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System Reliability Allocation Based on Bayesian Network

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Abstract: In this paper, system reliability allocation using BN(Bayesian Network) was researched. The relationship between system failure and component failure can be expressed by probability importance degree, structure importance degree and key importance degree in FTA (Fault Tree Analysis). BN supplies a new method to reflect the importance of components in system. That is the conditional failure probability which points out the most possible reason of system failure and is more reasonable and reliable. Some examples are given to allocate the reliability of systems by BN and the results show that BN is useful and effective.

Keywords: Reliability allocation, Bayesian Network, Importance degree, Conditional failure probability.

1. Introduction

For complex system, it is a difficult work to allocate its reliability. The importance of components in a system are different. The relationship between system failure and components failure is expressed by probability importance degree, structure importance degree and key importance degree in FTA [1,2,15]. They reflect the importance measurements of a component in whole system from different aspects. Probability importance degree is failure probability of system when one and only one certain component fails. It reflects that the system change caused by some small changes of a certain component and applies middle characteristic parameter for calculating structure importance degree and key importance degree. Structure importance degree is a particular result of probability importance degree and mostly used to allocate reliability. Key importance degree reflects the change rate of system failure probability caused by the change rate of a certain component failure probability. Key importance is mostly used to design system reliability parameters and arrange diagnosis and check-up sequence list[3–5].

Bayesian Network also called Bayesian Belief Networks (BBN), is a graphical method, sometimes known as a directed acyclic graph (DAG). The random variables are denoted by nodes and the directed arcs represent the conditional dependencies among the nodes. Each node has a probability density function associated with it. The arc emanates from a parent node to a child node. A node without any arcs linking into it is known as a root node, and a node with arrows linking into it is known as a child node. A child node without any arcs leading out is a leaf node. Each child node thus carries a conditional probability density function, given the value of the parent node. The entire BN can be represented using a joint probability density function[6–9].

According to Bayesian Formula:

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)} \quad (1)$$

Suppose A is a variable, and there are n states a_1, a_2, a_n, then from the total probability formula we can get:

$$P(B) = \sum_{i=1}^{n} P(B/A = a_i)P(A = a_i) \quad (2)$$

So we can get the posterior probability from Bayesian Formula.

BN can not only do forward reasoning, that means to get the posterior probability through the prior probability,

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but also can do backward reasoning, getting prior probability through the posterior probability. BN can ratiocinate the causes from the results. The reasoning methods of BN comprise accurate and approximate methods[10–13].

In Bayesian network root node can be got easily through its reasoning algorithm (such as Clique Tree Propagation and Bucket-Elimination etc.)[14,16,17]. The calculation formulas of three kinds of importance corresponding to the bottom event E_i in FTA are as follows.

Probability importance degree:

$$I_i^{Pr} = P(T=1|E_i=1) - P(T=1|E_i=0) \quad (3)$$

Structure importance degree:

$$I_i^{St} = P(T=1|E_i=1, P(E_j=1)=0.5, 1 \le j \ne i \le N) - P(T=1|E_i=0, P(E_j=1)=0.5, 1 \le j \ne i \le N) \quad (4)$$

Key importance degree:

$$I_i^{Pr} = \frac{P(E_i=1)P(T=1|E_i=1) - P(T=1|E_i=0)}{P(T=1)} \quad (5)$$

2. Component importance degree and conditional probability of bayesian network

The influence of components failure to the system can be expressed with three importance degrees in a FT of a system; they reflect the degrees of importance of components to the system. The physical significance of probability importance degree is the failure probability that the component x_i and only the component x_i failed, it reflects how the change of a component can cause the system change and it provides a necessary parameter to the calculation of structure importance degree and key importance degree. Structure importance degree equals to probability importance degree under particular condition and it can be a base to do reliability allocation. Key importance degree reflects the change rate of system fault probability causing by the change rate of the component fault probability and it can be used in reliability allocation, reliability design and fault diagnosing.

In FTA, to compute the probability of occurrence of the top event and middle event, it is necessary to get all the minimize cut sets firstly, and than one can compute it precisely through inclusive-exclusive theorem or through some approximate methods. While we calculate the probability of occurrence of a node, it is not necessary to get the minimize cut sets of the system. The system failure probability can be obtained through joint probability distribution as Equation 6.

