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Generation of Single Qubit Rotation Gates using Superadiabatic Approach

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Abstract: We discuss the quantum rotation gates in tripod system. We show that Stimulated Raman Adiabatic Passage (STIRAP) requires high Rabi frequencies to have a perfect rotation gate. Moreover, we improve this process by using superadiabatic approach. This approach requires additional Hamiltonian that can be implemented by driving the tripod with additional fields. Furthermore, we show that it is robust to the decay of the excited state, but not to the dephasing caused by collisions or phase fluctuations of the driving fields.

Keywords: Tripod, superadiabatic, STIRAP, rotation gates

1 Introduction

Shor proposed an efficient quantum algorithm for factorizing prime numbers [1] demonstrating that quantum computer can perform interesting computations much faster than any classical computer. The physical realization of quantum computer requires universally quantum gates to perform quantum operations. The most common quantum gate is the Hadamard gate, which is defined in the computational basis $\{|0\rangle, |1\rangle\}$ by the transformation

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \qquad (1)$$

The realization of this gate can be implemented adiabatically by combining two loops in parameter space or non-adiabatically by a single pulse pair [2]. The adiabatic implementation of the gate is based on stimulated Raman adiabatic passage (STIRAP). This STIRAP requires strong fields which is a disadvantage in many experiments. To overcome this requirement superadiabatic or transitionless approach have been recently proposed [3–6]. L Giannelli and E Arimondo [6] have discussed the robustness of superadiabatic transfer in three-level systems. They have shown that superadiabatic transfer overcome the difficulties associated with the adiabatic transfer (STIRAP).

In this paper we show how to generate single qubit rotation gates using superadiabatic approach. We begins in Section 2 by reviewing the basic concepts of quantum rotation gates. Section 3 describes the close system where all damping rates are neglected. In section 4, we study the effect of dephasing on the performance of the rotation gate. At the end a conclusion is given in section 5.

2 Background

The process of stimulated Raman adiabatic passage is one of the important techniques used to implement quantum gates. It is based on the adiabatic theorem which states that if the time-dependent Hamiltonian varies slowly and the system starts in one of its eigenstates, it will follow adiabatically this eigenstate [7]. Lacour et. al. [8] proposed an elegant experiment technique to implement generalized single-qubit Rotation gates in three-level lambda systems

$$R(a,\phi) = \begin{bmatrix} \cos a & e^{i\phi}\sin a \\ -e^{-i\phi}\sin a & \cos a \end{bmatrix}, \qquad (2)$$

where a is the angle of rotation and ϕ is the phase of the gate. This technique uses two STIRAPs. The first STIRAP is a reversed STIRAP, while the second STIRAP is a standard STIRAP. Each STIRAP has two pulses separated in time. The driving fields have large Rabi

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frequencies and large detuning so that the excited state can be adiabatically eliminated, and the system is left with only two states which form the desired qubit. If the two STIRAPs have the same pulse shapes with the same delay, the rotation gate (2) can be obtained up to a global phase. We extend their idea and use a tripod system rather than a three-level lambda system. The tripod consists of four-level system driven by three resonant laser fields with Rabi frequencies Ω_0, Ω_1, Ω_2. These laser fields couple the three lower levels $|0\rangle$, $|1\rangle$, and $|2\rangle$ to the upper level $|e\rangle$ as depicted in Fig. 1. The laser fields are modulated by Gaussian pulses with width δ, amplitudes A_j, phase ϕ_j, and time delay t_j

$$\Omega_j(t-t_j) = A_j e^{i\phi_j}\, e^{-\dfrac{(t-t_j)^2}{\delta^2}}. \tag{3}$$

In this paper, all parameters are scaled with respect to the width of the Gaussian pulses.

