Divisible On-Line/Off-Line Proxy Re-Signature

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Abstract: To improve the real-time efficiency of proxy re-signature schemes, a notion called divisible on-line/off-line proxy re-signatures is introduced in this paper. The idea of this notion is to split the re-signing algorithm into two phases: the off-line and on-line phases. The proxy can perform the bulk of re-signature computation in the off-line phase before seeing the message to be re-signed. The results of this pre-computation are saved and then used in the on-line phase after the message to be re-signed is given. Divisible on-line/off-line proxy re-signatures are very useful in a particular scenario where the proxy must respond quickly once the message to be re-signed is presented. In a divisible on-line/off-line proxy re-signature scheme, the partial re-signature might be exposed before the message to be re-signed arrives. We then propose a generic divisible on-line/off-line proxy re-signature scheme, which can transform any proxy re-signature scheme into a highly efficient divisible on-line/off-line proxy re-signature scheme. Compared to existing proxy re-signature schemes, our scheme demonstrates higher efficiency. The security of the scheme relies on the unforgeability of its underlying proxy re-signature scheme and the difficulty to solve the discrete logarithm problem.

Keywords: divisible on-line/off-line proxy re-signature, on-line/off-line proxy re-signature, unforgeability

1 Introduction

Proxy re-signature, introduced by Blaze, Bleumer and Strauss [1], allows a semi-trusted proxy to convert a delegatee’s signature into a delegator’s signature on the same message by using some additional information (a.k.a., re-signature key). However, the proxy does not obtain any signing key and cannot sign arbitrary messages on behalf of the delegatee or the delegator on its own. In a bidirectional scheme, the re-signature key allows the proxy to transform a delegatee’s signature into a delegator’s signature as well as a delegator’s signature into a delegatee’s signature. In a unidirectional scheme, the re-signature key allows the proxy to transform a delegatee’s signature into a delegator’s signature, but not vice versa. Due to the transformation function, proxy re-signature schemes are very useful and can be applied in many applications, including simplifying key management, forming weak group signatures, and constructing digital rights management (DRM) interoperable system, etc. Many constructions of proxy re-signature schemes appear in the literature [2,3,4,5,6,7,8,9,10], but most of these schemes are not fast enough for many practical applications. Proxy re-signature schemes that are efficient and provably secure are interesting both from a practical and a theoretical point of view.

In this paper, we introduce a notion called divisible on-line/off-line proxy re-signatures to improve the performance of proxy re-signature schemes, and we show how to construct such proxy re-signatures from existing proxy re-signature schemes. The idea of this notion is to divide the re-signature generation algorithm into two phases. The first phase is performed off-line (without knowing the message to be re-signed) and the second phase is performed on-line (after knowing the message to be re-signed). In the off-line phase, the proxy handle the most costly computations. Once the message to be re-signed is presented, the proxy utilizes the results of the pre-computation and produces a corresponding re-signature in a very short time. At that point, the computation of the actual re-signature requires very little effort. When re-signing a message $m^\prime$, a partial re-signature of $m^\prime$ called an off-line re-signature token is first computed in the off-line phase. The remaining part of the re-signature of $m^\prime$, called the on-line re-signature token, is generated in the on-line phase when the message is known. The off-line re-signature token is allowed to be exposed in the off-line phase. In other word, a divisible on-line/off-line proxy re-signature scheme is still secure if the adversary is allowed to query the signing/re-signing

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oracle with a message depending on this message’s off-line re-signature token.

Motivations: There are two reasons to consider divisible on-line/off-line proxy re-signatures:

1. Divisible on-line/off-line proxy re-signatures could be suitable for a scenario where the proxy must respond quickly once the message to be re-signed is presented. For example, divisible on-line/off-line proxy re-signature schemes are particularly useful in smart card applications: The off-line phase is performed either during the card manufacturing process or as a background computation whenever the card is connected to power, and the on-line phase utilizes the stored results of the off-line phase to re-sign actual messages. The on-line phase is typically very fast, and hence can be executed efficiently even on a weak processor.

2. The divisible on-line/off-line proxy re-signature scheme can send the off-line re-signature token in the off-line phase instead of in the on-line phase. This reduces the on-line bandwidth of the communication channel. For example, the proxy can pre-compute a series of off-line re-signature tokens and transmits these tokens when the communication channel is not busy.

Related work: The primitive of proxy re-signature was introduced at Eurocrypt’98 by Blaze, Bleumer and Strauss [1]. They proposed the first proxy re-signature scheme that is bidirectional and multi-use. However, from the re-signature key (which is public), the delegatee can easily get the delegator’s signing key or vice versa. In 2005, Ateniese and Hohenberger [2] first formalized the definition of security for proxy re-signature, and then presented two proxy re-signature schemes. The first scheme $S_{Bi}$ is bidirectional and multi-use, and the second scheme $S_{Umb}$ is unidirectional and single-use. The security of all schemes are analyzed in the random oracle model. However, Gennaro et al. [11] point out that some schemes are proven secure in the random oracle model, but they are trivially insecure under any instantiation of the oracle. Later, Shao et al. [12] proposed a bidirectional and multi-use proxy re-signature scheme $S_{mb}$ without random oracle, and Chow and Phan [13] proposed a unidirectional and single-use proxy re-signature scheme $CP$-PRS in the standard model. Libert and Vergnaud [14] proposed a unidirectional and multi-use proxy re-signature scheme $LV$-PRS in the standard model. To our knowledge, there is no divisible on-line/off-line proxy re-signature scheme existed in the literature yet.

