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Solving Multiple Traveling Salesman Problem using the Gravitational Emulation Local Search Algorithm

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Abstract: Multiple Travelling Salesman Problem (mTSP) is one of the most popular and widely used combinatorial optimization problems in the operational research. Many complex problems can be modeled and solved by the mTSP. To solve the mTSP, deterministic algorithms cannot be used as the mTSP is an NP-hard optimization problem. Hence, heuristics approaches are usually applied. In this paper, the Gravitational Emulation Local Search (GELS) algorithm is modified to solve the symmetric mTSP. The GELS algorithm is based on the local search concept and uses two main parameters in physics, velocity and gravity. Performance of the modified GELS has been compared with well-known optimization algorithms such as the genetic algorithm (GA) and ant colony optimization (ACO). Simulation results show superiority of the modified GELS over the other common optimization algorithms.

Keywords: MTSP, GELS, Optimization, Velocity

1 Introduction

The Travelling Salesman Problem (TSP) is one of the well-known problems in combinatorial optimization and many researchers have tried to solve this problem with different schemes so far. In the TSP problem, the objective is on finding the shortest path between a set of n randomly located cities in which each city is visited only once [1,2]. The mTSP is a generalization of the well-known TSP, where one or more salesman can be used in the solution [3]. As the TSP is an NP-hard problem, heuristic methods are needed to solve this problem [4]. In recent years, some well-known methods used to solve this problem are the evolutionary algorithms including the genetic algorithm (GA), stimulated annealing (SA), ant colony optimization (ACO), artificial neural networks (ANN) and particle swarm optimization (PSO).

In [3], a review of the mTSP problem and its applications was presented to highlight some formulations and to describe exact and heuristic solution procedures proposed for this problem. In [4], an interpretable representation based algorithm is proposed to solve the mTSP. Performance of the proposed algorithm is analyzed using different round tour types in simulation. [5] suggests a crossover operator titled two-part chromosome crossover (TCX) to solve the mTSP applying a GA for near-optimal solutions. The TCX representation technique minimized the size of the problem search space and its limitations were overcome. In [6], an effective neural network algorithm was developed to solve the mTSP based on transforming the mTSP to the TSP. This algorithm was checked on numerous problems with up to 30 cities and 5 salesmen, and in each test case, it always converged to acceptable solutions. An enhanced genetic algorithm was demonstrated to prepare an alternative and efficient solution to the problem in [7]. The initial population was generated by greedy strategy. Convergence speed increases and at the same time complexity is significantly reduced. The proper genetic operators were proposed in [8] to apply for the goal by design-of-experiments (DOE) which were examined in this article and it could be utilized to use in the genetic algorithms which applies the TC encoding method. An effort was made in [9] to present the ant colony optimization algorithm (ACO) that can be applied to the mTSP with ability restriction. The ACO was compared with modified genetic algorithm by examining various standard problems from TSPLIB. The

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TSPLIB is a library of TSP examples and related problems from several sources and of various kinds. An enhanced genetic algorithm for the mTSP was offered in [10]. In this algorithm, a pheromone-based crossover operator was designed, and a local search procedure was used to act as the mutation operator. An ant colony optimization algorithm for the mTSP with two goals was offered in [11]. A local hybrid algorithm was presented in [12], which was modified by sweep and ant colony algorithms (SW+ AS_{elite}) for solving the mTSP. An optimized model for the balanced multi-salesman problem with time and capacity constraints was proposed in [13] which required a salesman visits each vertex at least once and returned to the starting vertex within given time. H. Singh, R. Kaur [14] deal with the GA and also provided details how to solve the mTSP Using the GA. A columnar competitive model (CCM) of neural networks incorporates with a winner-take-all learning rule was employed to solve the mTSP [15]. Stability conditions of CCM were exploited by mathematical analysis. A new heuristic method called randomized gravitational emulation search (RGES) algorithm was presented to solve the symmetric TSP in [16]. It was found upon introducing randomization concept along with velocity and gravity, through swapping in terms of groups by using random numbers in the existing local search algorithm, it can be avoid local minima and yield global minimum for STSP. In [17], a novel hybrid method (i.e. Hr-GSA) was proposed by combining gravitational search algorithm (GSA) with the simulated annealing (SA) method. Simulation results show that Hr-GSA was more robust and efficient than other traditional population based algorithms, such as the genetic algorithm, particle swarm optimization, and the artificial immune system.

