Estimation of Stress-Strength Reliability Model Using Finite Mixture of Two Parameter Lindley Distributions

Adil H. Khan* and T. R. Jan

Department of Statistics, University of Kashmir, Srinagar 190006, India

Received: 14 Oct. 2014, Revised: 5 Dec. 2014, Accepted: 8 Dec. 2014
Published online: 1 Mar. 2015

Abstract: The term “stress-strength reliability” when it appears in statistical literature typically refers to the quantity $P(X > Y)$, where a system with random strength $X$ is subjected to a random stress $Y$ such that a system fails, if the stress exceeds the strength. Stress–Strength reliability is considered in this paper where the strength follows finite mixture of two parameter Lindley distribution and stress follows exponential, Lindley distribution and mixture of two parameter Lindley distribution. The general expressions for the reliabilities of a system are obtained. Estimates of parameters are obtained by maximum likelihood estimation method. For different values of stress and strength parameters, reliability has been obtained. Special cases are also discussed.

Key Words: Lindley distribution, Exponential distribution, Reliability function and Maximum likelihood estimation.

1 Introduction

The problem of increasing reliability of any system become more significant in many fields of industry, transport, communications technology, etc, with the complex mechanization and automation of industrial processes. Underestimation and overestimation of factors associated with reliability may engender great losses. The term "stress-strength reliability" when it appears in statistical literature, typically refers to the quantity $R=P(Y < X)$, where a system with random strength $X$ is subjected to a random stress $Y$ such that the system fails if the stress exceeds the strength. The term stress-strength was first introduced by Church and Harris. Some authors have considered different choices for stress and strength distributions. The stress-strength reliability and its estimation problems for several distributions are discussed in the works of Church and Harris [8], Woodward and Kelley [18], Beg and Singh [14], Awad and Gharraf [1], Surles and Padgett [6,7], Raqab and Kundu [13], Mokhlis[15], and Saraçoğlu et al. [3]. Recently, Kotz et al. [17] have presented a review of all methods and results on the stress-strength reliability in the last four decades. Adil H. khan and T.R Jan[2]obtained Bayes estimators of the parameters of the Consul, Geeta and Size-biased Geeta distributions and associated reliability function.
A. Khan, T. Jan: Estimation of Stress-Strength Reliability

Lindley [4] proposed “Lindley distribution” (LD) in the context of Bayesian statistics, as a counterexample of fiducial statistics. Lindley distribution belongs to an exponential family and it can be written as a mixture of an exponential and a gamma distribution with shape parameter 2. Therefore, many properties of the mixture distribution can be translated for the Lindley distribution. The Lindley distribution, in spite of little attention in the statistical literature, is important for studying stress-strength reliability modeling. Besides, some researchers have proposed new classes of distributions based on modifications of the Lindley distribution, including also their properties. Sankaran [12] introduced the discrete Poisson-Lindley distribution by combining the Poisson and Lindley distributions. Adamidis and Loukas [9] introduced a two-parameter lifetime distribution with decreasing failure rate by compounding exponential and geometric distributions, which was named exponential geometric (EG) distribution. Ghitany et al. [11] investigated most of the statistical properties of the Lindley distribution, showing this distribution may provide a better fitting than the exponential distribution. Bakouch et al. [5] introduced a new extension of the Lindley distribution, called extended Lindley (EL) distribution, which offers a more flexible model for lifetime data.

A two-parameter Lindley distribution with parameters $\lambda$ and $\alpha$ is defined by its probability density function (p.d.f.)

$$f(x; \lambda, \alpha) = \frac{\lambda^2}{\lambda + \alpha} (1 + \alpha x) e^{-\lambda x} ; \quad x > 0, \lambda > 0, \alpha > -\lambda$$

(1)

It can easily be seen that at $\alpha = 1$, the distribution (1) reduces to the one parameter LD and at $\alpha = 0$, it reduces to the exponential distribution with parameters $\lambda$.

In this paper we consider three cases

1) Stress follows exponential distribution and strength follows finite mixture of Lindley distributions.
2) Stress follows Lindley distribution and strength follows finite mixture of Lindley distributions.
3) Stress and strength both follow finite mixture of Lindley distributions.

We discuss the estimation procedure for finite mixture of two parameter Lindley distributions by the method of maximum likelihood estimation and also estimation of stress-strength reliability.

2 Statistical model

In this model propose that strength ($X$) and stress ($Y$) are independent random variables and the values of strength and stress are non-negative. The reliability of a component with strength $X$ and stress $Y$ imposed on it is given by

$$R = P(X > Y) = \int_0^\infty \int_0^x g(y) dy \left( p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x) \right) dx \quad \text{where } f(x) \text{ and } g(y) \text{ are pdf of strength and stress respectively.}$$

(2)

A finite mixture of 2-parameter Lindley distributions with $k$ components can be represented as

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x) ; p_i > 0, i = 1, 2, \ldots, k \sum_{i=1}^k p_i = 1$$

The $r^{th}$ moment of mixture of two 2-parameter Lindley distributions is

$$E(x^r) = \int_0^\infty x^r \left( p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} \right) dx$$

where, $p_1 + p_2 = 1$
\[ \frac{1}{\lambda_1 + \alpha_1} \left[ \Gamma(r + 1) \frac{\Gamma(r + 2)}{\lambda_1^{r+1}} + \alpha_1 \right] + \frac{p_2 \lambda_2^2}{\lambda_2 + \alpha_2} \left[ \Gamma(r + 1) \frac{\Gamma(r + 2)}{\lambda_2^{r+2}} + \alpha_2 \right] \]