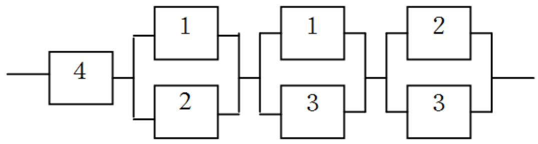

Figure 1 System reliability block-diagram.

$$P(T=1) = \sum_{E_1,\ldots,E_{M-1}} P(E_1=e_1,\ldots,E_{M-1}=e_{M-1},T=1) \quad (6)$$

While node $E_i(1<i<M-1)$ express the middle events and bottom events in a fault tree, $e_i \in 0,1$ express the occurrence of event E_i, M is the number of nodes in a BN. In addition to this, more information can be obtained through BN, such as when a event E_i happened, posterior probability of other events happens:

$$P_{B/A} = \sum_{E_1,\ldots,E_{i-1},\ldots,E_{j-1},\ldots,E_M} P(E_k=e_k, E_i=1, E_j=1)$$
$$P(E_i=1|E_j=1) = \frac{P_{B/A}}{P(E_j=1)} \quad (7)$$

While $1 \le k \ne M \le N, k \ne i, k \ne j$.

BN has the advantages and characters of applying bidirectional reasoning algorithm to identify the weak elements of system. Now an example is used to illustrate it. In Figure 1, suppose that the system and components all only have two states: normal or failure. The failure probabilities of components X_1, X_2, X_3, X_4 are 0.15, 0.2, 0.2 and 0.1 respectively. I_i^{Pr} represents the probability importance degree, I_i^{Cr} represents the key importance degree, I_i^{Sr} represents the structure importance degree, P_{is} represents system failure probability as component X_i fails. P_{si} represents component X_i failure probability as the system fails. Calculation results of these parameters are shown in Table 1. The sequences of each importance measurements and system conditional probability calculated by BN are shown in Table 2.

From Table 1 and Table 2, structure importance degree is only related to the position of the component in the structure. It has no relationship with component probability. Components X_1, X_2 and X_3 have the same positions so their structure importance degrees are the same. The sequence of system failure probability as component fails is the same as the sequence of component probability importance.

Table 1 Results of importance degree measurements.

Component Xi	X1	X2	X3	X4
I_i^{Pr}	2	3	3	1
I_i^{Cr}	3	2	2	1
I_i^{Sr}	2	2	2	1
P_{is}	2	3	3	1
P_{si}	3	2	2	1

Table 2 Sequence of importance degree measurements.

I_i^{Pr}	0.3352	0.2951	0.2951	0.9033
I_i^{Cr}	0.2688	0.3155	0.3155	0.4829
I_i^{Sr}	0.2500	0.2500	0.2500	0.5000
P_{is}	0.4240	0.3880	0.3880	1.0000
P_{si}	0.3550	0.4330	0.4330	0.5580

The sequence of component probability importance degree is more reasonable. Though the positions are the same of components X_1, X_2 and X_3, the failure probability X_1 is low and the failure probabilities X_2 and X_3 are high, it is exigent to improve the reliability of X_2 and $_3$.

Following is an example shows how to analyze a parallel and series system as Figure 2. The reliability of each component is: $R_1 = 0.97$, $R_2 = 0.87$, $R_3 = 0.81$, $R_4 = 0.93$ and the BN model is shown in Figure 3. The system reliability can be obtained from the BN model.

Furthermore the condition failure probability can be obtained through above BN model shown in Figure 5. The three kind of importance degree and component conditional probability which obtained through BN model are shown in Table 1. The Figure 6 shows the relations between the conditional failure probability of the four nodes under the condition that the system failed and component important degree, key importance degree and structure importance degree. From Figure 6 the component X_2 and X_3 has the same site, but the failure probability of component X_2 is less than that of component X_3, it is much more urgent to improve the reliability of component X_3 and component X_3 has a big impact to the system reliability. So using the conditional failure probability of a component to estimate which component is more important to the system reliability is reasonable.

3. Reliability allocation method

The conditional failure probability of each component in system has close relationship with the system reliability. The conditional failure probability is the function of system reliability. Conversely, we can use the conditional failure probability of component to allocate system reliabili-

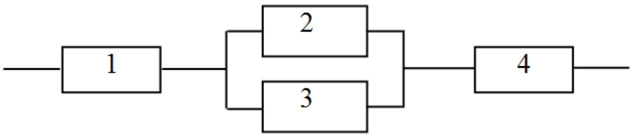

Figure 2 Parallel and series system.

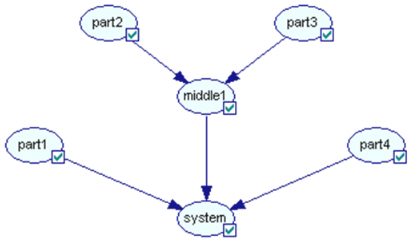

Figure 3 BN model of parallel and series system.