The basic idea of our scheme makes use of chameleon hashing to construct divisible on-line/off-line proxy re-signatures. A chameleon hash function is a special hash function endowed with a public hash key and a secret trapdoor key. The function is collision-resistant unless one knows the secret trapdoor key. Knowledge of the public hash key allows one to compute the hash function, while knowledge of the secret trapdoor key allows one to find arbitrary collisions. For many chameleon hash functions [15, 16, 17, 18, 19], the collision-finding procedure is very efficient, requiring only several modular multiplications.

Our contribution: We first explicitly give and exemplify the notion of divisible on-line/off-line proxy re-signatures. We show how to combine any proxy re-signature scheme with a specific chameleon hash function based on the discrete logarithm problem, and then present a generic divisible on-line/off-line proxy re-signature scheme, which can convert any proxy re-signature scheme into a divisible on-line/off-line proxy re-signature scheme. Our scheme is proven secure without resorting to the random oracle model. When compared to the existing proxy re-signature schemes, the proposed scheme requires less computation cost. The on-line complexity of our scheme is equivalent to two modular subtraction and two modular multiplication.

Organization: Our paper is organized as follows. We review some definitions in Section 2. The security model of the divisible on-line/off-line proxy re-signature scheme is presented in Section 3. The proposed scheme is given in Section 4. Conclusion is offered in Section 5.

2 Preliminaries

In this section, we review some background knowledge used in this paper, including negligible function, the discrete logarithm assumption and the formal definition of the proxy re-signature scheme.

Definition 1 (Negligible function). A function $\psi : \mathbb{N} \to \mathbb{R}$ is negligible if for all $\varepsilon > 0$, $\psi(k) < 1/k^\varepsilon$ for all sufficiently large $k$.

Definition 2 (Discrete logarithm assumption). Given a group $G$ of prime order $p$ with a generator $g$ and element $g^x$ where $x$ is chosen at random from $\mathbb{Z}_p^*$, the discrete logarithm problem in $G$ is to compute $x$. We say that the $(\varepsilon, t)$ discrete logarithm assumption holds in a group $G$ if no algorithm running in time at most $t$ can solve the discrete logarithm problem in $G$ with probability at least $\varepsilon$.

Definition 3 (Proxy re-signature scheme). A proxy re-signature scheme $PRS$ consists of algorithms KeyGen, ReKey, Sign, ReSign and Verify.

$\langle$KeyGen, Sign, Verify$\rangle$ form the key generation, signing and verification algorithms of a standard signature scheme.

$rk_{A \rightarrow B} \leftarrow$ ReKey$(sk_A, sk_B, pk_A, pk_B)$ is the re-signature key generation algorithm. On input public parameters $cp$, an (optional) delegatee’s private key $sk_A$, a delegator’s private key $sk_B$, and the corresponding public keys $(pk_A, pk_B)$, this algorithm outputs a re-signature key $rk_{A \rightarrow B}$ for the proxy to convert delegatee’s signatures into delegator’s signatures. If the input $sk_A$ is mandatory, the scheme is interactive.
–$\sigma^B \leftarrow \text{ReSign}(r_{KA}\rightarrow B, pk_A, m, \sigma_A)$ is the re-signing algorithm. On input public parameters $cp$, a re-signature key $r_{KA}\rightarrow B$, a delegatee’s public key $pk_A$, a message $m$ and a signature $\sigma_A$, this algorithm outputs a re-signature $\sigma^B$ if $\text{Verify}(pk_A, m, \sigma_A) = 1$ and outputs ⊥ otherwise.

### 3 Security model and security notions

We extend the definition of a proxy re-signature scheme to define a divisible on-line/off-line proxy re-signature scheme.

**Definition 4 (Divisible on-line/off-line proxy re-signature scheme).** A divisible on-line/off-line proxy re-signature scheme DOPRS = ($\text{KeyGen}$, $\text{ReKey}^{\text{on/off}}$, $\text{Sign}$, $\text{ReSign}^{\text{off}}$, $\text{ReSign}^{\text{on}}$, $\text{Ver}$) consists of the following algorithms:

–$(sk, pk) \leftarrow \text{KeyGen}(1^i)$ is the key generation algorithm. On input a security parameter $k \in \mathbb{N}$, this algorithm outputs signer’s secret/public key pair $(sk, pk)$.

–$RSK_{A\rightarrow B} \leftarrow \text{ReKey}^{\text{on/off}}(sk_A, sk_B, pk_A, pk_B)$ is the re-signature key generation algorithm. On input global parameters $params$, an (optional) delegatee’s private key $sk_A$, a delegator’s private key $sk_B$, and the corresponding public keys $(pk_A, pk_B)$, this algorithm outputs a re-signature key $RSK_{A\rightarrow B}$ for the proxy to convert delegator’s signatures into delegatee’s signatures.

–$\sigma \leftarrow \text{Sign}(sk, m)$ is the signing algorithm. On input global parameters $params$, a secret key $sk$ and a message $m$, this algorithm outputs a signature $\sigma$ on the message $m$.

–$(\sigma^{off}_B, St) \leftarrow \text{ReSign}^{\text{off}}(RSK_{A\rightarrow B}, pk_A, m)$ is the off-line re-signing algorithm. On input global parameters $params$, a delegator’s public key $pk_A$, a delegator’s public key $pk_B$ and a re-signature key $RSK_{A\rightarrow B}$, this algorithm outputs a (public) off-line re-signature token $\sigma^{off}_B$ and a (secret) state information $St$. The state information is kept secret and will be transmitted to the execution of the on-line re-signing algorithm.