In this paper, the Gravitational Emulation Local Search (GELS) algorithm is modified to solve the symmetric mTSP. Then, performance of the modified GELS has been compared with well-known optimization algorithms such as the GA and the ACO. Simulation results show superiority of the modified GELS over the other existing optimization algorithms in the literature. As Table 1 clearly presents, several exact solution procedures exist, consisting mainly of branch-and-bound type methods, which are limited to solving only problems of reasonable sizes. On the other hand, we observe that the literature has a tendency on heuristic solution techniques, of which Neural Network-based procedures seem to be the most popular. There also exist some transformation-based procedures. Solution procedures based on transforming the mTSP to the standard TSP do not seem efcient, since the result- ing TSP is highly degenerate, especially with the increasing number of salesman.

The rest of this paper is organized as follows. Section 2 introduces the mTSP problem. In Section 3, the GELS algorithm is explained. In Section 4, the GELS is used to solve the symmetric mTSP. In Section 5, simulation results are shown and performance of the GELS is compared to other existing optimization algorithms in the literature. Finally, a conclusion is given in Section 6.

2 MULTIPLE TRAVELLING SALESMAN PROBLEM

In the mTSP, there are m salesmen travelling a set of n cities, and each salesman is defined to start and end at the same city. In simulations, each city must be visited exactly once by only one salesman and its objective is to minimum total distances travelled by all the salesmen. As the number of salesmen is not fixed, each salesman has a related fixed cost incurring any time which is applied in the solution. This causes decreasing in the number of salesmen which should be activated in the solution. Time windows are frequently combined into the mTSP, where it is also specified that particular points should be met in certain time periods. The major purpose is on declining the entire traveling cost of the problem that is frequently developed as the following, based on integer linear programming [4]:

$$\min \sum_{i}^{n} \sum_{j}^{n} C_{ij} X_{ij} + mc_m \quad (1)$$

subject to

$$\sum_{j=2}^{n} X_{1j} = m \quad (2)$$

$$\sum_{j=2}^{n} X_{j1} = m \quad (3)$$

$$\sum_{i=1}^{n} X_{ij} = 1, \ldots, n \quad (4)$$

$$\sum_{j=1}^{n} X_{ij} = 1, \ldots, n \quad (5)$$

$$\text{subtour elimination constraints} \quad (6)$$

where $X_{ij} \in \{0,1\}$ is a binary variable indicating a used arch on the tour, C_{ij} refers to the cost related to the distances between the i^{th} and j^{th} nodes, and C_m represents the cost of one salesman's participation.

An example of the mTSP is depicted in Fig. 1, where $m = 3$ and $n = 7$. Transforming the mTSP with n cities into the TSP with$n+m-1$ cities by introduction of $m-1$ artificial points $(n+1,\ldots,n+m-1)$ was suggested in [9]. This transformation is illustrated in Fig. 2. When an mTSP is transformed to a single TSP, the resulting problem is more difficult to solve than an ordinary TSP with the same number of cities [39]. While the general objective of the mTSP is to minimize the total distance, generally, m-1 cities always exist to be chosen as the

Table 1: Solution procedures proposed for the mTSP

Type of approach	Solution procedure
Exact solution	Formulations and to describe exact and heuristic solution [3]
	Graph Theory [9]
	Integer linear programming formulations [18, 19]
	Cutting plane [20]
	Branch and Bound [21, 22]
	Lagrange an relaxation + branch and bound [23]
Heuristics	Ant Colony [8, 11]
	Sweep Algorithm [12]
	Particle Swarm Optimization [13]
	Columnar competitive model + neural networks [15]
	Simple heuristics [24, 25]
	Evolutionary algorithm [26]
	Simulated annealing [27]
	Genetic algorithms [4, 5, 7, 9, 14, 28, 29]
	Neural networks [6, 30, 31, 32]
	Tabu search [33]
Transformations	Asymmetric mTSP to asymmetric TSP [34]
	Symmetric mTSP to symmetric TSP [35, 36]
	Multi-depot mTSP to TSP [37, 38]

nearest cities for a round trip. As a result, a TSP formed by the remaining $n-m+1$ cities needs to be solved. For the mTSP with m salesmen, there are $m-1$ salesmen travelling only one city, and one salesman needs to travel the left $n+m-1$ cities. In reality, every salesman has the same abilities and limitations. Hence, the mTSP with ability constraint is more appropriate in the real world problems [40].