When \( r = 1 \), \( E(x) = \frac{p_1}{\lambda_1 + \alpha_1} \left[ 1 + \frac{2\alpha_1}{\lambda_1} \right] + \frac{p_2}{\lambda_2 + \alpha_2} \left[ 1 + \frac{2\alpha_2}{\lambda_2} \right] ; \quad p_1 + p_2 = 1 \)

Thus variance is given by
\[
V(x) = \frac{2p_1}{\lambda_1(\lambda_1 + \alpha_1)} \left[ 1 + \frac{3\alpha_1}{\lambda_1} \right] + \frac{2p_2}{\lambda_2(\lambda_2 + \alpha_2)} \left[ 1 + \frac{3\alpha_2}{\lambda_2} \right] - \left[ \frac{p_1}{\lambda_1 + \alpha_1} \left[ 1 + \frac{2\alpha_1}{\lambda_1} \right] + \frac{p_2}{\lambda_2 + \alpha_2} \left[ 1 + \frac{2\alpha_2}{\lambda_2} \right] \right]^2
\]

3 Reliability Computations

Let \( X \) be the strength of the \( k \)-components with probability density functions \( f_i(x); i = 1,2,\ldots, k \). The pdf of \( X \) which follows finite mixture of two parameter Lindley distributions is
\[
f_i(x) = \frac{\lambda_i^2}{\lambda_i + \alpha_i} (1 + \alpha_i x) e^{-\lambda_i x}; \quad x > 0, \lambda_i > 0, \alpha_i > -\lambda_i, p_i > 0, i = 1,2,\ldots, k, \sum_{i=1}^{k} p_i = 1
\]

**Case I: The stress \( Y \) follows exponential distribution**

As \( Y \) follows exponential distribution, pdf of \( Y \) is given by
\[
g(y) = \lambda e^{-\lambda y}, \lambda > 0, y > 0
\]

For two components \( k = 2 \) and
\[
f(x) = \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x}; \quad p_1 + p_2 = 1, \lambda_1, \lambda_2, x > 0
\]

As \( X \) and \( Y \) are independent then from (2), Reliability function \( R_2 \) is
\[
R_2 = \int_{0}^{\infty} \int_{0}^{x} \lambda e^{-\lambda y} \left[ \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} \right] dx dy
\]
\[
= - \frac{p_1 \lambda_1^2}{\lambda_3 + \alpha_1} \left[ \frac{1}{\lambda + \lambda_3 + \frac{1}{\lambda_1}} + \alpha_1 \left( \frac{1}{\lambda + \lambda_3} \right) \frac{1}{\lambda_1} \right] - \frac{p_2 \lambda_2^2}{\lambda_3 + \alpha_2} \left[ \frac{1}{\lambda + \lambda_3} + \frac{1}{\lambda_2} \right] \left[ \frac{1}{\lambda + \lambda_3} \right] \frac{1}{\lambda_2} ; \quad \sum_{i=1}^{2} p_i = 1
\]

For three components \( k = 3 \), we have
\[
f(x) = \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x} + \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} + \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x}
\]
\[ R_3 = \int_0^\infty \int_0^\infty e^{-\lambda y} \left[ \frac{p_1 \lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + \frac{p_2 \lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} + \frac{p_3 \lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x} \right] \, dx \, dy \]

\[ = - \frac{p_1 \lambda_1^2}{\lambda_1 + \alpha_1} \left( \frac{1}{\lambda + \lambda_1} - \frac{1}{\lambda_1} \right) + \alpha_1 \left( \frac{1}{\lambda + \lambda_1} - \frac{1}{\lambda_1^2} \right) - \frac{p_2 \lambda_2^2}{\lambda_2 + \alpha_2} \left( \frac{1}{\lambda + \lambda_2} - \frac{1}{\lambda_2} \right) + \alpha_2 \left( \frac{1}{\lambda + \lambda_2} - \frac{1}{\lambda_2^2} \right) - \alpha_3 \left( \frac{1}{\lambda + \lambda_3} - \frac{1}{\lambda_3} \right) + \alpha_3 \left( \frac{1}{\lambda + \lambda_3} - \frac{1}{\lambda_3^2} \right) \]

\[ R_3 = - \sum_{i=1}^3 \frac{p_i \lambda_i^2}{\lambda_i + \alpha_i} \left( \frac{1}{\lambda + \lambda_i} - \frac{1}{\lambda_i} \right) + \alpha_i \left( \frac{1}{\lambda + \lambda_i} - \frac{1}{\lambda_i^2} \right) ; \quad \sum_{i=1}^3 p_i = 1 \]

In general for k-components, \( f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x) \); \( \sum_{i=1}^k p_i = 1 \), we have

\[ R_k = - \sum_{i=1}^k \frac{p_i \lambda_i^2}{\lambda_i + \alpha_i} \left( \frac{1}{\lambda + \lambda_i} - \frac{1}{\lambda_i} \right) + \alpha_i \left( \frac{1}{\lambda + \lambda_i} - \frac{1}{\lambda_i^2} \right) \]

\[ R_k = \sum_{i=1}^k \frac{p_i \lambda_i}{\lambda_i + \alpha_i} \left[ 1 + \alpha_i \left( \frac{1}{\lambda_i} + \frac{1}{\lambda + \lambda_i} \right) \right] \]

**Special case**

1. When \( \alpha_i = 0 \), two parameter Lindley distribution reduces to exponential distribution and then \( R_k \) is the reliability function when \( X \) follows exponential distribution and \( Y \) follows mixture of exponential distributions and is given as

\[ R_k = \sum_{i=1}^k \frac{p_i \lambda_i}{\lambda + \lambda_i} = 1 - \sum_{i=1}^k \frac{p_i \lambda_i}{\lambda + \lambda_i} ; \quad \sum_{i=1}^k p_i = 1 \] (See Sandhya and Umamaheswari, [10])