–$\sigma^{on}_B \leftarrow \text{ReSign}^{\text{on}}(RSK_{A\rightarrow B}, St, pk_A, m, \sigma_A)$ is the on-line re-signing algorithm. On input global parameters $params$, a re-signature key $RSK_{A\rightarrow B}$, a state information $St$, a delegator’s public key $pk_A$, a message $m$ and a signature $\sigma_A$, this algorithm first checks that $\sigma_A$ is valid w.r.t. $pk_A$. It outputs an on-line re-signature token $\sigma^{on}_B$ if $\text{Ver}(pk_A, m, \sigma_B) = 1$ and ⊥ otherwise. The re-signature for $m$ is defined as $\sigma_B = (\sigma^{off}_B, \sigma^{on}_B)$ which is verified under a delegator’s public key $pk_B$.

–$0/1 \leftarrow \text{Verify}(pk, m, \sigma)$ is the verification algorithm. On input global parameters $params$, a public key $pk$, a message $m$ and a purported signature $\sigma$, this algorithm outputs 1 if $\sigma$ is a valid signature under $pk$ and 0 otherwise.

**Completeness:** For any message $m$ in the message space, the following two conditions must hold:

$\text{Ver}(pk_A, m, \sigma_A) = 1$ and $\text{Ver}(pk_B, m, \sigma_B) = 1$,

where $\sigma_B = (\sigma^{off}_B, \sigma^{on}_B)$.

**Remark 1:** In a divisible on-line/off-line proxy re-signature scheme, a signature manifests in two types: the original signature and the re-signature. An original signature can be computed only by the owner of the signing key, while a re-signature can be computed not only by the owner of the signing key, but also by collaboration between his proxy and delegatee. Namely, an original signature is one which is only outputted by the algorithm $\text{Sign}$.

**Remark 2:** $\text{ReSign}^{\text{off}}$ and $\text{ReSign}^{\text{on}}$ can be viewed as sub-algorithms of a complete re-signing algorithm. For simplicity, we use the notation $(\sigma^{off}_B, \sigma^{on}_B) \leftarrow \text{ReSign}^{\text{off}}(RSK_{A\rightarrow B}, pk_A, m)$ to denote such a complete re-signing process: $(\sigma^{off}_B, St) \leftarrow \text{ReSign}^{\text{off}}(RSK_{A\rightarrow B}, pk_A, m)$ and $(\sigma^{on}_B) \leftarrow \text{ReSign}^{\text{on}}(RSK_{A\rightarrow B}, St, pk_A, m, \sigma_A)$.

In the following, we define a security model for divisible on-line/off-line proxy re-signature schemes, which is an extension of the security model for proxy re-signature schemes presented by Ateniese and Hohenberger [2], and Shao et al. [20]. The major difference between two models is that the attacker in the new model is allowed to adaptively select the query messages depended on their off-line re-signature tokens.

In the security model of a divisible on-line/off-line proxy re-signature scheme, the adversary has obtained the off-line re-signature token of a message before he queries this message. The adversary $\mathcal{A}$ is allowed to make queries to an off-line re-signing oracle $\mathcal{O}_{\text{ReSign}^{\text{off}}}$ and an on-line re-signing oracle $\mathcal{O}_{\text{ReSign}^{\text{on}}}$.

We assume that if $\mathcal{A}$ makes the $i$-th on-line re-signature query, then $\mathcal{A}$ has already made the $i$-th off-line re-signature query. The security model for divisible on-line/off-line proxy re-signature schemes contains two aspects: the external security and the internal security. The details are as follows.

**External Security:** This security notion deals with adversaries outside the system (i.e., neither the proxy nor the delegation parties). A divisible on-line/off-line proxy re-signature scheme is said to be external-secure if for the security parameter $k$, any non-zero $n$, and all probabilistic polynomial-time algorithms $\mathcal{A}$, the following probability is negligible:

$Pr\{\{sk_i\}_{i \in \{1,...,n\}} \leftarrow \text{KeyGen}(1^i)\} \cdot Pr[\mathcal{A}((i^*, m^*, \sigma^*) \leftarrow \mathcal{O}_{\text{Sign}(\cdot, \cdot)}(\cdot, \cdot) \cdot \mathcal{O}_{\text{ReSign}^{\text{off}}}(\cdot) \cdot \mathcal{O}_{\text{ReSign}^{\text{on}}}(\cdot, \cdot, \cdot)\{pk_i\}_{i \in \{1,...,n\}}):\mathcal{V er}(pk_{i^*}, m^*, \sigma^*) = 1 \land (i^*, m^*) \notin \mathcal{Z}]]$
of $\sigma \leftarrow \text{Sign}(sk_i, m)$; the off-line re-signing oracle $\mathcal{O}_{\text{ReSign}}(\cdot, \cdot)$ takes as input two distinct indices $i, j \in \{1, \ldots, n\}$, and returns the output of $(\sigma_{ij}^{\text{off}}, St) \leftarrow \text{ReSign}^{\text{off}}(\text{ReKey}^{\text{on/off}}(sk_i, sk_j, pk_i, pk_j), pk_i, pk_j)$; the on-line re-signing oracle $\mathcal{O}_{\text{ReSign}}^{\text{on}}(\cdot, \cdot)$ takes as input two distinct indices $i, j \in \{1, \ldots, n\}$, a message $m$ and a signature $\sigma$, this oracle first retrieves the state information $St$ from the memory, then returns the output of $\sigma_i^{\text{on}} \leftarrow \text{ReSign}^{\text{on}}(\text{ReKey}^{\text{on/off}}(sk_i, sk_j, pk_i, pk_j), St, pk_i, m, \sigma_i)$; and $\mathcal{D}$ is defined as the set of (index, messages) pairs $(i, m)$ queried the oracle $\mathcal{O}_{\text{Sign}}(\cdot)$ or $\mathcal{O}_{\text{ReSign}}^{\text{on}}(\cdot, \cdot, \cdot)$. This security notion only makes sense if the re-signing key is kept private by the proxy.