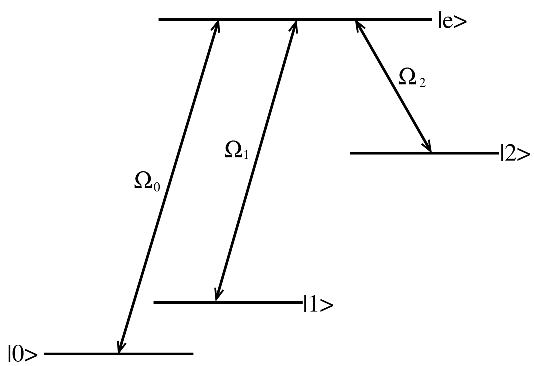

Fig. 1: Energy level for a four-level tripod. The three ground levels $|0\rangle$, $|1\rangle$ and $|2\rangle$ are coupled to the excited level $|e\rangle$ by three different lasers. The two ground states $|0\rangle$ and $|1\rangle$ are the states of the desired qubit.

Following Ref. [8], the two Rabi frequencies Ω_0 and Ω_1 are given by

$$\begin{aligned}\Omega_0 &= \Omega(t+T-\tau)+\Omega(t+T+\tau)\cos a,\\ \Omega_1 &= \Omega(t+T+\tau)\sin a.\end{aligned} \tag{4}$$

These fields represent two STIRAP processes separated by T in time, and each STIRAP has two pulses separated by τ in time. The first STIRAP is a reversed STIRAP starting with a constant ratio $\Omega_0/\Omega_1 \to \cot a$, while The second STIRAP process is a standard STIRAP where the pulses are switched on counter-intuitively and switched off in a given constant ratio $\Omega_0/\Omega_1 \to \tan a$. In addition to these STIRAP processes another STIRAP which consists of two pulses separated in time such that it starts before the two STIRAPs and ends after them (see Fig. 2).

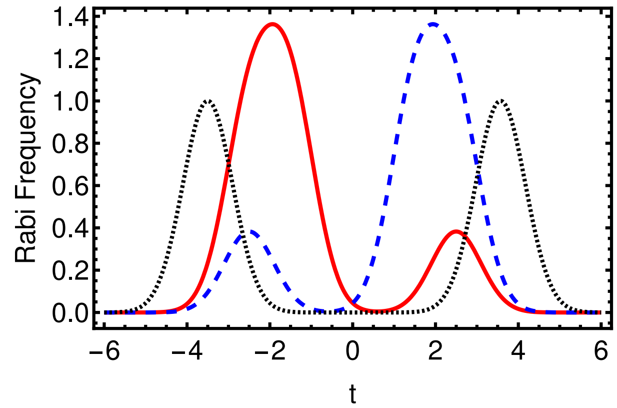

Fig. 2: The Rabi frequencies Ω_0 (solid line), Ω_1 (dashed line), and Ω_2 (dotted line), as a function of time. The parameters are: $A_0 = A_1 = A_2 = 1$, $\tau = 0.5$, $T = 2$, $a = \pi/8$. The time delay of the first(second) pulse of Ω_2 is $-3.5(3.57)$.

3 Close system

Closed system is a system which does not interact with the environment. That is, the decay rates of all atomic levels are ignored. In the close system, the time evolution of a quantum system is governed by the Schrodinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle. \tag{5}$$

where the Hamiltonian H. In the next section we describe the STIRAP process and show that the rotation gates can be obtained only for High Rabi frequencies.

3.1 STIRAP

In the close system the Hamiltonian H is given by

$$H = \frac{1}{2}\begin{bmatrix} 0 & 0 & \Omega_0 & 0 \\ 0 & 0 & -\Omega_1 & 0 \\ \Omega_0 & -\Omega_1 & 0 & \Omega_2 e^{-i\phi_2} \\ 0 & 0 & \Omega_2 e^{i\phi_2} & 0 \end{bmatrix}, \tag{6}$$

Where are the Rabi frequencies Ω_i are real numbers. This Hamiltonian was considered in Ref. [9]. Its has four eigenvalues. They are called the instantaneous adiabatic eigenvalues [9]

$$\lambda_{\pm} = \pm\frac{1}{2}\sqrt{\Omega_0^2+\Omega_1^2+\Omega_2^2}\,, \qquad \lambda_i = 0 \ (i=1,2). \tag{7}$$