2. When \( \alpha_i = 1 \), two parameter Lindley distribution reduces to one parameter Lindley distribution and then \( R_k \) is the reliability function when \( X \) follows exponential distribution and \( Y \) follows mixture of one parameter Lindley distributions and is given as

\[ R_k = \sum_{i=1}^k \frac{p_i \lambda_i}{(\lambda_i + 1)(\lambda + \lambda_i)} \left[ 1 + \frac{1}{\lambda_i} + \frac{1}{\lambda + \lambda_i} \right] \]

**Case II: The stress Y followstwo parameter Lindley distribution**

As \( Y \) follows Lindley distribution, pdf of \( Y \) is given by

\[ g(y) = \frac{\lambda^2}{\lambda + \alpha} (1 + \alpha y) e^{-\lambda y} \quad ; \quad y > 0, \lambda > 0, \alpha > -\lambda \]

For two components \( k = 2 \)

\[ f(x) = p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} \quad ; \quad p_1 + p_2 = 1, \lambda_1, \lambda_2, x > 0 \]

As \( X \) and \( Y \) are independent then from (2), Reliability function \( R_2 \) is
\[ R_2 = \int_0^\infty \int_0^x \frac{\lambda^2}{\lambda + \alpha} (1 + \alpha y) e^{-\lambda y} \left[ p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} \right] \, dx \, dy \]

\[ R_2 = \sum_{i=1}^2 \int_0^\infty \int_0^x \frac{p_i \lambda_i^2}{(\lambda + \alpha)(\lambda_i + \alpha_i)} (1 + \alpha y) e^{-\lambda y} (1 + \alpha_i x) e^{-\lambda_i x} \, dx \, dy \]

\[ R_2 = \sum_{i=1}^2 \frac{p_i \lambda_i^2}{(\lambda + \alpha)(\lambda_i + \alpha_i)} \left\{ \lambda \left[ \frac{1}{\lambda_i} - \frac{1}{\lambda + \lambda_i} \right] + \alpha_i \left[ \frac{1}{(\lambda_i)^2} - \frac{1}{(\lambda + \lambda_i)^2} \right] \right\} \]

\[ + \alpha \left[ \frac{1}{\lambda_i} - \frac{1}{\lambda + \lambda_i} - \frac{\lambda}{(\lambda + \lambda_i)^2} \right] + \alpha_i \left[ \frac{1}{(\lambda_i)^2} - \frac{1}{(\lambda + \lambda_i)^2} - \frac{2\lambda}{(\lambda + \lambda_i)^3} \right] \}

For three components k = 3, we have

\[ f(x) = p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} + p_3 \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x} \]

\[ ; \quad p_1 + p_2 + p_3 = 1, \lambda_1, \lambda_2, \lambda_3, x > 0 \]

\[ R_3 = \int_0^\infty \int_0^x \frac{\lambda^2}{\lambda + \alpha} (1 + \alpha y) e^{-\lambda y} \left[ p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} + p_3 \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x} \right] \, dx \, dy \]

\[ R_3 = \sum_{i=1}^3 \frac{p_i \lambda_i^2}{(\lambda + \alpha)(\lambda_i + \alpha_i)} \left\{ \lambda \left[ \frac{1}{\lambda_i} - \frac{1}{\lambda + \lambda_i} \right] + \alpha_i \left[ \frac{1}{(\lambda_i)^2} - \frac{1}{(\lambda + \lambda_i)^2} \right] \right\} \]

\[ + \alpha \left[ \frac{1}{\lambda_i} - \frac{1}{\lambda + \lambda_i} - \frac{\lambda}{(\lambda + \lambda_i)^2} \right] + \alpha_i \left[ \frac{1}{(\lambda_i)^2} - \frac{1}{(\lambda + \lambda_i)^2} - \frac{2\lambda}{(\lambda + \lambda_i)^3} \right] \}

In general for k-components,

\[ \sum_{i=1}^k \lambda_i = \sum_{i=1}^k \alpha_i = 0, \text{ we have} \]

\[ R_k = \sum_{i=1}^k \frac{p_i \lambda_i^2}{(\lambda + \alpha)(\lambda_i + \alpha_i)} \left\{ \lambda \left[ \frac{1}{\lambda_i} - \frac{1}{\lambda + \lambda_i} \right] + \alpha_i \left[ \frac{1}{(\lambda_i)^2} - \frac{1}{(\lambda + \lambda_i)^2} \right] \right\} \]

\[ + \alpha \left[ \frac{1}{\lambda_i} - \frac{1}{\lambda + \lambda_i} - \frac{\lambda}{(\lambda + \lambda_i)^2} \right] + \alpha_i \left[ \frac{1}{(\lambda_i)^2} - \frac{1}{(\lambda + \lambda_i)^2} - \frac{2\lambda}{(\lambda + \lambda_i)^3} \right] \}

\[ R_k = \sum_{i=1}^k \frac{\lambda p_i \lambda_i^2}{(\lambda + \alpha)(\lambda_i + \alpha_i)} \left[ \frac{\lambda + \alpha}{\lambda_i(\lambda + \lambda_i)} \left( 1 + \alpha_i \left[ \frac{1}{\lambda_i} + \frac{1}{\lambda + \lambda_i} \right] - \frac{\alpha}{(\lambda + \lambda_i)^2} \left( 1 + \frac{2\alpha_i}{\lambda_i} \right) \right) \right] \}

