sr, Sc and Br. It is also found that an increase in temperature is more for Br when compared to the other parameters.

Figs. 6-10 are presented to examine the behavior of embedded parameters on the concentration. Decrease in concentration is observed when M and Du increase (see Figs. 6 & 7). Concentration also decreases when Sr, Sc and Br are increased (Figs. 8-10). Again through these Figs. it is clear that decrease in concentration is abrupt for variation in M, Sc and Sr but is slow for the case of Du. Further it is seen that the fluid with variable viscosity has a higher value of concentration compared to that of fluid with constant viscosity.

Behavior of heat transfer coefficient Z for various parameters is shown in the Figs. 11-15. As expected, Z shows an oscillatory behavior which is because of peristalsis. It is also noted that there is no variation in Z for amplitude ratio (d) between 0 and 0.3. It is observed from Fig. 11 that Z increases when M is increased. The absolute values of Z in variable viscosity fluid are more than the constant viscosity fluid. Figs 12-15 show that Z increases with the increase in Du, Sr, Sc and Br. Effects of Du, Sr, Sc and Br on Z are opposite to that of M.

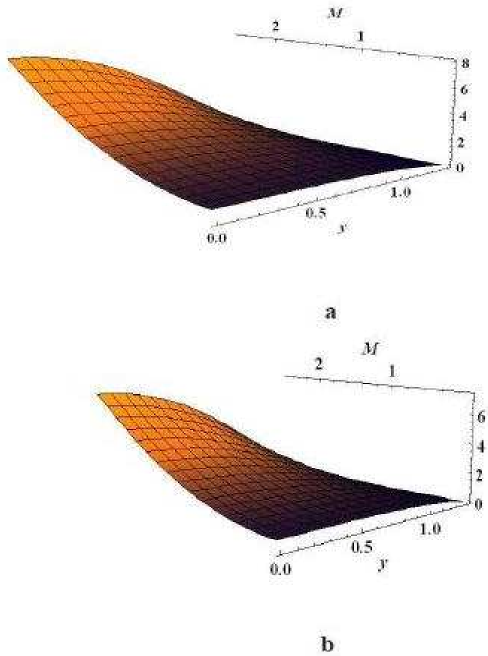

Fig. 1: (a and b) Effect of M on θ when Du=0.5, Sr=0.5, Sc=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

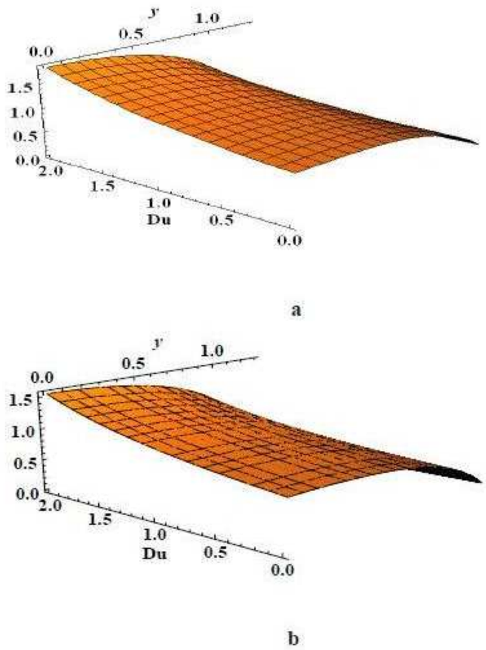

Fig. 2: (a and b) Effect of Du on θ when M=0.5, Sr=0.5, Sc=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

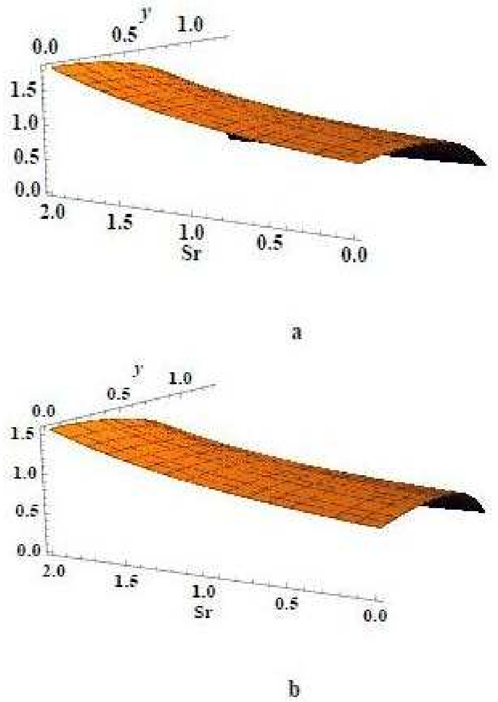

Fig. 3: (a and b) Effect of Sr on θ when Du=0.5, M=0.5, Sc=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