Internal Security: This security notion protects a user, as much as possible, from adversaries inside the system, such as dishonest proxy and colluding delegation partners. It can be classified into the following three types:

1. Limited Proxy Security: This security notion guarantees that the proxy cannot sign messages on behalf of the delegatee or produce signatures for the delegator unless messages were first signed by the latter's delegates. Limited proxy security is very similar to external security described above except that the adversary queries a re-signature key generation oracle $\mathcal{O}_{\text{ReKey}^{\text{on/off}}}$ instead of the re-signing oracle (including $\mathcal{O}_{\text{ReSign}}^{\text{on}}$ and $\mathcal{O}_{\text{ReSign}}^{\text{on/off}}$). A divisible on-line/off-line proxy re-signature scheme is said to be limited-proxy-secure if for the security parameter $k$, any non-zero $n$, and all probabilistic polynomial-time algorithms $A'$, the following probability is negligible:

$$\Pr[(pk_i, sk_i) \leftarrow \text{KeyGen}(1^k)_{i \in \{0, \ldots, n\}}, (i^*, m^*, \sigma^*) \leftarrow A'_{\text{Sign}}(\cdot), \mathcal{O}_{\text{ReKey}^{\text{on/off}}}(\cdot)_{i \in \{0, \ldots, n\}}, (pk_i)_{i \in \{1, \ldots, n\}}) :$$

$$\text{Ver}(pk_k, m^*, \sigma^*) = 1 \land (i^*, m^*) \notin \mathcal{D}$$

where $\mathcal{O}_{\text{Sign}}(\cdot)$ is the same as that in external security; the re-signature key generation oracle $\mathcal{O}_{\text{ReKey}^{\text{on/off}}}(i, j)$ takes as input two distinct indices $i, j \in \{1, \ldots, n\}$ and returns the output of $\text{RSK}_{i \rightarrow j} \leftarrow \text{ReKey}^{\text{on/off}}(sk_i, sk_j, pk_i, pk_j)$; and $\mathcal{D}$ is the set including any message $m$ corresponding to which $A'$ has queried $\mathcal{O}_{\text{Sign}}(\cdot)$ for $\sigma = i^*$ or any $\sigma$ where $\mathcal{O}_{\text{ReKey}^{\text{on/off}}}(\cdot, \sigma)$ has been queried.

2. Delegatee Security: This notion protects the delegatee from a collusion between the delegator and the proxy. Delegatee security guarantees that their collusion cannot generate any signatures on behalf of the delegatee. Namely, the delegatee is assigned the index 0. A divisible on-line/off-line proxy re-signature scheme is said to be delegatee-secure if for the security parameter $k$, any non-zero $n$, and all probabilistic polynomial-time algorithms $A'$, the following probability is negligible:

$$\Pr[(pk_i, sk_i) \leftarrow \text{KeyGen}(1^k)_{i \in \{0, \ldots, n\}}, (m^*, \sigma^*) \leftarrow A'_{\text{Sign}}(\cdot), \mathcal{O}_{\text{ReKey}^{\text{on/off}}}(\cdot)_{i \in \{0, \ldots, n\}}, (pk_i)_{i \in \{1, \ldots, n\}}) :$$

$$\text{Ver}(pk_k, m^*, \sigma^*) = 1 \land (m^*, \sigma^*) \notin \mathcal{D}$$

where $\mathcal{O}_{\text{Sign}}(\cdot)$ is an original signature, $\mathcal{O}_{\text{Sign}}(\cdot)$ is the same as that in external security, $\mathcal{O}_{\text{ReKey}^{\text{on/off}}}(\cdot, \cdot)$ is the same as that in limited proxy security, and $\mathcal{D}_{\text{attr}}$ is the set of messages $m$ queried to $\mathcal{O}_{\text{Sign}}(0, m)$.

Since the delegatee and the delegator mutually delegate in a bidirectional scheme, the properties of delegatee security and delegator security do not apply. We give the following security notions.

Definition 5. A bidirectional divisible on-line/off-line proxy re-signature scheme is secure if it is external-secure and limited-proxy-secure.

Definition 6. A unidirectional divisible on-line/off-line proxy re-signature scheme is secure if it is external-secure, limited-proxy-secure, delegatee-secure, and delegator-secure.

4 A divisible on-line/off-line proxy re-signature scheme

In this section, we present a generic divisible on-line/off-line proxy re-signature scheme, and show how the existing proxy re-signature scheme can be used in performing the computation of the off-line proxy re-signature token. The proposed scheme is proven secure without random oracles. We assume that the message space is $\mathbb{Z}_q$. Note that using a collision-resistant hash function $H: \{0, 1\}^* \rightarrow \mathbb{Z}_q$, one can extend the message domain to $\{0, 1\}^*$. 