**Special case**

1. When \( \alpha = \alpha_i = 0 \), generalized Lindley distribution reduces to exponential distribution and then \( R_k \) is the reliability function when \( X \) follows exponential distribution and \( Y \) follows mixture of exponential distributions and is given as

\[ R_k = 1 - \sum_{i=1}^k p_i \frac{\lambda_i}{\lambda + \lambda_i}; \quad \sum_{i=1}^k p_i = 1 \]
When $\alpha = \alpha_1 = 1$, two parameter Lindley distribution reduces to one parameter Lindley distribution and then $R_k$ is the reliability function when $X$ follows one parameter Lindley distribution and $Y$ follows mixture of one parameter Lindley distribution and is given as

$$R_k = \sum_{i=1}^{k} \frac{\lambda_1 p_1^{\lambda_2}}{(\lambda + 1)(\lambda_1 + 1)} \left\{ \frac{\lambda + 1}{\lambda_1(\lambda + \lambda_1)} \left[ 1 + \frac{1}{\lambda} + \frac{1}{\lambda_1} \right] - \frac{1}{(\lambda + \lambda_1)^2} \left( 1 + \frac{2}{\lambda + \lambda_1} \right) \right\}$$

**Case III: The stress $Y$ follows mixture of two parameter Lindley distributions**

As $X$ and $Y$ both follows mixture of Lindley distributions, pdf of $X$ and $Y$ is given by

For two components, $k = 2$

$$f(x) = p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x)e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x)e^{-\lambda_2 x} ; \quad p_1 + p_2 = 1, \lambda_1, \lambda_2, x > 0$$

$$g(y) = p_3 \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 y)e^{-\lambda_3 y} + p_4 \frac{\lambda_4^2}{\lambda_4 + \alpha_4} (1 + \alpha_4 y)e^{-\lambda_4 y} ; \quad p_3 + p_4 = 1, \lambda_3, \lambda_4, y > 0$$

As $X$ and $Y$ are independent then from (2), Reliability function $R_2$ is

$$R_2 = \int_0^\infty \int_0^x \left( \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x)e^{-\lambda_1 x} + \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x)e^{-\lambda_2 x} \right) \left( \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 y)e^{-\lambda_3 y} + \frac{\lambda_4^2}{\lambda_4 + \alpha_4} (1 + \alpha_4 y)e^{-\lambda_4 y} \right) \, dx \, dy$$

$$R_2 = \sum_{j=1+2}^{n} \sum_{i=1}^{2} \int_0^\infty \int_0^x \left( \frac{p_1 \lambda_1^2 \lambda_2^2}{\lambda_1 + \alpha_1 (\lambda_1 + \alpha_1)} \right) \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_1 + \lambda_1} \right) + \alpha_1 \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_1 + \lambda_1} \right) + \alpha_1 \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_1 + \lambda_1} \right) \, dx \, dy$$

For three components $k = 3$, we have

$$f(x) = p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x)e^{-\lambda_1 x} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x)e^{-\lambda_2 x} + p_3 \frac{\lambda_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x)e^{-\lambda_3 x} ; \quad p_1 + p_2 + p_3 = 1, \lambda_1, \lambda_2, \lambda_3, x > 0$$

$$g(y) = p_4 \frac{\lambda_4^2}{\lambda_4 + \alpha_4} (1 + \alpha_4 y)e^{-\lambda_4 y} + p_5 \frac{\lambda_5^2}{\lambda_5 + \alpha_5} (1 + \alpha_5 y)e^{-\lambda_5 y} + p_6 \frac{\lambda_6^2}{\lambda_6 + \alpha_6} (1 + \alpha_6 y)e^{-\lambda_6 y} ; \quad p_4 + p_5 + p_6 = 1, \lambda_4, \lambda_5, \lambda_6, y > 0$$
X and Y are independent then from (1), Reliability function \( R \) is

\[
R_3 = \int_0^\infty \int_0^\infty \left( \frac{p_1^2 \alpha_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 x) e^{-\lambda_1 x} + \frac{p_2^2 \alpha_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 x) e^{-\lambda_2 x} + \frac{p_3^2 \alpha_3^2}{\lambda_3 + \alpha_3} (1 + \alpha_3 x) e^{-\lambda_3 x} \right) \\
\times \left( \frac{p_4 \alpha_4}{\lambda_4 + \alpha_4} (1 + \alpha_4 y) e^{-\lambda_4 y} + \frac{p_5 \alpha_5}{\lambda_5 + \alpha_5} (1 + \alpha_5 y) e^{-\lambda_5 y} + \frac{p_6 \alpha_6}{\lambda_6 + \alpha_6} (1 + \alpha_6 y) e^{-\lambda_6 y} \right) \, dx \, dy
\]

\[
R_3 = \sum_{j=1}^{6} \sum_{i=1}^{3} \int_0^\infty \int_0^\infty \left( \frac{p_1 p_2 \alpha_2^2}{(\lambda_1 + \alpha_1)(\lambda_1 + \alpha_2)} \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_1 + \lambda_2} + \alpha_1 \left( \frac{1}{\lambda_1^2} - \frac{1}{(\lambda_1 + \lambda_2)^2} \right) \right) \\
+ \alpha_1 \left( \frac{1}{\lambda_1^2} - \frac{1}{(\lambda_1 + \lambda_2)^2} - \frac{2\lambda_2}{(\lambda_1 + \lambda_2)^3} \right) \right) \right) dx \, dy
\]