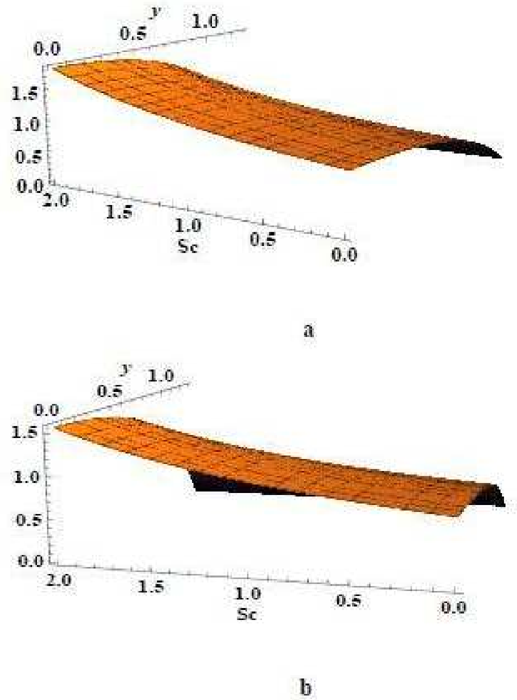

Fig. 4: (a and b) Effect of Sc on θ when Du=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

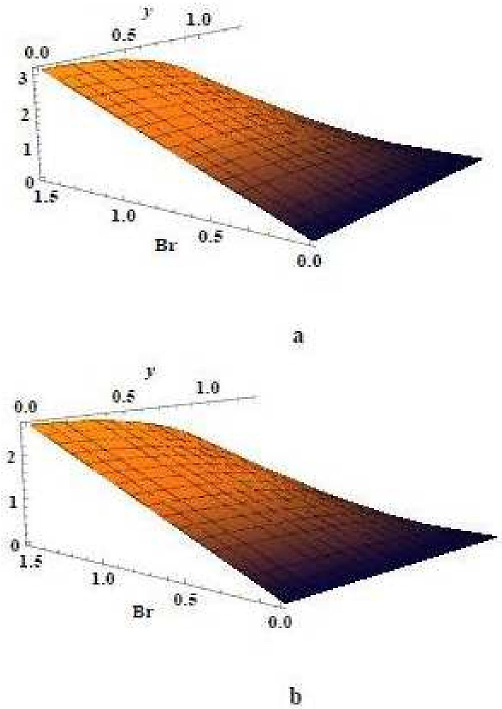

Fig. 5: (a and b) Effect of Br on θ when Du=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$, Sc=0.5 and x=0.

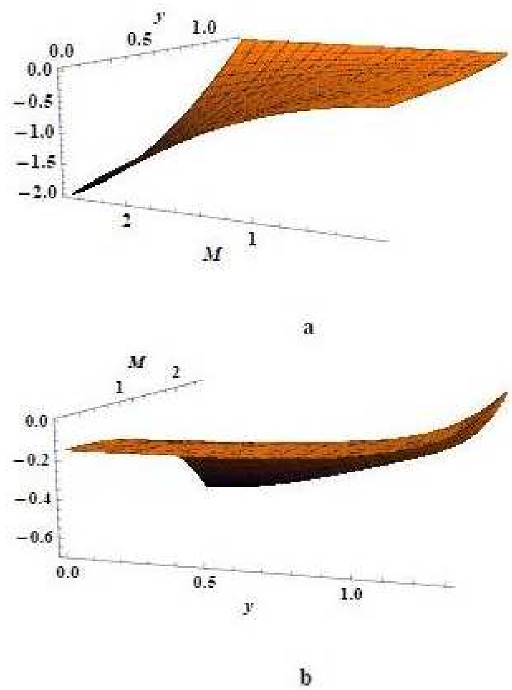

Fig. 6: (a and b) Effect of M on ϕ when Du=0.5, Sr=0.5, Sc=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

