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The Lane Squeezing Behaviors in the Lane Reduction Bottleneck with an Optimal Velocity Model

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Abstract: As a kind of typical human factor influencing the transportation seriously, the lane squeezing behaviors in the lane reduction bottleneck is investigated with the optimal velocity model. Two kinds of vehicles are introduced. The asymmetric lane changing rules in the slowdown section and the lane squeezing behaviors near the merging point are taken into account. Under the periodic boundary condition, the numerical simulations are performed, which shows that the asymmetric lane changing rules, especially the lane squeezing behaviors influence the current distribution of right and left lanes in the slowdown section. And an interesting phenomenon of ratio inversion appears. Besides, the jam queue can also be affected by the length and the speed limit of section B obviously.

Keywords: Traffic flow, lane squeezing behaviors, lane reduction, optimal velocity model.

1. Introduction

Recently, traffic has become a global exasperating problem, and the increasing traffic jams and traffic pollution are exerting great pressure on society and causing enormous economic losses. Many researchers from different disciplines have been trying to understand the fundamental principals governing the traffic based on three different approaches, namely: the macroscopic, mesoscopic and microscopic description. On the one hand, various traffic models, such as car-following models, cellular automaton models, gas kinetic models and hydrodynamic models, have been suggested to explore the traffic mechanism of all kinds of phenomena etc [1–5]; on the other hand, other related topics, e.g., how to improve the transportation ability and how to decrease traveling time, have also attracted considerable attention of scientists [6].

The main reason for studying traffic is that new problems are being continuously generated. As we know, the jams are mainly caused by many factors such as vehicle fleet, allocation of road, infrastructure and supporting facilities and traffic management. However, another significant reason is human factors, including reckless jaywalking, non-motor vehicles moving on the motorway, motor vehicles changing lanes or making a U-turn randomly, taxies stopping casually, etc.. A recent survey revealed that the influence of human factors on traffic has exceeded over 30% [7]. Moreover, the blockages from human factors are becoming an increasingly key reason influencing traffic accessibility. Especially in China, with a large population and mixture traffic, this influence is more serious. A question is raised that how to make effective use of existing transportation resources with the aid of scientific theories so as to ease traffic pressure.

The answer to this question requires a deep investigation. Among all kinds of bottleneck, lane reduction is common in road system. Some scholars studied this bottleneck from different aspects [8–10]. While as shown in Fig.1, two-lane road merges into single-lane road at the merging point M. Near the merging point, the maximum velocity is restricted and such behaviors as lane changing, especially lane squeezing are much more obvious, which affects the capacity seriously. According to our knowledge, these behaviors have seldom been investigated.

In this paper, our primary aim is to propose a modified optimal velocity model to investigate traffic properties upstream of the bottleneck by analyzing the characteristics of human factors from traffic psychology, which can contribute to a better understanding to the

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traffic mechanism. The rest of the present paper is organized as follows. In Section 2, the modified model is set up by introducing asymmetric lane changing rules in the slowdown section and the lane squeezing rules near the merging point. Section 3 gives the simulation results and corresponding discussion. The conclusion is drawn in Section 4.

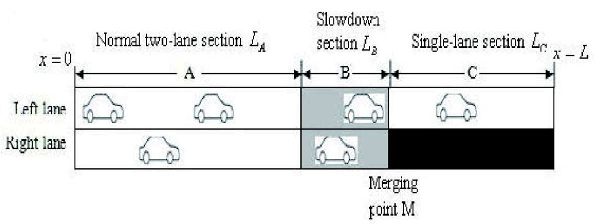

Figure 1 Schematic illustration of the lane reduction bottleneck (the road is divided into three sections: A, B, and C. In section A, the symmetric lane changing rules are used; in section B (illustrated by gray color), special lane changing rules and the lane squeezing rules near the merging point are adopted; in section C, only the left lane is available, the right lane illustrated by black color is closed).

2. Model

Here, two kinds of vehicles: fast vehicle (the fraction is f) and slow vehicle (the fraction is $1-f$) are introduced. We consider the traffic situation illustrated in Fig. 1, in which the road is divided into three sections: A, B, and C. Section A is far away from the merging point M, so it is a normal two-lane section and is not influenced by section C. Since section B is near the merging point M, it is usually a slowdown section and seriously influenced by section C. Section C is a single-lane section. We set the sections of A, B and C with length L_A , L_B and L_C, respectively.