In general for \( k \)-components,

\[
f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x) ; \quad \sum_{i=1}^{k} p_i = 1
\]

\[
g(y) = p_{k+1} f_{k+1}(y) + p_{k+2} f_{k+2}(y) + \cdots + p_{2k} f_{2k}(y) ; \quad \sum_{i=k+1}^{2k} p_i = 1
\]

\[
R_k = \int_0^\infty \int_0^\infty \left( p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x) \right) \left( p_{k+1} f_{k+1}(y) + p_{k+2} f_{k+2}(y) + \cdots + p_{2k} f_{2k}(y) \right) dx \, dy
\]

\[
R_k = \sum_{j=1}^{2k} \sum_{i=1}^{k} \int_0^\infty \int_0^\infty \left( \frac{p_1 p_2 \alpha_2^2}{(\lambda_1 + \alpha_1)(\lambda_1 + \alpha_2)} \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_1 + \lambda_2} + \alpha_1 \left( \frac{1}{\lambda_1^2} - \frac{1}{(\lambda_1 + \lambda_2)^2} \right) \right) \\
+ \alpha_1 \left( \frac{1}{\lambda_1^2} - \frac{1}{(\lambda_1 + \lambda_2)^2} - \frac{2\lambda_2}{(\lambda_1 + \lambda_2)^3} \right) \right) dx \, dy
\]

**Special case**

1) When \( \alpha = \alpha = 0 \), generalized Lindley distribution reduces to exponential distribution and then \( R_k \) is the reliability function when \( X \) and \( Y \) both follows mixture of exponential distributions and is given as

\[
R_k = 1 - \sum_{j=1}^{2k} \sum_{i=1}^{k} p_j \frac{\lambda_j}{(\lambda_i + \lambda_j)} ; \quad \sum_{j=k+1}^{2k} p_j = \sum_{i=1}^{k} p_i = 1
\]
2) When \( \alpha_i = \alpha_j = 1 \), two parameter Lindley distribution reduces to one parameter Lindley distribution and then \( R_k \) is the reliability function when \( X \) follows mixture of one parameter Lindley distribution and \( Y \) follows mixture of one parameter Lindley distribution and is given as

\[
R_k = \sum_{j=1}^{2k} \sum_{i=k}^{k} \frac{p_i p_j \lambda_1^2 \lambda_1}{(\lambda_1 + 1)(\lambda_1 + \lambda_j)} \left( \frac{\lambda_1 + 1}{\lambda_1} + \frac{1}{\lambda_1 + \lambda_j} \right) - \frac{1}{(\lambda_1 + \lambda_j)^2} \left( 1 + \frac{2}{\lambda_1 + \lambda_j} \right)
\]

4 Hazard Rate

Let \( S(t) \) be the survival function of a component then the survival function of the model

For two components is given by

\[
S(t) = p_1 \left( \frac{\lambda_1 + \alpha_1 + \lambda_2 \alpha_1}{\lambda_1 + \alpha_1} \right) e^{-\lambda_1 t} + p_2 \left( \frac{\lambda_2 + \alpha_2 + \lambda_2 \alpha_2}{\lambda_2 + \alpha_2} \right) e^{-\lambda_2 t}
\]

then

\[
h(t) = \frac{f(t)}{S(t)} = \frac{p_1 \lambda_1^2 \lambda_1}{\lambda_1 + \alpha_1} (1 + \alpha_1 t) e^{-\lambda_1 t} + p_2 \lambda_2^2 \lambda_2 (1 + \alpha_2 t) e^{-\lambda_2 t}
\]

In general for \( k \)-components

\[
h(t) = \frac{\sum_{i=1}^{k} p_i \lambda_i^2 \lambda_i}{\sum_{i=1}^{k} (\lambda_i + \alpha_i)} e^{-\lambda_i t}
\]

5 Estimation of Parameters

\[
L(\lambda_1, \lambda_2, p_1, y) = \prod_{j=1}^{n_1} \left[ p_1 \frac{\lambda_1^2}{\lambda_1 + \alpha_1} (1 + \alpha_1 y_i) e^{-\lambda_1 y_i} + p_2 \frac{\lambda_2^2}{\lambda_2 + \alpha_2} (1 + \alpha_2 y_i) e^{-\lambda_2 y_i} \right]
\]

\[
= \frac{n_1! n_2!}{n_1! n_2!} \left( \frac{p_1 \lambda_1^2}{\lambda_1 + \alpha_1} \right)^{n_1} \left( \frac{p_2 \lambda_2^2}{\lambda_2 + \alpha_2} \right)^{n_2} \prod_{j=1}^{n_1} (1 + \alpha_1 y_{1j}) e^{-\lambda_1 y_{1j}} \prod_{j=1}^{n_2} (1 + \alpha_2 y_{2j}) e^{-\lambda_2 y_{2j}}
\]

\[
\log L(\lambda_1, \lambda_2, p_1, y) = \log \frac{n_1! n_2!}{n_1! n_2!} + n_1 \log p_1 \lambda_1^2 - n_1 \log (\lambda_1 + \alpha_1) + n_2 \log p_2 \lambda_2^2 - n_2 \log (\lambda_2 + \alpha_2)
\]

\[
- \lambda_1 \sum_{j=1}^{n_1} y_{1j} - \lambda_2 \sum_{j=1}^{n_2} y_{2j} + \sum_{j=1}^{n_1} \log (1 + \alpha_1 y_{1j}) + \sum_{j=1}^{n_2} \log (1 + \alpha_2 y_{2j})
\]