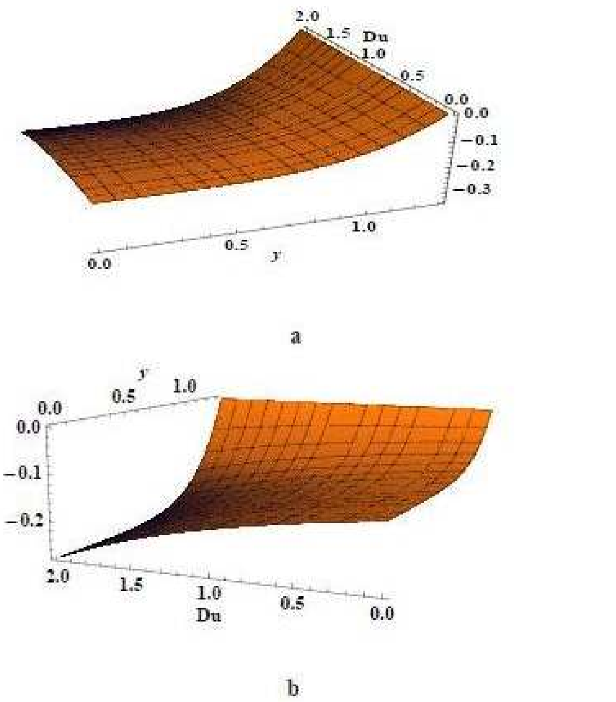

Fig. 7: (a and b) Effect of Du on ϕ when M=0.5, Sr=0.5, Sc=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

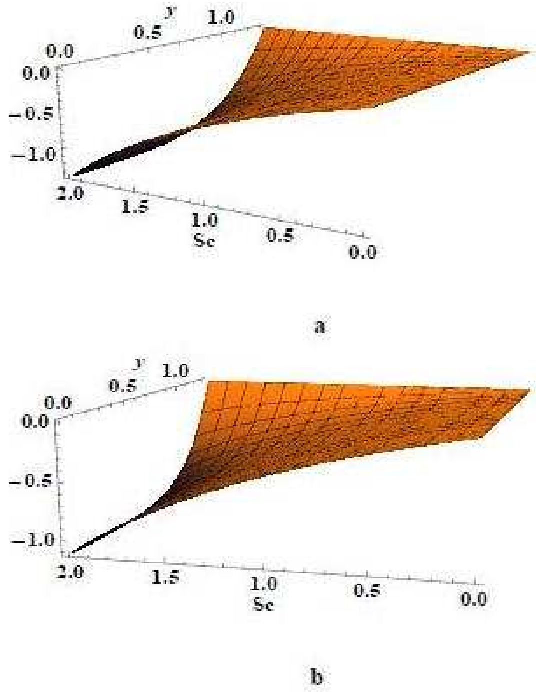

Fig. 9: (a and b) Effect of Sc on ϕ when Du=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

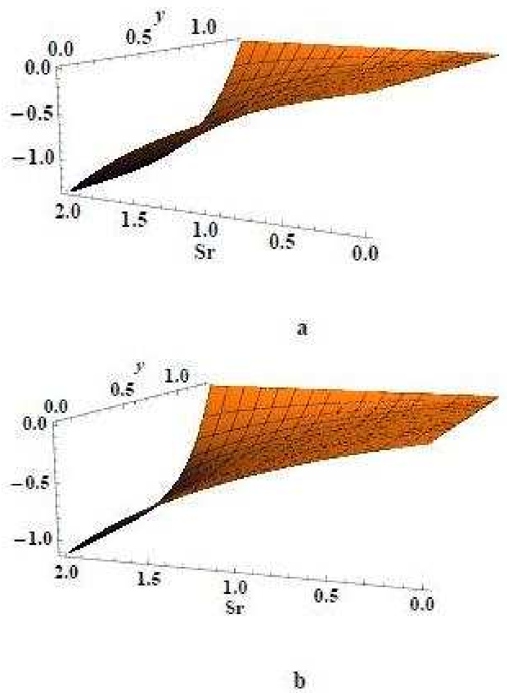

Fig. 8: (a and b) Effect of Sr on ϕ when Du=0.5, M=0.5, Sc=0.5, $\phi = 0.3$, $\eta = 1.4$, Br=0.5 and x=0.