We assume the vehicular movement is divided into sideways movement and forward movement. Lane changing is implemented as a sideways movement. We apply the following equation of optimal velocity model to the forward movement [11]:

$$\frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} = a\left\{V\left(\Delta x_i\right) - \frac{\mathrm{d}x_i}{\mathrm{d}t}\right\} \tag{1}$$

where $V(\Delta x_i)$ is the optimal velocity function; $x_i(t)$ denotes the position and the headway of vehicle i at time t; $\Delta x_t(t)$ $(= x_{i+1}(t) - x_i(t))$ is the position and the headway of vehicle i at time t and a is the sensitivity (the inverse of the delay time) [1].

In sections A, C, vehicles move with normal velocity; while in section B, vehicles with the forced speed. The corresponding optimal velocity function of vehicles are respectively given by

$$V(\Delta x_i) = \frac{v^{AC}_{f(s),\max}}{2}[\tanh(\Delta x_i - x_{f(s),c}) + \tanh(x_{f(s),c})] \tag{2}$$

$$V(\Delta x_i) = \frac{v^{B}_{f(s),\max}}{2}[\tanh(\Delta x_i - x_{f(s),c}) + \tanh(x_{f(s),c})] \tag{3}$$

where $v^{AC}_{f,\max}(v^{AC}_{s,\max})$ and $v^{B}_{f,\max}(v^{B}_{s,\max})$ are the maximal velocity of fast (slow) vehicles in section A, C and B, respectively. Note that $v^{AC}_{f,\max} > v^{AC}_{s,\max}$ and $v^{B}_{f(s),\max} = \min(v_{B,\max}, v^{AC}_{f(s),\max})$ ($v_{B,\max}$ is speed limits of the slowdown section B). $x_{f,c}(x_{s,c})$ is the position of the turning point of fast (slow) vehicle, also the safety distance [1]. Here we assume the safe distances of different kinds of vehicles (fast and slow) are different, but for the same kind of vehicles, the safe distances are taken as a constant regardless of the properties of section. Fig. 2 shows the schematic illustration of lane changing on the two-lane road.

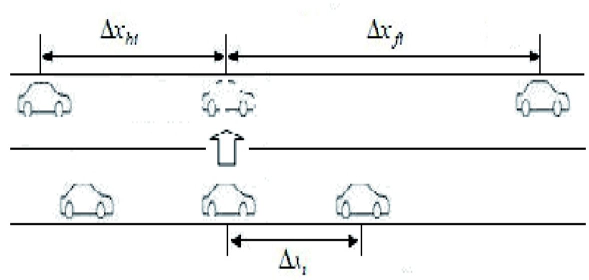

Figure 2 Schematic illustration of lane changing on the two-lane road. The incoming vehicle on the one lane enters into the middle position between two vehicles ahead and behind on the other one.

We consider the lane changing criteria. In section A, the following symmetric rule is adopted:

$$\Delta x_i < 2x_{f(s),c} \quad \text{for the incentive criterion} \tag{4}$$

$$\Delta x_{fi} > \Delta x_i, \Delta x_{bi} > x_{f,c} \quad \text{for the security criterion} \tag{5}$$

$$rand() \le p_A \quad \text{for the probability of changing lane} \tag{6}$$

where $\Delta_{fi}(\Delta_{bi})$ is the headway between vehicle i and the vehicle ahead (behind) on the target lane; p_A is the changing probability when the incentive criterion and the security criterion are both satisfied.