4.1 Construction

Let $\text{PRS}=(\text{KeyGen}, \text{ReKey}, \text{Sign}, \text{ReSign}, \text{Verify})$ be a proxy re-signature scheme. The resulting divisible
on-line/off-line proxy re-signature scheme is defined as

\[
\text{DOPRS} = (\text{KeyGen}, \text{ReKey}^{\text{on/off}}, \text{Sign}, \text{ReSign}^{\text{off}}, \text{ReSign}^{\text{on}}, \text{Ver}),
\]

- **KeyGen**: Choose two primes \( p, q \) such that \( q | p - 1 \), where \( p \)'s length depends on the security parameter \( k \in \mathbb{N} \). Pick an element \( g \) of order \( q \) in \( \mathbb{Z}_p^* \). Let \( G \) be the subgroup of \( \mathbb{Z}_p^* \) generated by \( g \). Run the key generation algorithm **KeyGen** of the original proxy re-signature scheme \( \text{PRS} \) to generate signer's secret/public key pair \((sk, pk)\). Let \( c_p \) be the set of the public parameters of \( \text{PRS} \). The proxy chooses at random \( y, z \in \mathbb{Z}_q^* \), and computes \( h_1 = g^y, h_2 = g^z \) and \( Y = y^{-1} \mod q \). The set of the public parameters is

\[
\text{params} = \{c_p, p, q, g, h_1, h_2\}.
\]

- **ReKey\(^{\text{on/off}}\)**: On input an (optional) delegatee's private key \( sk_A \), a delegator's private key \( sk_B \), and the corresponding public keys \((pk_A, pk_B)\), the proxy first runs the re-signature key generation algorithm **ReKey** of \( \text{PRS} \) with \((sk_A, sk_B, pk_A, pk_B)\) to obtain a re-signature key \( rk_{A-B} \) for \( \text{PRS} \). The proxy then retains \( \text{RSK}_{A-B} = (rk_{A-B}, y, z, Y) \) as its own local re-signature key for **DOPRS**.

- **Sign**: On input a secret signing key \( sk \) and a message \( m \), it runs the signing algorithm **Sign** of \( \text{PRS} \) with \( sk \) to obtain a signature \( \sigma = \text{Sign}(sk, m) \).

- **ReSign\(^{\text{off}}\)**: In order to generate an off-line re-signature token \( \sigma_B^{\text{off}} \), this algorithm runs as follows:

  1. The proxy chooses at random \( m, r, s, t \in \mathbb{Z}_q^* \) and sends \( \text{Com} = g^m h_1^r h_2^s \) to a delegatee.
  2. The delegatee runs the algorithm **Sign** of **PRS** with \( sk_A \) to compute a signature \( \sigma_A \) on the message \( \text{Com} \), and sends \( \sigma_A \) to the proxy.
  3. The proxy checks \( \text{Verify}(pk_A, \text{Com}, \sigma_A) \neq 1 \). If the equation holds, the proxy runs the re-signing algorithm **ReSign** of **PRS** with \((rk_{A-B}, \text{Com}, \sigma_A)\) to obtain a re-signature \( \sigma_B \) on \( \text{Com} \).
  4. The proxy computes \( t = m + r + (-s \cdot z) \mod q \), and stores the state information \( St = t \). The off-line re-signature token \( \sigma_B^{\text{off}} = (pk_A, \sigma_B) \).

- **ReSign\(^{\text{on}}\)**: On input a re-signature key \( \text{RSK}_{A-B} \), a delegatee’s public key \( pk_A \), a message \( m' \) and a signature \( \sigma_A' \), the proxy first retrieves the state information \( St \) from the memory, and then computes \( r' = (St - m') \cdot Y \mod q \). The on-line re-signature token on \( m' \) is \( \sigma_B^{\text{on}} = (r', s, \sigma_A) \). The re-signature for \( m' \) is given by \( \sigma_B = (\sigma_B^{\text{off}} : \sigma_B^{\text{on}}) = (pk_A, \sigma_B, r', s, \sigma_A') \).

- **Ver**: Given a public key \( pk \), a message \( m' \) and a purported signature \( \sigma \), the verification algorithm performs as follows:

  - If \( \sigma \) is an original signature under \( pk \), it runs the verification algorithm **Verify** of **PRS** to check that \( \text{Verify}(pk, m', \sigma) \neq 1 \). If this equation holds, it outputs 1; otherwise, it outputs 0.

- If \( \sigma \) is a re-signature \( \sigma = (\sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4) \) under \( pk \), it checks that

\[
\text{Verify}(pk, g^m h_1^r h_2^s, \sigma_4) = 1\quad \text{and} \quad \text{Verify}(\sigma_0, m', \sigma_4) = 1.
\]

If the above two equations both hold, it outputs 1; otherwise, it outputs 0.

**Correctness**: Let \( \sigma'_A \) be an original signature on a message \( m' \) corresponding to a delegatee’s public key \( pk_A \) and \( \sigma_B \) be a re-signature on \( \text{Com} = g^m h_1^r h_2^s \) under a delegatee’s public key \( pk_B \). Let \( \sigma_B' = (\sigma_B^{\text{off}}, \sigma_B^{\text{on}}) = (pk_A, \sigma_B, r', s, \sigma_A') \) be a re-signature on the same message \( m' \) under \( pk_B \). The proposed scheme has the correctness due to the following equations.

\[
g^m h_1^r h_2^s = g^m (g^y)^r (g^z)^s = g^{m' + y'r' + z's'}
\]

\[
= g^{m' + y(St - m')Y + z's'} = g^{m' + (r - m')y + z's'}
\]

\[
= g^{m' + m + r + (-s \cdot z) - m' + z's'} = g^{m' + r + z's'} = \text{Com}
\]

\[
\text{Verify}(pk_B, g^m h_1^r h_2^s, \sigma_B^{\text{on}} = \sigma_B) = 1
\]

\[
\text{Verify}(\sigma_B, m', \sigma_A') = 1
\]