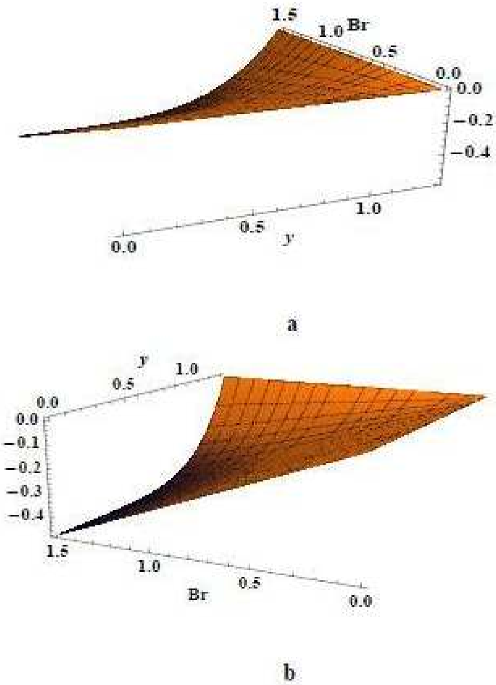

Fig. 10: (a and b) Effect of Br on ϕ when Du=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$, Sc=0.5 and x=0.

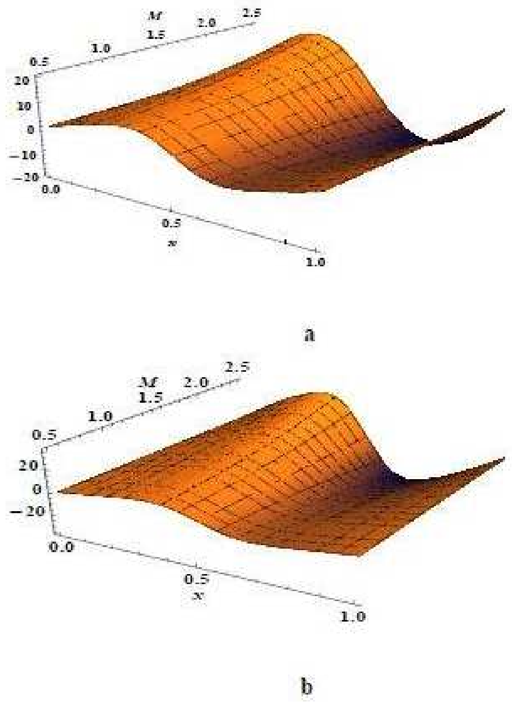

Fig. 11: (a and b) Effect of M on Z when Du=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$ and Br=0.5.

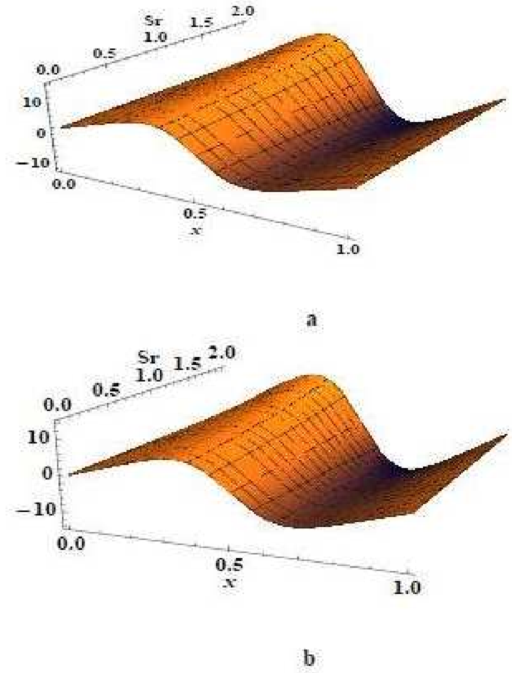

Fig. 13: (a and b) Effect of Sr on Z when Sc=0.5, Du=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$ and Br=0.5.