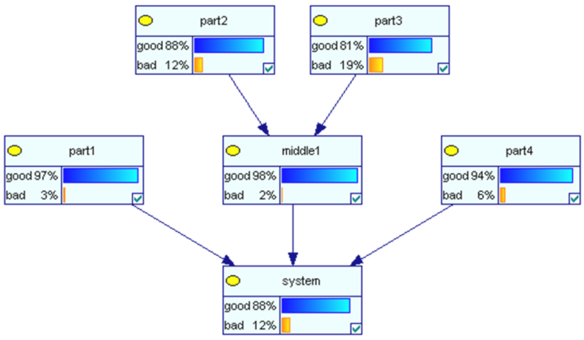

Figure 4 Reliability calculation of parallel and series system.

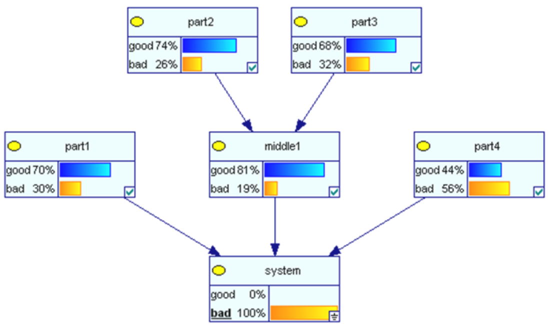

Figure 5 Component conditional failure probability of parallel and series system.

Table 3 Component conditional failure probability and importance degrees.

Components	1	2	3	4
Probability importance degrees	0.92	0.16	0.10	0.94
Conditional failure probability	0.30	0.26	0.32	0.56
Critical importance degrees	0.23	0.16	0.158	0.47
Structure importance degrees	0.37	0.13	.0.13	0.37

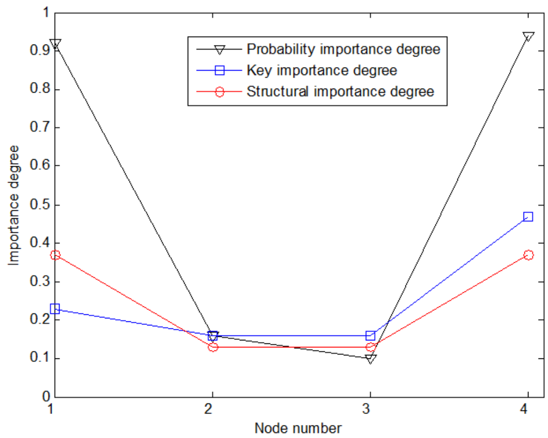

Figure 6 Curves of condition failure probability and component importance degrees.

ty. Normally, for a component in system, bigger the failure rate, higher the manufacturing cost, so a smaller reliability could be allocated. On the contrary, the component could be allocated bigger reliability as its failure rate is less. The conditional failure probability of component represents that as system fails which components are more likely to fail.

From this, the new method based on BN is proposed to allocate reliability. For the series system, conditional failure probabilities of all components $\omega_i(i = 1, 2, , n)$ should be obtained first by BN reliability model, and the sum of them $\sum \omega_i$. As the design reliability of system is R_s, the failure probability of each component being allocated is $F_i = 1 - R_s\omega_i \sum \omega_i$ and reliability of each component is $R_i = (1 - F_i)$.

Now an example is used to illustrate the method to allocate reliability by conditional failure probability.

A certain parallel and series system is shown in Figure 2. The reliabilities of each component allocated by experiences are $R_1 = 0.97$, $R_2 = 0.88$, $R_3 = 0.81$, $R_4 = 0.86$, $R_5 = 0.91$, $R_6 = 0.94$ respectively. Require that the system reliability R_s is 0.94 and allocate reliability to the system. The BN reliability model is shown in Figure 3.

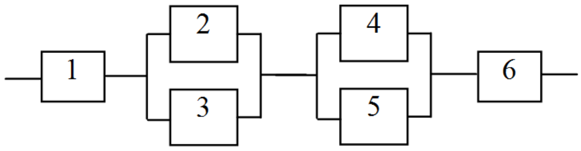

Figure 7 An example of parallel and series system diagram.

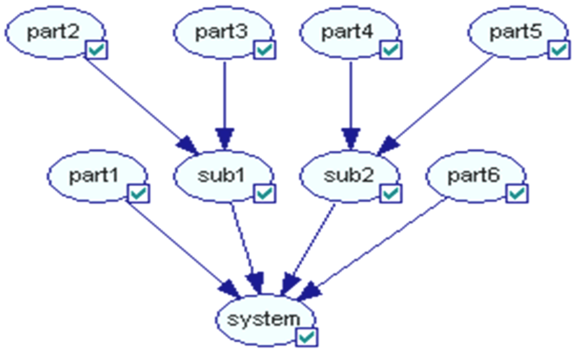

Figure 8 BN model of system reliability.