Now,

\[
\frac{\partial \log L}{\partial \lambda_1} = 0
\]

\[
\Rightarrow \lambda_1^2 - \left( \frac{n_1}{\sum_{j=1}^{n_1} y_{1j} - \alpha_1} \right) \lambda_1 - \frac{2n_1 \alpha_1}{\sum_{j=1}^{n_1} y_{1j}} = 0
\]
\[ \hat{\lambda}_1 = \frac{1}{2} \left( \frac{n_1}{\sum_{j=1}^{n_1} y_{ij}} - \alpha_1 \right) + \frac{n_1}{\sum_{j=1}^{n_1} y_{ij}} \left( \frac{n_1}{\sum_{j=1}^{n_1} y_{ij}} - \alpha_1 \right)^2 + \frac{8n_1 \alpha_1}{\sum_{j=1}^{n_1} y_{ij}} \] 

Similarly, 

\[ \hat{\lambda}_2 = \frac{1}{2} \left( \frac{n_2}{\sum_{j=1}^{n_2} y_{2j}} - \alpha_2 \right) + \frac{n_2}{\sum_{j=1}^{n_2} y_{2j}} \left( \frac{n_2}{\sum_{j=1}^{n_2} y_{2j}} - \alpha_2 \right)^2 + \frac{8n_2 \alpha_2}{\sum_{j=1}^{n_2} y_{2j}} \] 

and, 

\[ \frac{\partial \log L}{\partial p_1} = 0 \] 

\[ \Rightarrow \hat{p}_1 = \frac{n_1}{n} \] 

and \( n = n_1 + n_2 \) 

Generalizing the above results for \( k \)-components we get 

\[ \hat{\lambda}_1 = \frac{1}{2} \left( \frac{n_i}{\sum_{j=1}^{n_i} y_{ij}} - \alpha_i \right) + \frac{n_i}{\sum_{j=1}^{n_i} y_{ij}} \left( \frac{n_i}{\sum_{j=1}^{n_i} y_{ij}} - \alpha_i \right)^2 + \frac{8n_i \alpha_i}{\sum_{j=1}^{n_i} y_{ij}} \] 

and \( \hat{p}_1 = \frac{n_i}{n} ; \quad n = \sum_{i=1}^{k} n_i \) 

**Estimation of Stress-Strength Reliability**

1) The M.L.E of \( R \) when the strength \( X \) follows finite mixture of Lindley distributions with parameter \( \lambda_i, \alpha_i \) (known), and \( p_i \) and stress \( Y \) follows exponential distribution with parameter \( \lambda \) is given as 

\[ \hat{R}_k = \sum_{i=1}^{k} \hat{p}_i \frac{\lambda_i^2 \lambda}{(\lambda_i + \alpha_i)(\lambda + \hat{\lambda}_i)} \left[ 1 + \alpha_i \left( \frac{1}{\hat{\lambda}_i} + \frac{1}{\lambda + \hat{\lambda}_i} \right) \right] \]

2) The M.L.E of \( R \) when the strength \( X \) follows finite mixture of Lindley distributions with parameter \( \lambda_i, \alpha_i \) (known), and \( p_i \) and stress \( Y \) follows Lindley distribution with parameter \( \lambda \) and \( \alpha \) (known) is given as 

\[ \hat{R}_k = \sum_{i=1}^{k} \hat{p}_i \frac{\lambda_i}{(\lambda + \alpha)(\hat{\lambda}_i + \alpha_i)} \left[ 1 + \alpha_i \left( \frac{1}{\hat{\lambda}_i} + \frac{1}{\lambda + \hat{\lambda}_i} \right) \right] - \alpha_i \left( \frac{1}{\hat{\lambda}_i} + \frac{2\alpha_i}{\lambda + \hat{\lambda}_i} \right) \left( \frac{1}{\lambda + \hat{\lambda}_i} + \frac{2\alpha_i}{\lambda + \hat{\lambda}_i} \right) \]

3) The M.L.E of \( R \) when the strength \( X \) and \( Y \) follows finite mixture of Lindley distributions with parameter \( \lambda_i, \alpha_i \) (known), and \( p_i \) is given as 

\[ \hat{R}_k = \sum_{j=1}^{k} \sum_{i=1}^{k} \frac{\hat{p}_j \hat{p}_i \lambda_i^2 \lambda_j}{(\hat{\lambda}_i + \alpha_i)(\hat{\lambda}_j + \alpha_j)} \left[ 1 + \alpha_i \left( \frac{1}{\hat{\lambda}_i} + \frac{1}{\lambda + \hat{\lambda}_i} \right) \right] - \alpha_i \left( \frac{1}{\hat{\lambda}_i} + \frac{2\alpha_i}{\lambda + \hat{\lambda}_i} \right) \left( \frac{1}{\lambda + \hat{\lambda}_i} + \frac{2\alpha_i}{\lambda + \hat{\lambda}_i} \right) \]
6 Numerical Evaluation

For some specific values of the parameters involved in the expression of $R$ in two component systems, we have evaluated system reliability $R$ for different cases of two parameter Lindley distribution. We consider a data on life to death of two groups of leukaemia patients which is given in table 7. Table 8 provides the values of estimates for different values of $\alpha_1$ and $\alpha_2$ by M.L.E method. Also, table 9 and 10 provides estimates of survival function and hazard rate function, respectively, for different values of $\alpha_1$ and $\alpha_2$ at various time points.