**Remark 3**: **Sign, ReSign\(^{\text{off}}\)**, ReSign\(^{\text{on}}\) and **Ver** are performed per message.

**Remark 4**: To reduce the computation cost of the on-line re-signing algorithm, we move the validity of the original signature \( \sigma'_A \) to the verification algorithm **Ver**. As a result, \( \sigma'_A \) is attached as part of the on-line re-signature token. To save the on-line bandwidth, the transmission of the whole re-signature \( \sigma_B = (\sigma_B^{\text{off}}, \sigma_B^{\text{on}}) \) is broken into two stages. The on-line re-signature token \( \sigma_B^{\text{on}} = (pk_A, \sigma_B, \sigma_A') \) on a message \( m' \) is transmitted to the recipient at the end of the on-line phase, while the off-line re-signature token \( \sigma_B^{\text{off}} = (pk_B, \sigma_B) \) is transmitted in the off-line phase.

**Remark 5**: If the underlying proxy re-signature scheme **PRS** is bidirectional, the re-signature key \( rk_{A-B} \) can be used to transform delegatee’s signatures into delegator’s or vice versa. If the underlying scheme **PRS** is unidirectional, \( rk_{A-B} \) can be used to transform delegatee’s signatures into delegator’s, but not vice versa. Moreover, \((y, z, Y)\) are three secret trapdoor keys of the chameleon hash function \( g^m h_1^r h_2^s \). Hence, the proposed divisible on-line/off-line proxy re-signature scheme **DOPRS** with \( \text{RSK}_{A-B} = (rk_{A-B}, y, z, Y) \) is bidirectional (resp. unidirectional) if so is its underlying proxy re-signature scheme **PRS**.

### 4.2 Security

The following theorem shows that the security of the proposed on-line/off-line proxy re-signature
scheme is reduced to the security of its underlying proxy re-signature scheme together with computational infeasibility of the discrete logarithm assumption. In essence, two schemes hold the same security properties. For example, if an underlying bidirectional proxy re-signature scheme is external-secure and limited-proxy-secure, the resulting divisible on-line/off-line proxy re-signature scheme is also external-secure and limited-proxy-secure.

Theorem 1Assuming that PRS=(KeyGen, ReKey, Sign, ReSign, Verify) is a secure proxy re-signature scheme and the discrete logarithm assumption holds, our divisible on-line/off-line proxy re-signature scheme DOPRS = (KeyGen, ReKey^on/off, Sign, ReSign^off, ReSign^on, Ver) is also secure.

Proof. We prove this theorem by contradiction. Assume that there exists an adversary A that breaks the unforgeability of our scheme DOPRS and we show how to exploit it to break either the unforgeability of the underlying proxy re-signature scheme PRS or the discrete logarithm assumption. In other words, we can construct an efficient algorithm B that, using A as a black box, succeeds in above mentioned tasks.

Without loss of generality, we assume that A makes $q_s$ signature queries, $q_{off}$ off-line re-signature queries and $q_{on}$ on-line re-signature queries on messages $\{m_i\}_{i=1}^{q_{on}}$, where $q_{on} \leq q_{off}$ and $q_{on} \leq q_s$. Let $\{(\sigma_0^i, \sigma_1^i, r^i, s^i, \sigma^A_i)\}_{i=1}^{q_{on}}$ be $q_{on}$ full re-signatures returned by the re-signing oracle (including the off-line and on-line re-signing oracles). Finally, A outputs a valid forgery $\sigma^*$ on a new message $m^*$ with probability $\epsilon$. If $\sigma^*$ is an original signature, then $\sigma^*$ is also a valid forgery of the underlying proxy re-signature scheme PRS. Hence, for simplicity, we assume that $\sigma^*=(\sigma_0^*, \sigma_1^*, r^*, s^*, \sigma^A_*)$ is a re-signature. Notice that, any valid forgery must be one of the following types:

Case 1: $g^{m^*}h_1^{r^*}h_2^{s^*} \neq g^{m_i}h_1^{r^i}h_2^{s^i}$ for all $i \in \{1, \ldots, q_{on}\}$.

Case 2: $g^{m^*}h_1^{r^*}h_2^{s^*} = g^{m_i}h_1^{r^i}h_2^{s^i}$ for some $i \in \{1, \ldots, q_{on}\}$ and $r^* \neq r^i$.

Case 3: $g^{m^*}h_1^{r^*}h_2^{s^*} = g^{m_i}h_1^{r^i}h_2^{s^i}$ for some $i \in \{1, \ldots, q_{on}\}$, $r^* = r^i$ and $s^* \neq s^i$.

If the first case holds, we build an algorithm B against the unforgeability of PRS with probability at least $\epsilon/3$. B is given the set $cp$ of the public parameters of PRS. B’s goal is to use the forgery produced by the adversary A to contradict the existential unforgeability of PRS. B works as follows:

Setup: B chooses two random numbers $y, z \in \mathbb{Z}_p^*$ and computes $h_1 = g^y, h_2 = g^z$. The public key of the target delegate is defined as $pk_A$ and the public key of the target delegator is defined as $pk_B$. Then, B sends $\{cp, p, q, g, pk_A, pk_B, h_1, h_2\} \rightarrow A$. When A issues a key generation query, B first forward this query to the challenger of PRS; the challenger returns the resulting public key or secret/public key pair to B; B then sends it to A.