The right lane of the section C is closed, drivers tend to drive on the left lane in section B. Therefore we adopt the different lane changing rules for vehicles on two lanes. For the vehicles on the left lane,

$$\Delta x_i < \frac{1}{2}x_{f(s),c} \quad \text{for the incentive criterion} \tag{7}$$

$$\Delta x_{fi} > 2x_{f(s),c}, \Delta x_{bi} > x_{f,c} \quad \text{for the security criterion} \tag{8}$$

$$rand() \le p_B \quad \text{for the probability of changing lane} \tag{9}$$

For the right lane of section B, its downstream section is closed, and the vehicles on it have no priority comparatively, so we consider the asymmetric lane changing rule from two cases:
case 1:

$$\Delta x_i \le \Delta x_{fi} \quad \text{for the incentive criterion} \tag{10}$$

$$\Delta x_{bi} > \frac{1}{2}x_{f,c} \quad \text{for the security criterion} \tag{11}$$

case 2:

$$\Delta x_i > \Delta x_{fi}, \Delta x_i < \frac{1}{2}x_{f(s),c}, \Delta x_i - \Delta x_{fi} < \frac{1}{2}x_{f(s),c} \quad \text{for the incentive criterion} \tag{12}$$

$$\Delta x_{bi} > \frac{1}{2}x_{f,c} \quad \text{for the security criterion} \tag{13}$$

$$rand() \le 1 - p_B \quad \text{for the probability of changing lane} \tag{14}$$

As approaching the merging point M, the serious lane squeezing occurs between the leading vehicles on both lanes, as shown in Fig. 3. Here we consider three cases. Each case is divided into three sub-cases according to the positions of the two leading vehicles. Here $x^L_{\text{leading}}(x^R_{\text{leading}})$ represents the position of the leading vehicle on the left (right) lane of section B.

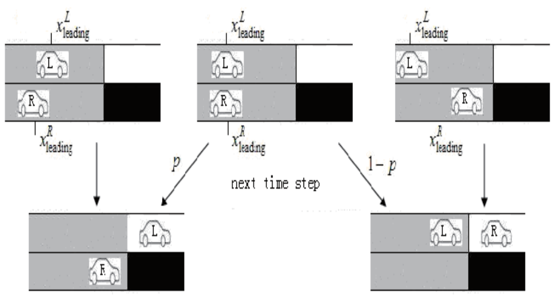

Figure 3 Schematic illustration of lane squeezing at the merging point M. At next time step, only one of two squeezing leading vehicles can enter the section C.

case 1: when the types of two leading vehicles are same, the lane squeezing rules are as follows.

$$x^R_{\text{leading}} - x^L_{\text{leading}} \le 0, \quad G = 1; \tag{15}$$

$$0 < x^R_{\text{leading}} - x^L_{\text{leading}} \le \frac{1}{2}x_{f(s),c}, \quad G = \begin{cases} 1, & rand() \le p_1 \\ -1 & rand() \le 1 - p_1 \end{cases} \tag{16}$$

$$x^R_{\text{leading}} - x^L_{\text{leading}} > \frac{1}{2}x_{f(s),c}, \quad G = -1 \tag{17}$$

where G represents the passing state at the bottleneck, $G = 1(G = -1)$ represents that the leading vehicle of the left (right) lane enters the section C.

case 2: when the leading vehicle on the left lane is fast and the right is slow, we have:

$$x^R_{\text{leading}} - x^L_{\text{leading}} \le \frac{1}{3}x_{f,c}, \quad G = 1; \tag{18}$$

$$\frac{1}{3}x_{f,c} < x^R_{\text{leading}} - x^L_{\text{leading}} \le \frac{2}{3}x_{f,c}, \quad G = \begin{cases} 1, & rand() \le p_2 \\ -1 & rand() \le 1 - p_2 \end{cases} \tag{19}$$

$$x^R_{\text{leading}} - x^L_{\text{leading}} > \frac{2}{3}x_{f,c}, \quad G = -1 \tag{20}$$

case 3: when the leading vehicle on the left lane is slow and the right is fast, we have:

$$x^R_{\text{leading}} - x^L_{\text{leading}} \le -\frac{1}{3}x_{f,c}, \quad G = 1; \tag{21}$$

$$-\frac{1}{3}x_{s,c} < x^R_{\text{leading}} - x^L_{\text{leading}} \le \frac{1}{3}x_{s,c}, \quad G = \begin{cases} 1, & rand() \le p_3 \\ -1, & rand() \le 1 - p_3 \end{cases} \tag{22}$$

$$x^R_{\text{leading}} - x^L_{\text{leading}} > \frac{1}{3}x_{s,c}, \quad G = -1 \tag{23}$$

The dynamics is determined by Eqs. 1-23, then various dynamic states of traffic appear and traffic jam may occur. Below the critical point $(x_{f,c}, a_c)(a_c = v_{f,\max})$, a phase transition occurs [1]. In this paper, we restrict to such case that the sensitivity $a > a_c$, since we do not investigate spontaneous jams but study the traffic states induced by the lane reduction. In addition, based on the above analysis and realistic traffic condition, we give the approximate intervals $p_B \le 0.2$, $p_1 \ge 0.75$, $p_2 \ge 0.9$ and $p_3 \ge 0.5$.