The eigenstate corresponds to zero energy is a degenerate state. It is composed of two dark states

$$\begin{aligned}|D_1\rangle &= -\cos\theta_1 \sin\theta_0\,|0\rangle + \cos\theta_1\cos\theta_0\,|1\rangle + \sin\theta_1 e^{i\phi_2}\,|2\rangle,\\ |D_2\rangle &= \cos\theta_0\,|0\rangle + \sin\theta_0\,|1\rangle,\end{aligned} \tag{8}$$

where

$$\tan\theta_0 = \frac{\Omega_0}{\Omega_1}, \qquad \tan\theta_1 = \frac{\sqrt{\Omega_0^2+\Omega_1^2}}{\Omega_2}. \tag{9}$$

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The eigenstate that corresponds to non zero eigenvalue is called bright state. So, there are two bright states which correspond to the non zero eigenvalues $\lambda_\pm$

$$|\pm\rangle = \frac{1}{\sqrt{2}}[-\sin\theta_1\sin\theta_0\,|0\rangle - \sin\theta_1\cos\theta_0\,|1\rangle \pm|e\rangle + \cos\theta_1 e^{i\phi_2}\,|2\rangle\,]\,. \tag{10}$$

According the adiabatic theorem [7], a system remains in its instantaneous eigenstate if its time-dependent Hamiltonian varies slowly compared with the energy difference between eigenstates. If the tripod system starts in the superposition of the two dark states it remains in a superposition of these dark states at later time.

To measure the performance of the rotation gate we use the fidelity which is given by

$$F = \left|\langle\psi(t_f)|R(a,\phi)|\psi(t_i)\rangle\right|, \tag{11}$$

where $|\psi(t_i)\rangle$ represents the initial state at time t_i and $|\psi(t_f)\rangle$ is the final state at time t_f. The final state is obtained by solving the Schrodinger equation (5) at the end of the evolution. For numerical computations we focus on the generation of the rotation gate $R(\pi/4,0)$ and we set $\phi_2 = 0$.

In Figure 3 we plot the Maximum, minimum and average fidelity as a function of common amplitude A of the Gaussian pulses for the rotation gate with angle $a = \pi/4$ and phase $\phi = 0$. The fidelity are computed numerically for 1000 initial random states uniformly distributed on the Bloch Sphere as follows.

$$|\psi(t_i)\rangle = \cos(\pi u)|0\rangle + \sin(\pi u)\,e^{i\arccos(2v-1)}|1\rangle,$$

where u and v are two random numbers uniformly distributed on $[0,1]$. It is clear that the fidelity is close to 1 only for large values of A. Large Rabi frequencies is a disadvantage in many experimental applications. In the next section we show how to implement the rotation gate $R(\pi/4,0)$ with small Rabi frequencies by introducing superadiabatic approach (sa-STIRAP).

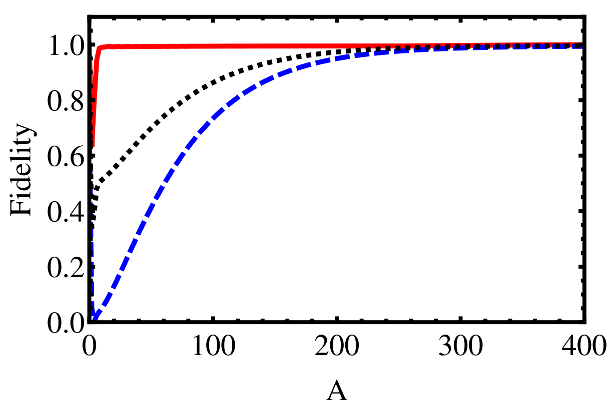

Fig. 3: Fidelity for rotation gate $R(\pi/4,0)$. The Maximum (solid line), the average (dotted line) and the minimum (dashed line) fidelity as a function of A. The fidelity for the STIRAP process approaches 1 only for high Rabi frequencies.

3.2 Superadiabatic process

The process of superadiabatic or transitionless adiabatic passage is a process in which there is not transition between the adiabatic states. In recent paper [6] different superadiabatic corrections were discussed for three-level lambda systems. These corrections require the application of additional pulse which couples between the two lower states. This additional coupling can be implemented using magnetic field which has π-area or near π-area. They have shown that the application of sa-STIRAP will produce the desired transfer and its robustness is much larger than STIRAP.