Queries: B builds the following oracles:

- Re-signature key generation oracle $\mathcal{O}_{ReKey^{on/off}}$: When A issues a re-signature key generation query which consists of two public keys $(pk_i, pk_j)$, B submits the query $(pk_i, pk_j)$ to the re-signature key generation oracle of PRS to obtain a corresponding re-signature key $rk_{i\leftarrow j}$, and then B returns a re-signature key $RSK_{i\leftarrow j} = (rk_{i\leftarrow j}, y, z, z^{-1}) \rightarrow A$.

- Signature oracle $\mathcal{O}_{Sign}$: When A issues a signing query which consists of a public key $pk_i$ and a message $m_i$, B submits the query $(pk_i, m_i)$ to the signing oracle of PRS to obtain a corresponding signature $\sigma$ and sends $\sigma \rightarrow A$.

- Off-line re-signature oracle $\mathcal{O}_{ReSign^{off}}$: On input $(pk_j, pk_k)$, B first chooses two random numbers $r_i, s_i \in \mathbb{Z}_p^*$ and computes $Com_i = g^{m_i}h_1^{r_i}h_2^{s_i}$. B then queries the signing oracle of PRS to get a signature $\sigma_{A,i}$ on $Com_i$ under the public key $pk_k$. Furthermore, B queries the re-signing oracle of PRS providing $(pk_i, pk_j, Com_i, \sigma_{A,i})$ as input and obtains a resulting re-signature $\sigma_{B,i}$. Finally, B returns an off-line re-signature token $\sigma_{j\leftarrow i}^{off} = (pk_i, \sigma_{A,i}) \rightarrow A$ and stores the secret state information $(m_i, r_i, s_i, \sigma_{A,i})$.

- On-line re-signature oracle $\mathcal{O}_{ReSign^{on}}$: On input $(pk_j, pk_i, m_i, \sigma_{A,i}^*)$, B outputs $\bot$ if $\mathcal{O}(pk_i, m_i, \sigma_{A,i}) = 0$. Otherwise, B computes $r_i = r_i + (m_i + z_s - m_i' - z_s') \mod q$, and then returns an on-line re-signature token $\sigma_{j\leftarrow i}^{on} = (r_i, s_i, \sigma_{A,i}^*)$. It can be verified that $(pk_i, \sigma_{B,i}, r_i, s_i, \sigma_{A,i}^*)$ is a valid signature on the message $m_i$ under the public key $pk_j$.

Forgery: The simulated oracles are perfectly indistinguishable from the real ones for A. At the end, B outputs a valid forgery $\sigma^*=(\sigma_0^*, \sigma_1^*, r^*, s^*, \sigma^A_*)$ on a message $m^*$ satisfying the condition in Case 1. This means that $m^*$ was never queried by B to the signing oracle and the re-signing oracle of PRS. But in this case, B has forged a signature $\sigma_{A,i}^*$ on a message $g^{m_i}h_1^{r_i}h_2^{s_i}$ with respect to the underlying proxy re-signature scheme PRS. Thus, B succeeds in breaking the unforgeability of the underlying proxy re-signature scheme.

[CASE 2]}

If the second case holds, we build an algorithm B that breaks the discrete logarithm assumption. To do so, B is given a couple $(g, h) \in G^2$. His goal is to use the forgery produced by A to compute the discrete log of $h$ in base $g$. B works as follows:

Setup: B chooses two random elements $\hat{y}, z \in \mathbb{Z}_p^*$, and sets $h_1 = h, h_2 = g^z$. The other public parameters are same as those in the first case. Note that $\hat{y}$ differs from $y$. Then, B sends $\{cp, p, q, g, pk_A, pk_B, h_1, h_2\} \rightarrow A$. 
Queries: \( \mathcal{B} \) builds the following oracles:

- \( O_{Sign} \) is the same as that in the first case.

- Re-signature key generation oracle \( O_{ReKey^{on/off}} \): When \( \mathcal{A} \) issues a re-signature key generation query which consists of two public keys \((pk_i, pk_j)\), \( \mathcal{B} \) submits the query \((pk_i, pk_j)\) to the re-signature key generation oracle of PRS to obtain a corresponding re-signature key \( rk_{i\rightarrow j} \), and then \( \mathcal{B} \) returns a re-signature key \( RSK_{i\rightarrow j} = (rk_{i\rightarrow j}, y, z, z^{-1}) \) to \( \mathcal{A} \).

- Off-line re-signature oracle \( O_{ReSign^{off}} \): On input \((pk_i, pk_j)\), \( \mathcal{B} \) first chooses \( m_i, r_i, s_i \in \mathbb{Z}_q \), sets \( s'_i = (s_i + (m - m')z^{-1})(\text{mod } q) \), and computes \( Com_i = g^m h_{i1}^r h_{i2}^s \). \( \mathcal{B} \) then queries the signing oracle of PRS to get a signature \( \sigma_{A_i} \) on \( Com_i \) under the public key \( pk_i \). Furthermore, \( \mathcal{B} \) queries the re-signing oracle of PRS providing \((pk_i, pk_j, Com_i, \sigma_{A_i})\) as input and obtains a resulting re-signature \( \sigma_{o_{i}} \). Finally, \( \mathcal{B} \) returns an off-line re-signature token \( \sigma^{off}_i = (pk_i, \sigma_{B_i}) \) to \( \mathcal{A} \) and stores the secret state information \((m_i, r_i, s_i, s'_i)\).