3. Simulations results and discussion

We perform computer simulations under the periodic boundary condition and solve numerically Eq.1 with optimal velocity functions (2) and (3) by using fourth-order Runge-Kutta method where the time interval is $\Delta t = \frac{1}{20}$.

We carry out simulation by varying the initial headway for the vehicles on the roads. The values of each parameter are selected as follows: $a = 3.0$, $f = 0.9$,

$v_{f,\max}^{AC} = 2.0$, $v_{s,\max}^{AC} = 2.0$, $v_{B,\max} = 1.2$, $x_{f,c} = 4$, $x_{s,c} = 3$, $p_A = 0.7$, $p_B = 0.2$, $p_1 = 0.75$, $p_2 = 0.9$ and $p_3 = 0.5$. Initially, we put the all vehicles in the road system with same headway Δx_{int}. The length of road is $L = 1600$, $L_A = 800$, $L_B = 200$ and $L_C = 600$ unless otherwise stated.

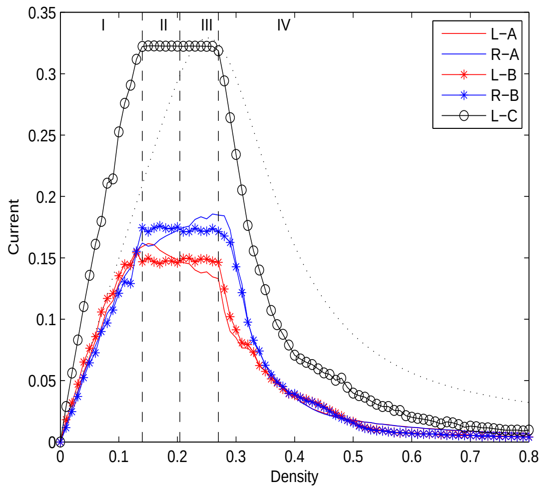

Figure 4 Plots of traffic current against density for each lane of every section in Fig.1, where L (R) indicate the left (right) lane; A, B and C the different sections. The dashed line is the theoretical current curve for all slowdown sections of single-lane with $V_{s,\max} = 1.5$.

The current of each lane in every section against density is shown in Fig. 4. The theoretical current is given by $J = \frac{V(\Delta x_{\text{int}})}{\Delta x_{\text{int}}}$, where Δx_{int} is the initial headway. From Fig. 4, we can see the current on the left lane is not consistent with that on the right lane either in section A and B, and the current of the section A is not consistent with that of the section B. The whole curve can be divided into four regions. In region I, the vehicular density is low and the current of every lane in each section increases linearly with density. Then, the currents of the section B and C almost saturate and keep a constant value in region II and III. An interesting phenomenon appears that in section B, the saturated current of right lane is larger than that of left lane, which means that due to the lane-squeezing behavior, drivers on the right lane tend to "cut in" the left lane. As a result, the current of right lane increases. In region IV at high density, the currents decrease with increasing density. As $\rho > 0.4$, the currents both of section A and B including left and right lanes are nearly same. In such case, the lane changing probability is zero at higher density, which is in accordance with the realistic traffic. Section C is a single lane, but its saturation is not consistent with the theoretical current value for all slowdown sections of single-lane with $V_{s,\max} = 1.5$.

Fig. 5 gives the profiles of fast vehicle ratio against density. The fast vehicle ratios of two lanes in either section A or B are not consistent with each other when the density is not very high. Generally, the ratio of the left lane is larger than that of the right lane in section A, but smaller in section B. In addition, the fast vehicle ratio difference in section B on regions II and III becomes obvious. This phenomenon is closely analogous to density inversion [8], which can be treated as ratio inversion. The ratio difference in section B between two lanes is greater than that in section A, which is caused by the asymmetric lane changing behaviors in section B as well as the lane-squeezing factor near the merging point. When density is high, this ratio inversion phenomenon disappears in both sections A and B.