**Case I:** Strength has mixture of two parameter Lindley distribution and Stress has Exponential distribution:

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$\lambda_1 = \lambda_2$</th>
<th>$\alpha_1 = \alpha_2$</th>
<th>$\lambda$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.6</td>
<td>0.9591</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.6</td>
<td>0.8839</td>
</tr>
<tr>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
<td>0.8055</td>
</tr>
<tr>
<td>0.1</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7333</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.6694</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6</td>
<td>0.5</td>
<td>0.6</td>
<td>0.6136</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5650</td>
</tr>
<tr>
<td>0.1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5227</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.5</td>
<td>0.6</td>
<td>0.4857</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>0.5</td>
<td>0.6</td>
<td>0.4531</td>
</tr>
</tbody>
</table>

**Case II:** Strength has mixture of two parameter Lindley distribution and Stress has Lindley distributions:

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$\lambda_1 = \lambda_2$</th>
<th>$\alpha_1 = \alpha_2$</th>
<th>$\lambda$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
<td>0.7372</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.3</td>
<td>0.8378</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.4</td>
<td>0.8888</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.9182</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.6</td>
<td>0.9366</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9490</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.8</td>
<td>0.9578</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.9</td>
<td>0.9642</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>1</td>
<td>0.9691</td>
</tr>
</tbody>
</table>

**Case III:** Stress -Strength has mixture of two parameter Lindley distributions:

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$\lambda_1 = \lambda_2$</th>
<th>$\alpha_1 = \alpha_2$</th>
<th>$\lambda_3 = \lambda_4$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
<td>0.7372</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3771</td>
</tr>
<tr>
<td>0.1</td>
<td>0.4</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2686</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2107</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1708</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1708</td>
</tr>
</tbody>
</table>

Table 1: Variation in $R$ for constant Stress

Table 2: Variation in $R$ for constant Stress

Table 3: Variation in $R$ for constant Strength

Table 4: Variation in $R$ for constant Strength

Table 5: Variation in $R$ for constant Stress

Table 6: Variation in $R$ for constant Stress
The following data are taken from Feigl and Zelen [16]. They refer to 33 leukaemia patients, classified as either “AG positive” or “AG negative” (positive values being defined by the presence of Auer rods and/or significant granulature of the leukaemic cells in the bone marrow at diagnosis, and negative values if both Auer rods and granulature are absent). The initial white blood cell counts and the survival times in weeks are given. We are interested in what we can say about how long a patient is likely to survive, given the white blood cell count (WBC) and the AG group.

Table 7: Survival times of leukaemia patients

<table>
<thead>
<tr>
<th>WBC</th>
<th>TIME(t)</th>
<th>WBC</th>
<th>TIME(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2300</td>
<td>65</td>
<td>4400</td>
<td>56</td>
</tr>
<tr>
<td>750</td>
<td>156</td>
<td>3000</td>
<td>65</td>
</tr>
<tr>
<td>4300</td>
<td>100</td>
<td>4000</td>
<td>17</td>
</tr>
<tr>
<td>2600</td>
<td>134</td>
<td>1500</td>
<td>7</td>
</tr>
<tr>
<td>6000</td>
<td>16</td>
<td>9000</td>
<td>16</td>
</tr>
<tr>
<td>10500</td>
<td>108</td>
<td>5300</td>
<td>22</td>
</tr>
<tr>
<td>10000</td>
<td>121</td>
<td>10000</td>
<td>3</td>
</tr>
<tr>
<td>17000</td>
<td>4</td>
<td>19000</td>
<td>4</td>
</tr>
<tr>
<td>5400</td>
<td>39</td>
<td>27000</td>
<td>2</td>
</tr>
<tr>
<td>7000</td>
<td>143</td>
<td>28000</td>
<td>3</td>
</tr>
<tr>
<td>9400</td>
<td>56</td>
<td>31000</td>
<td>8</td>
</tr>
<tr>
<td>32000</td>
<td>26</td>
<td>26000</td>
<td>4</td>
</tr>
<tr>
<td>35000</td>
<td>22</td>
<td>21000</td>
<td>3</td>
</tr>
<tr>
<td>100000</td>
<td>1</td>
<td>79000</td>
<td>30</td>
</tr>
<tr>
<td>100000</td>
<td>1</td>
<td>100000</td>
<td>4</td>
</tr>
<tr>
<td>52000</td>
<td>5</td>
<td>100000</td>
<td>43</td>
</tr>
<tr>
<td>100000</td>
<td>65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Estimation of parameters of survival times of leukaemia patients

<table>
<thead>
<tr>
<th>α1 = α2</th>
<th>( \hat{\lambda}_1 )</th>
<th>( \hat{\lambda}_2 )</th>
<th>( \hat{\beta}_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.01600</td>
<td>0.05574</td>
<td>0.515</td>
</tr>
<tr>
<td>0.1</td>
<td>0.02846</td>
<td>0.08576</td>
<td>0.515</td>
</tr>
<tr>
<td>0.2</td>
<td>0.02993</td>
<td>0.09371</td>
<td>0.515</td>
</tr>
<tr>
<td>0.3</td>
<td>0.03053</td>
<td>0.09779</td>
<td>0.515</td>
</tr>
<tr>
<td>0.4</td>
<td>0.03086</td>
<td>0.10031</td>
<td>0.515</td>
</tr>
<tr>
<td>0.5</td>
<td>0.03107</td>
<td>0.10204</td>
<td>0.515</td>
</tr>
<tr>
<td>0.6</td>
<td>0.03122</td>
<td>0.10330</td>
<td>0.515</td>
</tr>
<tr>
<td>0.7</td>
<td>0.03132</td>
<td>0.10427</td>
<td>0.515</td>
</tr>
<tr>
<td>0.8</td>
<td>0.03141</td>
<td>0.10502</td>
<td>0.515</td>
</tr>
<tr>
<td>0.9</td>
<td>0.03147</td>
<td>0.10564</td>
<td>0.515</td>
</tr>
<tr>
<td>1</td>
<td>0.03152</td>
<td>0.10614</td>
<td>0.515</td>
</tr>
</tbody>
</table>
Table 9: MaximumLikelihood estimate of survival probability at various time points for different values of $\alpha_1 = \alpha_2$