In the superadiabatic approach the total Hamiltonian is given by

$$H = H_0 + H_1, \tag{12}$$

where H_0 is given by eq. (6) and H_1 is the superadiabatic correction [3, 5, 6]

$$H_1 = i\sum_n [|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|] \tag{13}$$

where the sum is over all the instantaneous eigenstates. This Hamiltonian can be written in a matrix form as

$$H_1 = \begin{bmatrix} 0 & h_{0,1} & 0 & h_{0,2} \\ h^*_{0,1} & 0 & 0 & h_{1,2} \\ h^*_{0,2} & h^*_{1,2} & 0 & 0 \end{bmatrix}. \tag{14}$$

where

$$h_{0,1} = i\frac{\Omega_0\dot{\Omega}_1 - \Omega_1\dot{\Omega}_0}{\Omega_0^2 + \Omega_1^2},$$

$$h_{0,2} = i\Omega_0\frac{(\Omega_0\dot{\Omega}_0 + \Omega_1\dot{\Omega}_1)\Omega_2 - (\Omega_0^2+\Omega_1^2)\dot{\Omega}_2}{(\Omega_0^2+\Omega_1^2)(\Omega_0^2+\Omega_1^2+\Omega_2^2)},$$

$$h_{1,2} = i\Omega_1\frac{(\Omega_0\dot{\Omega}_0 + \Omega_1\dot{\Omega}_1)\Omega_2 - (\Omega_0^2+\Omega_1^2)\dot{\Omega}_2}{(\Omega_0^2+\Omega_1^2)(\Omega_0^2+\Omega_1^2+\Omega_2^2)}.$$

With our Gaussian pulses the term $h_{0,1} = 0$. That is, the Hamiltonian H_1 is equivalent to additional driving fields which couple the two lower levels $|0\rangle$ and $|1\rangle$ to the level $|1\rangle$.

In Figure. 4 we plot the fidelity as function of the amplitude A. It shows that sa-STIRAP leads to a perfect rotation gates for all Rabi frequencies. This is an important improvement over STIRAP which needs high Rabi frequencies.

4 Open system

Close system is an ideal system. In reality the tripod is interacting with the environment and its subject to decays.

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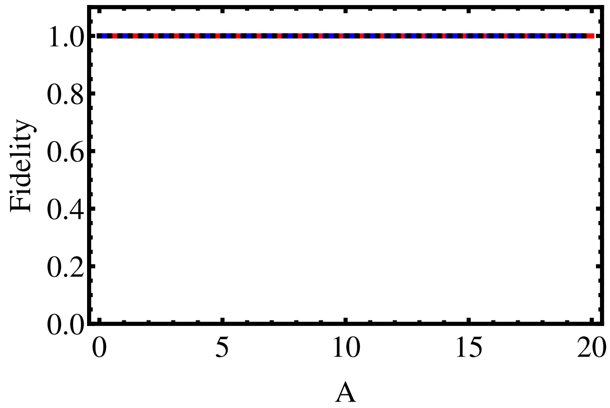

Fig. 4: Fidelity for rotation gate $R(\pi/4,0)$ as a function of A. The fidelity is 1 for all Rabi frequencies.

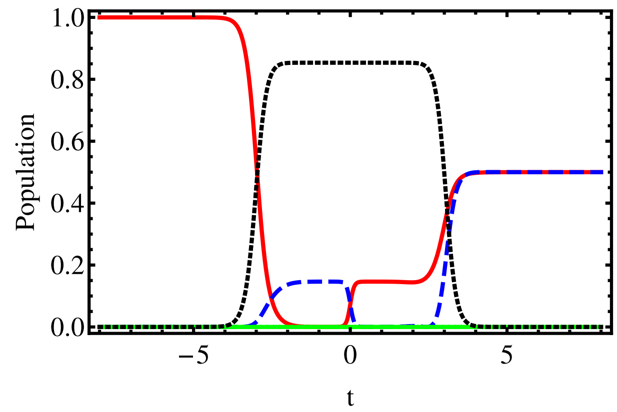

Fig. 5: Population of the atomic states. The atom is initially in the state $|0\rangle$. At the end of the evolution the population of the each of the lower states is 0.5. The excited state is unpopulated. There is no difference between the close system and the open system when the excited state loss is considered.