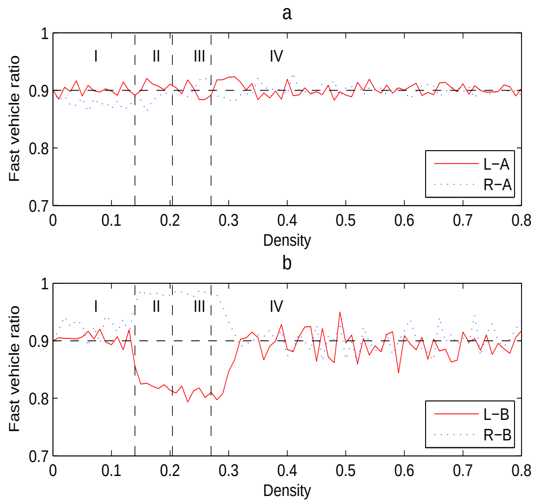

Figure 5 Plots of fast vehicle ratio against density in section A (a) and section B (b). The solid and dashed line indicates, respectively, the fast vehicle ratio profiles in the left and the right lane.

Fig. 6 shows the profiles of headway and velocity corresponding to regions II and III in Fig. 4 with saturated currents. The traffic road system is divided into four regions indicated by I-IV. The headway and velocity of the right lane are always greater than that of the left lane in region II and III, which means that the densities of two lanes are inconsistent. This inconsistency is in agreement with the results obtained by the cellular automata models [8]. A peak emerges on the velocity profiles of the right lane in region III, which may comes from the asymmetric lane changing rules in the slowdown section. The jammed state appears between $x = 540$ and $x = 1000$ at density $\rho = 0.18$, but between $x = 300$ and $x = 1000$ at density $\rho = 0.22$, which shows the traffic jam can spread backwards with increasing density when the current is saturated. The most serious jam is before the slowdown section B.

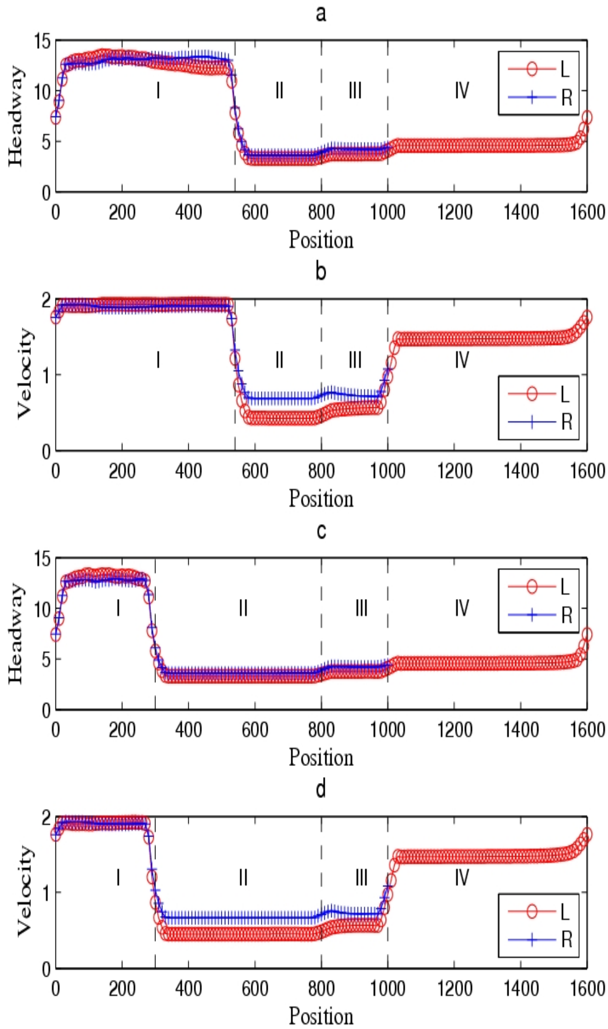

Figure 6 Plot of headway (velocity) against position at density $\rho = 0.18$ in region II (a) ((b)) and $\rho = 0.22$ in region III (c) ((d)) of Fig. 4.