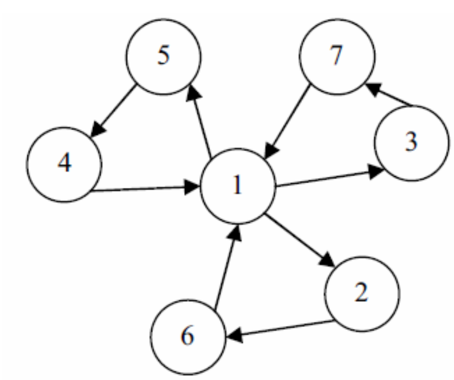

Fig. 1: Example solution of the mTSP [9]

3 THE GELS ALGORITHM

In 1995, an algorithm called GLS was proposed based on gravity. The GLS is an intelligent search scheme or a combinatorial optimization algorithm. Its main feature is the iterative use of a local search and information is

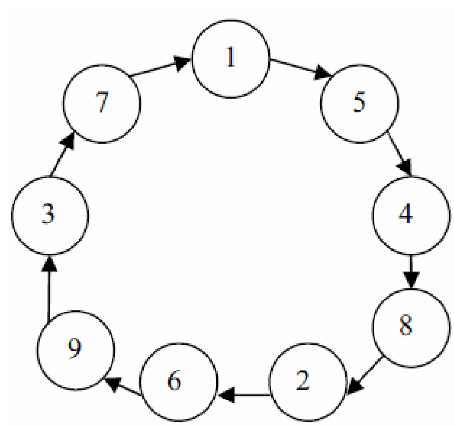

Fig. 2: Transformation from mTSP to TSP [9]

gathered from various sources and is exploited to guide local search toward promising parts of the search space [41].

In 2004, Webster [42] developed an algorithm to localize optimal solutions for difficult problems. This algorithm was called the Gravitational Emulation Local Search algorithm (GELS). The idea was based on gravity which causes objects attracted to each other, therefore; a heavier object has more gravity and attracts lighter objects. Of course, the distance between two objects affects this gravity according to the Newton's law in (7). If there are two objects with equal weights and different distances to the lighter object, the object with shorter distance to the lighter object will apply more gravity on it.

$$F = \frac{GM_1M_2}{R^2} \quad (7)$$

Where M_1 and M_2 respectively refer to the mass of object 1 and 2. G represents the gravitational constant and R is the distance between two objects.

The GELS emulates these natural processes to formulate a heuristic algorithm. The idea is on imaging the search space as being the universe. Contained within the search space are one or more valid solutions to the problem. Each solution has a mass which is represented by its objective function value. The best solution has the higher mass. Locations in the search space that do not have any solutions are considered as a zero mass. A small object represented as a pointer is moving through the search space. As it approaches a solution object, the mass of the solution object will cause the pointer object to be pulled towards it. Newtons law in (7) is used to define how much gravitational force exists between the pointer object and the solution object [43].

In the GELS, possible solutions in the search space are divided into categories based on a criterion depending on the type of the problem. Each of these new divided categories is called a dimension of the problem solution. A value as an initial velocity is determined for each dimension of the problem solution. The GELS includes a vector whose size determines the number of solution dimensions. The value of this vector reflects the relative velocity in each dimension. The algorithm starts with an initial solution, initial velocity vector, and the direction. A random number is chosen between one and the maximum of each dimension in the velocity vector. The initial solution, as the current solution, is produced by a user or randomly. A direction is chosen based on initial velocity vector of solution dimensions for each dimension in the initial velocity vector. The algorithm will be ended with the occurrence of one of the following conditions: all components of initial velocity vector equals zero or the number of iterations reaches to its maximum.

Gravity between two objects can also be calculated by (8). Eq. (8) is derived from (7) by replacing masses with a difference between the cost value of the candidate solution and the cost value of the current solution [42]:

$$F = \frac{G(CU - CA)}{R^2} \quad (8)$$

Where CU and CA are the cost values of the current and candidate solutions respectively. If the current solution cost value is greater than the candidate one, (8) has a positive value; otherwise, it is negative. Then, this force value will be added to the velocity vector in the current path. Hence, if the velocity value exceeds the maximum arrangement, it will reach the maximum value.