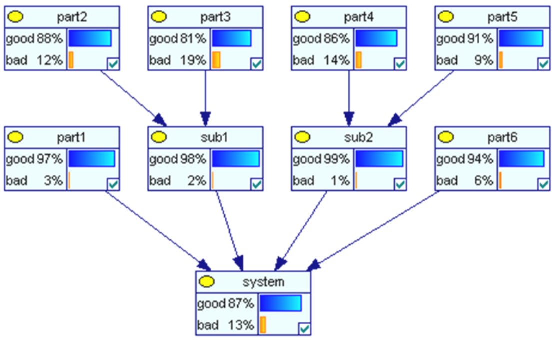

Figure 9 The results of the system reliability.

Forward reasoning of Bayesian network reliability model is shown in Figure 4, namely system reliability calculation and backward reasoning is shown in Figure 5, namely component failure probability as system fails.

Obtain from Figure 4 the system reliability $R_s = 0.8727$, which is smaller than required reliability so the system reliability needs to allocate again. First, calculate the failure probability of each component on the same system floor. Form the title, above the joint "system" there are four joints: "part1", "part2", "sub1" and "sub2". their conditional failure probability are $F_{part1|s} = 0.27$, $F_{sub1|s} = 0.17$, $F_{sub2|s} = 0.1$, $F_{part6|s} = 0.51$ respectively.

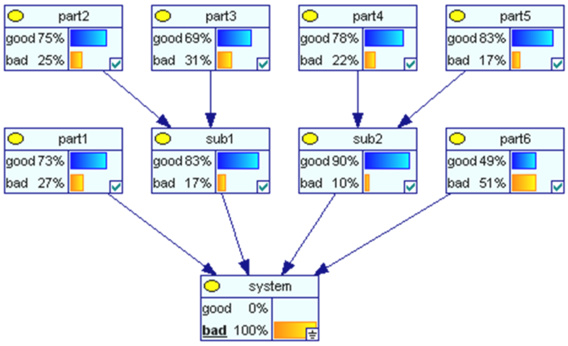

Figure 10 Components failure probability as system fails.

The sum of conditional failure probability of components:

$$F = \sum_{i=1}^{s} F_i^s = 0.27{+}0.17{+}0.1{+}0.51 = 1.05$$

The failure probabilities of components are allocated:

$$F_1 = F_s F_{part1|s}/F = 0.060.27/1.05 = 0.0154$$
$$F_{sub1} = F_s F_{sub1|s}/F = 0.170.27/1.05 = 0.0097$$
$$F_{sub2} = F_s F_{sub2|s}/F = 0.100.27/1.05 = 0.0057$$
$$F_6 = F_s F_{part6|s}/F = 0.060.51/1.05 = 0.0291$$

Next to allocate the sub system ”sub1” and ”sub2”. Which allocating method should be used according to the complex degree. If system can be transformed to series system the conditional failure probability can be continued to allocate reliability. For redundancy system the equal allocation method or proportion allocation method could be used. In this example we use the proportion allocation method to allocate reliability.

For $R_2 = 0.93$, $R_3 = 0.87$, $R_{sub1} = 0.9909$.
For $R_4 = 0.89$, $R_5 = 0.95$, $R_{sub2} = 0.9945$.
The new system reliability is:
$R_s = R_{part1} R_{part6} R_{sub1} R_{sub2} = 0.9420$,
so the result can satisfy the requirement of the system.

4. Conclusions

Reliable product is our needed not only in our life but also in industry. The reliability of product is assured by components. Only high reliability components can obtain high reliability system. How to allocate reasonable reliability to each component of system is very important and so that the components could be assured on design, manufacturing and experiments, etc. Through above research, it can be see that BN is much more flexible than common method such as FTA, RDB and other independent failure model. The ability of bidirectional reasoning and uncertainty assessment makes it a more suitable technique for multi-states systems and dependent failure system.

The relationship between system failure and component failure is expressed by probability importance degree, structure importance degree and key importance degree in FTA. Bayesian Network supplies a new method to reflect the importance of component in system. That is the conditional failure probability which points out the most impossible reason of system failure and is more reasonable and reliable. This new method is used to allocate the system reliability. Some examples are illustrated how to use conditional failure probability to allocate system reliability and the results proved that the method is valid.

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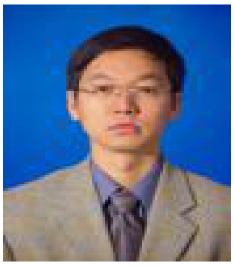

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