- On-line re-signature oracle \( O_{ReSign^{on}} \): On input \((pk_i, pk_j, m_i, s_i)\), \( \mathcal{B} \) outputs \( \perp \) if \( \text{Ver}(pk_i, m_i, \sigma'_i) = 0 \). Otherwise, \( \mathcal{B} \) computes \( r'_i = r_i \), and then returns an on-line re-signature token \( \sigma^{on}_i = (r'_i, s'_i, \sigma_i) \). It can be verified that \((pk_i, \sigma_{B_i}, r'_i, s'_i, \sigma_i)\) is a valid signature on the message \( m_i \) under the public key \( pk_j \).

Forgery: From \( \mathcal{A} \)'s view, the simulated oracles are indistinguishable from the real ones. If \( \mathcal{A} \) outputs a valid forgery \( \sigma^{*}=(\sigma_0^*, \sigma_1^*, r^*, s^*, \sigma_4^*) \) on a message \( m^* \) satisfying the condition in Case 2, then \( g^{m^*} h_{1}^{r^*} h_{2}^{s^*} \) and \( r^* \neq r_i' \) for some \( i \) hold. \( \mathcal{A} \) can check to find this \( i \) and obtain the discrete log \( dl_i h = ((m_i - m^*) + (s_i - s^*)z)(r^* - r_i')^{-1} \text{ (mod } q) \). Thus, \( \mathcal{B} \) can successfully solve the received discrete logarithm instance.

[CASE 3.]

If the third case holds, we construct an algorithm \( \mathcal{B} \) that breaks the discrete logarithm assumption. To do so, \( \mathcal{B} \) is given a couple \((g, h) \in G^2\). \( \mathcal{B} \)'s goal is to compute the discrete log \( dl_i h \) of \( h \) in base \( g \). We focus on describing the differences with the second case.

Setup: \( \mathcal{B} \) chooses two random elements \( y, z \in \mathbb{Z}_p^* \), and sets \( h_1 = g^y, h_2 = h \). Thus, \( \mathcal{B} \) knows \( y \) and a value \( z \) that differs from \( z \).

Queries: \( \mathcal{B} \) builds the following oracles:

- \( O_{ReKey^{on/off}} \) and \( O_{Sign} \) are identical to those in Case 2, by just switching the roles of \( z \) and \( y \).

- Off-line re-signature oracle \( O_{ReSign^{off}} \): On input \((pk_i, pk_j)\), \( \mathcal{B} \) chooses \( m_i, r_i, s_i \in \mathbb{Z}_q \), sets \( s'_i = s_i \), and computes \( Com_i = g^{m_i} h_{i1}^{r_i} h_{i2}^{s_i} \). The rest of the oracle is done exactly as in Case 2.

| Table 1 Comparison of computational complexity among proxy re-signature schemes |
|---------------------------------|----------------|----------------|----------------|----------------|
| S_M                             | Subtractions | Multiplications | Exponentiations | Pairings       |
| 0                               | 0             | 1              | 0              | 2              |
| S_uni                           | 0             | 1              | 3              | 2              |
| S_Nab                           | 0             | 0              | 2              | 3              |
| CP – PRS                        | 0             | 2              | 3              | 2              |
| LV – PRS                        | 0             | 6              | 3              |                |
| Our DOPRS                       | 1             | 1              | 0              | 0              |

- On-line re-signature oracle \( O_{ReSign^{on}} \): On input \((pk_i, pk_j, m_i, \sigma_i^*)\), \( \mathcal{B} \) outputs \( \perp \) if \( \text{Ver}(pk_i, m_i, \sigma_i^*) = 0 \). Otherwise, \( \mathcal{B} \) computes \( r'_i = r_i + (m_i - m_i')y^{-1} \text{ (mod } q) \), and then returns an on-line re-signature token \( \sigma^{on}_i = (r'_i, s_i') \).

Forgery: If \( \mathcal{A} \) outputs a valid forgery \( \sigma^{*}=(\sigma_0^*, \sigma_1^*, r^*, s^*, \sigma_4^*) \) on a message \( m^* \) satisfying the condition in Case 3, then \( g^{m^*} h_{1}^{r^*} h_{2}^{s^*} \) and \( s^* \neq s_i' \) for some \( i \) hold. \( \mathcal{B} \) then can easily get the discrete log \( dl_i h = ((m_i - m^*) + (r_i - r_i')y)(s^* - s_i')^{-1} \text{ (mod } q) \). Therefore, \( \mathcal{B} \) succeeds in breaking the received discrete logarithm challenge.

4.3 Comparison

We compare our divisible on-line/off-line proxy re-signature scheme with some known proxy re-signature schemes. We mainly analyze bit complexity of all schemes required by a proxy in computing a re-signature when a message to be re-signed arrives, and show the results in Table 1. However, schemes S_M and S_uni presented by Ateniese and Hohenberger [2], scheme S_Nab proposed by Shao et al. [12], scheme CP – PRS proposed by Chow and Phan [13], and scheme LV – PRS presented by Libert and Vergnaud [14] are not considered to be on-line/off-line proxy re-signature schemes because no pre-computation is performed. To achieve the same security level, we can set the modular parameter \( p \) in all schemes to be the same size.

From Table 1, we can see that the proxy performs one modular subtraction and one modular multiplication when computing a re-signature in our scheme. This is very efficient and comparable to the other schemes, since modular subtraction and modular multiplication are negligible when compared with exponentiation and pairing. Our scheme makes a tradeoff by incurring a large cost in the off-line phase to obtain a quick on-line phase.

5 Conclusion

We introduce a notion called divisible on-line/off-line proxy re-signatures, in which the off-line re-signature tokens can be sent to the recipient before the messages to
be re-signed are seen. We also present an efficient generic construction, and prove its security without random oracles.

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