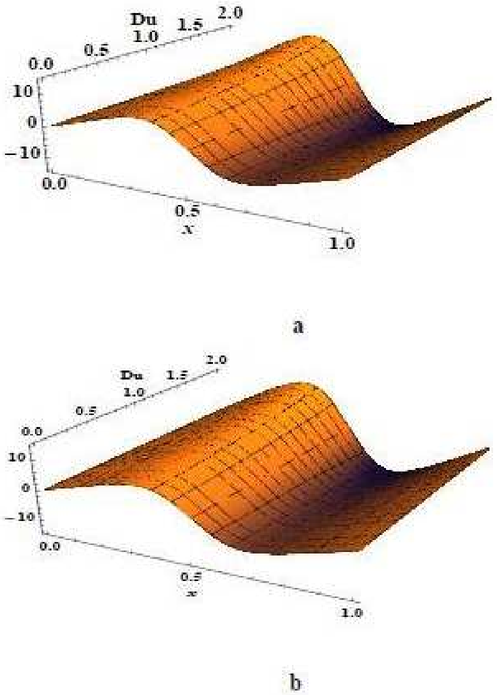

Fig. 12: (a and b) Effect of Du on Z when Sc=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$ and Br=0.5.

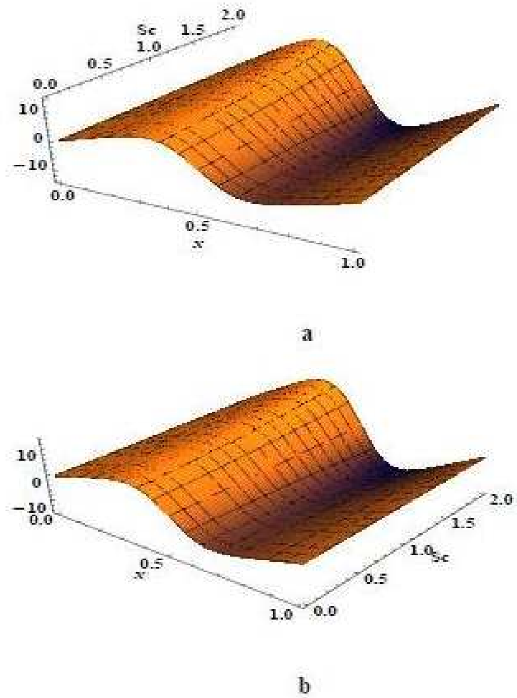

Fig. 14: (a and b) Effect of Sc on Z when Du=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$ and Br=0.5.

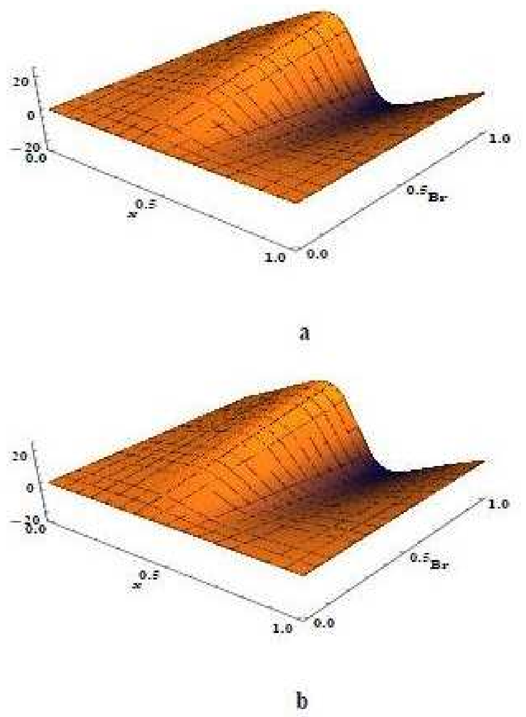

Fig. 15: (a and b) Effect of Br on Z when Sc=0.5, Sr=0.5, M=0.5, $d = 0.3$, $\eta = 1.4$ and Du = 0.5.