Fig. 7 shows the traffic patterns with saturated current under different parameters. (a)-(c) and (d)-(f) indicate the traffic states at density $\rho = 0.18$ in region II and $\rho = 0.22$ in region III of Fig. 4, respectively. The length of jam is longer at $\rho = 0.22$ than that at $\rho = 0.18$, which shows that the traffic jam can spread backwards with increasing density when the current is saturated. By comparison, the beginning position of traffic jam in diagram (a) (or (d)) is before that in diagram (b) (or (e)); the beginning position of the traffic jam in diagram (c) (or (f)) is before that in diagram (b) (or (e)). Consequently, we conclude that the jam queue is influenced by the length of section B L_B and the speed limit $v_{B,\max}$. For example, under the same length of section B L_B (see Fig.7a and b), the length of jam queue decreases with increasing $v_{B,\max}$, namely, the decrease of speed limit of section B will improve the road capacity. And under the same speed limit $v_{B,\max}$ (see Fig.7b and c), the increasing L_B also can cause the increase of the length of jam queue.

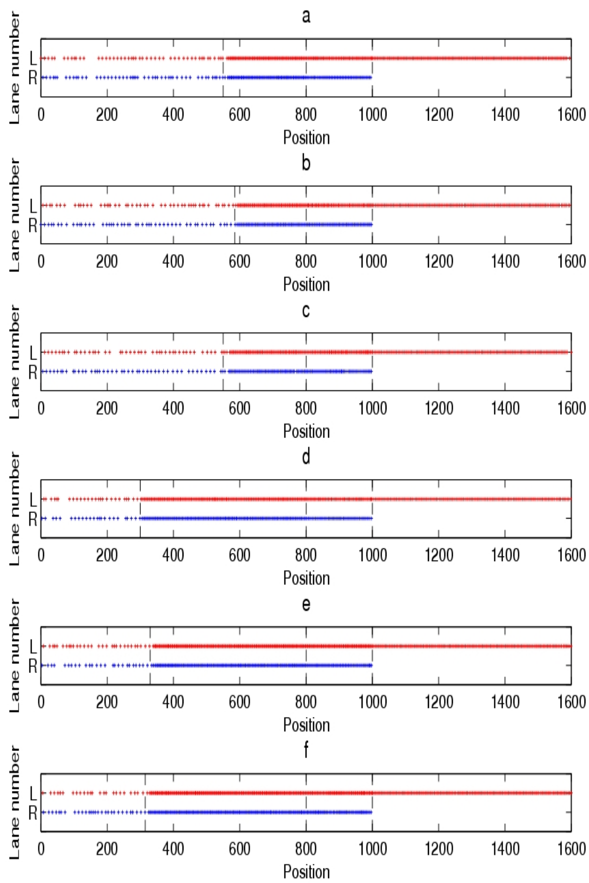

Figure 7 Traffic patterns for different lengths of section B L_B and speed limits $v_{B,\max}$ in section B at time $t = 5000$. Digrams (a)-(c) indicate, respectively, the patterns for $v_{B,\max} = 1.2$ and $L_B = 200$, $v_{B,\max} = 1.6$ and $L_B = 200$, and $v_{B,\max} = 1.6$ and $L_B = 400$ at density $\rho = 0.18$. Digrams (d)-(f) correspond to the patterns at density $\rho = 0.22$, respectively.

4. Conclusion

We have extended the classical optimal velocity model to investigate the traffic characteristic caused by the squeezing behaviors in the lane reduction bottleneck. Two kinds of vehicles including fast and slow vehicles are introduced. The asymmetric lane changing rules in the slowdown section and the lane squeezing rules at the bottleneck have been taken into account. The numerical simulations shows that the asymmetric lane changing rules, especially the lane squeezing behaviors cause the current of right lane to be larger than that of left lane in the slowdown section. An interesting phenomenon of ratio inversion that the fast vehicle distribution ratio of two lanes in section A is contrary to that in section B for low density appears. In addition, the length of section B and the speed limit of section B can influence the jam queue obviously.

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