<table>
<thead>
<tr>
<th>$\alpha_1 = \alpha_2$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.96553</td>
<td>0.99379</td>
<td>0.99300</td>
<td>0.99263</td>
<td>0.99242</td>
<td>0.99228</td>
</tr>
<tr>
<td>15</td>
<td>0.61531</td>
<td>0.46854</td>
<td>0.45260</td>
<td>0.44602</td>
<td>0.44241</td>
<td>0.44017</td>
</tr>
<tr>
<td>30</td>
<td>0.40977</td>
<td>0.24798</td>
<td>0.23665</td>
<td>0.23227</td>
<td>0.22995</td>
<td>0.22855</td>
</tr>
<tr>
<td>45</td>
<td>0.29015</td>
<td>0.14575</td>
<td>0.13824</td>
<td>0.13544</td>
<td>0.13398</td>
<td>0.13312</td>
</tr>
<tr>
<td>60</td>
<td>0.21430</td>
<td>0.08996</td>
<td>0.08463</td>
<td>0.08267</td>
<td>0.08166</td>
<td>0.08107</td>
</tr>
<tr>
<td>75</td>
<td>0.16253</td>
<td>0.05666</td>
<td>0.05274</td>
<td>0.05131</td>
<td>0.05057</td>
<td>0.05014</td>
</tr>
<tr>
<td>90</td>
<td>0.12523</td>
<td>0.03598</td>
<td>0.03308</td>
<td>0.03202</td>
<td>0.03148</td>
<td>0.03117</td>
</tr>
<tr>
<td>105</td>
<td>0.09737</td>
<td>0.02292</td>
<td>0.02079</td>
<td>0.02003</td>
<td>0.01963</td>
<td>0.01941</td>
</tr>
<tr>
<td>120</td>
<td>0.07610</td>
<td>0.01462</td>
<td>0.01308</td>
<td>0.01253</td>
<td>0.01225</td>
<td>0.01209</td>
</tr>
</tbody>
</table>

Table 10: MaximumLikelihood estimate of Hazard rate at various time points

<table>
<thead>
<tr>
<th>$\alpha_1 = \alpha_2$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0349</td>
<td>0.0183</td>
<td>0.0140</td>
<td>0.0119</td>
<td>0.0107</td>
<td>0.0100</td>
</tr>
<tr>
<td>15</td>
<td>0.0295</td>
<td>0.0413</td>
<td>0.0446</td>
<td>0.0462</td>
<td>0.0471</td>
<td>0.0477</td>
</tr>
<tr>
<td>30</td>
<td>0.0248</td>
<td>0.0476</td>
<td>0.0512</td>
<td>0.0526</td>
<td>0.0534</td>
<td>0.0538</td>
</tr>
<tr>
<td>45</td>
<td>0.0214</td>
<td>0.0504</td>
<td>0.0537</td>
<td>0.0548</td>
<td>0.0554</td>
<td>0.0558</td>
</tr>
<tr>
<td>60</td>
<td>0.0191</td>
<td>0.0557</td>
<td>0.0600</td>
<td>0.0613</td>
<td>0.0621</td>
<td>0.0626</td>
</tr>
<tr>
<td>75</td>
<td>0.0179</td>
<td>0.0635</td>
<td>0.0692</td>
<td>0.0713</td>
<td>0.0726</td>
<td>0.0732</td>
</tr>
<tr>
<td>90</td>
<td>0.0170</td>
<td>0.0731</td>
<td>0.0804</td>
<td>0.0834</td>
<td>0.0848</td>
<td>0.0856</td>
</tr>
<tr>
<td>105</td>
<td>0.0165</td>
<td>0.0837</td>
<td>0.0923</td>
<td>0.0958</td>
<td>0.0978</td>
<td>0.0990</td>
</tr>
<tr>
<td>120</td>
<td>0.0163</td>
<td>0.0944</td>
<td>0.1047</td>
<td>0.1093</td>
<td>0.1110</td>
<td>0.1125</td>
</tr>
</tbody>
</table>

Table 10: MaximumLikelihood estimate of Hazard rate at various time points

Fig. 1: Variation in R for constant Stress
Fig. 2: Variation in R for constant Strength

Fig. 3: Variation in R for constant Stress

Fig. 4: Variation in R for constant Strength
Fig. 5: Variation in R for constant Stress

Fig. 6: Variation in R for constant Strength

Fig. 7: Survival probability at various time points for different values of $\alpha_1 = \alpha_2$
7. Conclusions

In the proposed model, we have studied the stress-strength reliability for two parameter Lindley distribution when strength follows finite mixture of two parameter Lindley distribution and stress follows exponential, Lindley distribution and finite mixture of two parameter Lindley distribution. Estimates of parameters are obtained by maximum likelihood estimation method. The numerical evaluation indicates that when the stress has exponential and Lindley distributions then reliability increases when stress increases and vice versa. And when stress follows finite mixture of two parameter Lindley distribution reliability increases with increases in strength and decreases with decreases in stress. At the end we have discussed real life example and plot the graphs of reliability function and hazard rate for various values of $\alpha$.

Acknowledgement

The author would like to sincerely thank anonymous referees for insightful comments and suggestions that substantially improved the paper.

References