In the presence these decays the evolution of the system is given now by the Lindblad master equation

$$\dot{\rho} = -i[H,\rho] + \frac{1}{2}\sum_i \left(2C_i\rho C_i^\dagger - C_i^\dagger C_i\rho - \rho C_i^\dagger C_i\right), \quad (15)$$

where ρ is the atomic density operator, H is the Hamiltonian operator for the closed system, and C_i are the Lindblad operators associated with the decoherence.

Figure 5 shows the evolution of the population when the initial state is $|0\rangle$ and Figure 6 when the initial state is $|1\rangle$. Both of them do not show any difference between close system and open system in the presence of the decay rate of the excited state (we have used the damping rate of the excited state equal to 10). Thus, The rotation gate is robust under the loss of the excited state because the excited state is unpopulated during the evolution. This means that the decay rate of the exited state has a negligible effect.

So, it is interesting to check the robustness in the presence of dephasing caused by collisions or phase fluctuations of the fields. The effect of dephasing on the STIRAP has been explored by various authors. Ivanov *el al* [10] found that the population transfer efficiency of the STIRAP is found to decrease exponentially with the dephasing rate. Here we restrict ourselves to the dephasing of the ground state $|0\rangle$ which can be described by the Lindblad operators $C_0 = \sqrt{2\Gamma_0}|0\rangle\langle 0|$, where Γ_0 is the dephasing rate. Figure 7 shows the fidelity as a function of the dephasing rate Γ_0 for $A = 1$. One can see that the fidelity decrease linearly and the rotation gate become imperfect. So, the dephasing caused by collisions or phase fluctuations of the fields produces significant effect on the performance of the rotation gate.

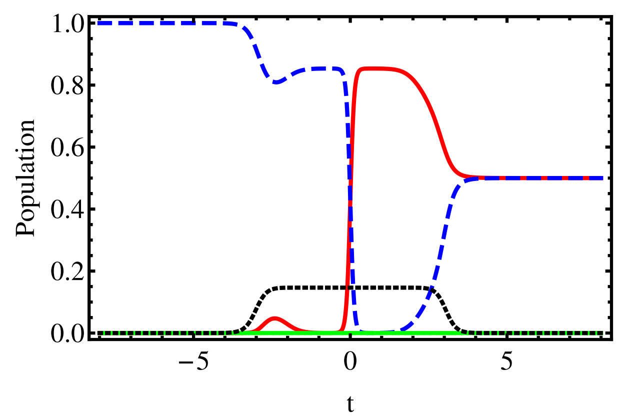

Fig. 6: Same as Figure.5 except the atom is initially in the state $|1\rangle$.

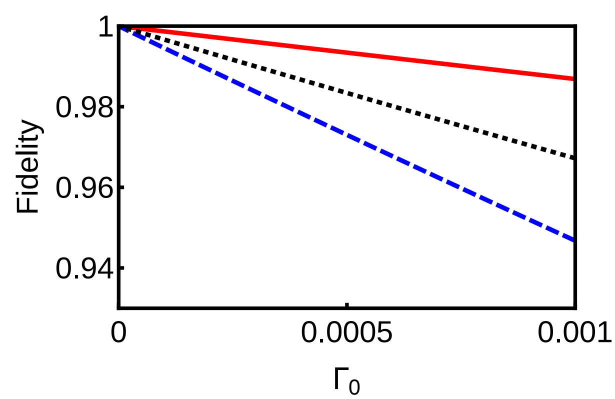

Fig. 7: Fidelity for rotation gate. The Maximum, the average and the minimum fidelity as a function of Γ_0 for $A = 1$.

5 Conclusion

In this paper we have discussed the generation of single qubit rotation gates.We have focused on the rotation gate

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with angle $a = \pi/4$ and phase $\phi = 0$. This study have shown that the STIRAP requires high Rabi frequencies to implement the rotation gates. To overcome this disadvantage we use superadiabatic approach that leads to a perfect gate for small Rabi frequencies.Furthermore,we have explored the effect of dephasing on the performance of the gate. We have shown that is robust to the decay rate of the excited state because the excited state is unpopulated during the evolution. However, the dephasing which cause by collisions or phase fluctuations of the field can leads to imperfect gate. Therefore, to get a perfect gate one must keep the dephasing as small as possible.

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