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Appendix

$A_1=\frac{FMy\cosh(hM)+y\sinh(hM)-(F+h)\sinh(My)}{hM\cosh(hM)-\sinh(hM)}$, $A_2=yA_3+(hMy\cosh(hM)+(h^3M^2-y)\sinh(hM))$
$2\sinh(My)$, $A_3=-1+2h^2M^2+\cosh(2hM)+2My\cosh(My)(-hM\cosh(hM)+\sinh(hM))$
$-2hM\sinh(2hM)$, $B_1=-8F-2h(4+h)-2F+h^2(F+2h)M^2-2h^2(F+h)^2M^4+2$
$(1+F(F+2h)M^2+(F+h)^2M^4)y^2+$
$(F^2(1+M^2)+2F(4+h+hM^2)-2y^2+h(8+h(3+M^2(1+2h^2-2y^2))))$
$\cosh(2hM)-(F+h)^2(1+M^2)\cosh(2My)-4hM(2F+h(2+h)-y^2)\sinh(2hM)+$
$16h(F+h)M\cosh(hM)(M(h-y)+\sinh(My))-16(F+h)\sinh(hM)(M(h-y)+\sinh(My))$,
$B_2=-8h^3(F+h)(h-y)\sinh(2hM)M^4-\frac{1}{2}(F+h)B_9\cosh(hM)-16h^4(F+h)M^4\cosh(hM)^3$
$+8h^3(F+h)M^4(h-y)\cosh(2hM)$, $B_3=\frac{1}{2}(F+h)(3(F+h)+3(F+h-8h^2)M^2+8h^4M^4)$
$\cosh(3hM)+16h^2(F+h)M^4y^2\cosh(hM)^2\cosh(My)-2h(F+h)^2M^2(1+M^2)y\cosh(hM)$
$\cosh(2My)$, $B_4=(F+h)MB_{10}\sinh(hM)$, $B_5=64F1h^2M^3\cosh(hM)^2\sinh(hM)+2h^3$
$(F+h)^2M^3(1+M^2)\cosh(2My)\sinh(hM)+2(F+h)^2M(1+M^2)y\cosh(2My)\sinh(hM)+$
$16(F+h)M^2y^2\cosh(My)\sinh(hM)^2-8h^4(F+h)M^5(h-y)\sinh(2hM)$, $B_6=hMB_{11}\cosh(hM)$
$\sinh(2hM)$, $B_7=hM\left(-4F+F^2-4h+2Fh+h^2+(F+h)^2(1+h^2)M^2+h^2(F+h)^2M^4\right)$
$\sinh(3hM)-16h^2(F+h)M^3y\cosh(hM)^2\sinh(My)+16h^3(F+h)M^3\sinh(hM)^2\sinh(My)-$
$16(F+h)My\sinh(hM)^2\sinh(My)-8h^4(F+h)M^4\sinh(2hM)\sinh(My)$,$B_8=16h(F+h)$
$M^2y\sinh(2hM)\sinh(My)+(F+h)^2(1+M^2)(-3+2M^2y^2)\sinh(hM)\sinh(2My)-h(F+h)M\cosh$
$(hM)B_{12}$,
$B_9=F(1+M^2)(3+4hM^2(-3h+2h^3M^2+2y-2M^2y^3))+h$
$(3+M^2(3+4h(h(-3+M^2(-3+2h(1+h+hM^2)))+2(-3+y)+2M^2y-2M^2(1+M^2)y^3)))$,
$B_{10}=-3h(4+F+h)+h(F(-3+5h^2)+h(-3+h(16+5h)))M^2+h^3(F+h)(5+4h^2)M^4+$
$4h^5(F+h)M^6+4(F+h)(1+M^2)y-4h^3(F+h)M^4(1+M^2)y^2-4(F+h)M^2(1+M^2)y^3$,
$B_{11}=-3(F+h)^2+(-3F^2-6Fh+32f1h+(-3+2F(8+F))h^2+4(4+F)h^3+2h^4)M^2+2h^2$
$(F+h)^2M^4$, $B_{12}=16M^2y^2\sinh(M(h-y))+(F+h)(1+M^2)(-3+2M^2y^2)\sinh(2My)+16M^2y^2$
$\sinh(M(h+y))$, $C_1=2C_{14}-16h(F+h)M^2(h-y)\cosh(hM)-$
$(F^2(1+M^2)+2F(4+h+hM^2)-2y^2+h(8+h(3+M^2(1+2h^2-2y^2))))\cosh(2hM)$,
$C_2=8F\cosh(M(h-y))-8h\cosh(M(h-y))+F^2\cosh(2My)+2Fh\cosh(2My)+h^2\cosh(2My)$
$+F^2M^2\cosh(2My)+2FhM^2\cosh(2My)+h^2M^2\cosh(2My)+8(F+h)\cosh(M(h+y))+$
$16FhM\sinh(hM)+16h^2M\sinh(hM)-16FMy\sinh(hM)-16hMy\sinh(hM)$,
$C_3=8FhM\sinh(2hM)+8h^2M\sinh(2hM)+4h^3M\sinh(2hM)-4hMy^2\sinh(2hM)+$
$8FhM\sinh(M(h-y))+8h^2M\sinh(M(h-y))-8h(F+h)M\sinh(M(h+y))$,
$C_4=(3(F+h)^2+(3F^2+6F(1-4h)h+h(64f1+3(1-8h)h))M^2)\cosh(3hM)$,
$C_5=2M\cosh(hM)^2C_{15}$, $C_6=-8h^3(F+h)M^4(h-y)+2(F+h)^2M(1+M^2)(h^3M^2+y)$
$\cosh(My)$, $C_7=2My\sinh(hM)+16(F+h)M^2y^2\cosh(My)\sinh(hM)^2$, $C_8=64F1hM$
$\cosh(hM)+32h^3(F+h)M^3(h-y)\cosh(2hM)-64F1hM\cosh(3hM)$,
$C_9=C_{16}\sinh(hM)$, $C_{10}=32h^4(F+h)M^4(h-y)\sinh(2hM)+$
$(-h(F+h)(-8+5F+5h+(5F+(5-16h)h)M^2)+8F1(4+F+h+(F+h-4h^2)M^2))$
$\sinh(3hM)$,$C_{11}=4MF1(4-4h^2M^2)+4M(F+h)(-3y+h^2M^2(3h+y))+4M$
$(F1(4-4h^2M^2)+4M(F+h)(y+h^2M^2(-h+y)))\cosh(2hM)+4M2h(F+h)M(h^3M^2-2y)$
$\sinh(2hM)\sinh(My)+4M(F+h)^2(1+M^2)(-3+2M^2y^2)\sinh(hM)\sinh(2My)$,
$C_{12}=\cosh(hM)(F+h)(F(1+M^2)(3+4hM^2(-3h+2h^3M^2+2y-2M^2y^3))$
$+\cosh(hM)h\left(3+M^2\left(3+4h^2\left(\begin{array}{c}-3+M^2(-3+2h(4+h+hM^2))+2\cosh(hM)(-3+y)\\ +2M^2y-2M^2(1+M^2)y^3\end{array}\right)\right)\right)$,
$C_{13}=64F1hM^2\cosh(2hM)+4h(F+h)^2M^2(1+M^2)y\cosh(2My)+2hM$
$(-32F1hM^2\sinh(2hM)+(F+h)C_{18})$, $C_{14}=h(4+h+h^3M^4)-(1+h^2M^4)y^2$
$+F^2M^2(1+M^2)(h-y)(h+y)+2F(2+hM^2(1+M^2)(h-y)(h+y))$, $C_{15}=$
$16h^2(F+h)M^3y^2\cosh(My)-$
$(h(F+h)(-8+5F+5h+(5F+(5-16h)h)M^2)+8F1(4+F+h+(F+h+4h^2)M^2))$
$\sinh(hM)-8(4f1(-1+h^2M^2)+(F+h)(y+h^2M^2(-h+y)))\sinh(My)$,$C_{16}=8F1$
$(4+F+h+(F+h-4h^2)M^2)+(F+h)C_{17}$,
$C_{17}=-56h+Fh(1+M^2)(-13+24h^2M^2+16h^4M^4)+$
$h^2(-13+M^2(-13+8h(10+h(1+M^2)(3+2h^2M^2))))$
$+16(F+h)(1+M^2)y-16h^3(F+h)M^4(1+M^2)y^2-16(F+h)M^2(1+M^2)y^3$,
$C_{18}=16M^2y^2\sinh(M(h-y))+(F+h)(1+M^2)(-3+2M^2y^2)\sinh(2My)+$
$16M^2y^2\sinh(M(h+y))$.

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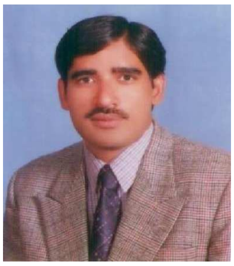

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