The GELS Parameters are as follow [43].
Maximum velocity: maximum value that can be assigned to each initial velocity vector components. This parameter avoids extra growing of this component.
Radius: the radius, or R, is used in gravity calculating formula.
Iteration: it determines the maximum number of iterations and ensures that the algorithm will be ended.

Fig. 3 shows the GELS flowchart. As it can be seen, first an initial response of the problem is created, and evaluation for each mass is achieved. Then, the problem is updated as a *G*, or *Best*, and or *Worst* and the parameters *m* and *a*, are calculated for each mass. Then *Speed* and *location* of each mass are also updated as well. Finally the algorithm will be terminated if the maximum number of iterations meets or all the initial velocity vector elements become zero. Otherwise, the algorithm goes back to step 2 and continues until, the optimal answer reaches [43,44].

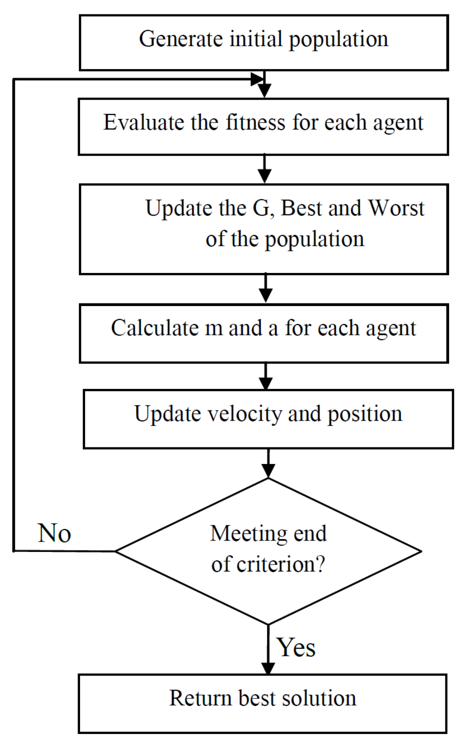

Fig. 3: The GELS flowchart [44]

4 MODIFYING THE GELS TO SOLVE THE MTSP

In this paper, the GELS algorithm is modified as a strategy to solve the mTSP. The goal is on finding the shortest path travelled by the salesmen between cities.

4.1 Solution Dimensions Definition

In the modified GELS, each solution dimension can be considered as a city. In fact, the number of solution dimensions equals the number of input cities for the mTSP problem. The neighbor of the current solution in the dimension equals the city which has the shortest distance, minimum time and the highest velocity toward the current solution for each salesman and the salesman has not traveled that city yet.

4.2 Neighborhood Definition

In the GELS algorithm, unlike the other gravitational search algorithms, search for the neighbor solution is not done randomly. In the GELS, each current solution has different neighbors and each neighbor is determined based on the direction toward the neighbor solution. In the modified GELS, to find a neighbor solution, we choose a city with the shortest distance, minimum time and the highest velocity toward the current solution as a neighbor and a candidate solution for every salesman.

4.3 Modified GELS

In this paper, we applied modified GELS which is the extension of GELS algorithm, in order to solve the above problem. modified GELS is initially classified using sweep algorithm of intended cities in the problem and then simultaneously solves each category created for each seller by separate TSPs method. In this way for each seller, our cities should be located within an array in per mutational manner, and then the cost of distance between cities should be calculated as current response for sellers. This is done by assigning a random value associated with experimental values of adjustable operating parameters. next modified GELS running is started and when it was ended, the output is the shortest distance between cities for each seller and also indicates the amount of time that is lasted until the distance between cities is traversed by sellers.

Using this method, we have received a relatively good initial solution of the problem and in order to improve this solution we will use record to record algorithm on the obtained solution by modified GELS compared to other methods. considering displacements which occur in cities for sellers with record to record algorithm in Tow-point and one-point manner, when we run record to record algorithm on any category for each seller, it will determine that each seller must go in which direction and cross the cities to traverse the minimum distance And therefore the solution will be improved.

In Fig. 4, the pseudo codes of the modified GELS are presented to solve the mTSP. In the first step, the parameters of the problem are defined and then, the CU and CA are generated. If the current response cost is greater than the candidate response cost, the obtained solutions are positive. Fig. 5 illustrates the flowchart of the modified GELS. First of all, the cities are divided into different groups. Then, each group of cities forms a TSP problem for each salesman and the problem will be solved by the modified GELS.

According to Fig. 4 and Fig. 5, the modified GELS starts from an initial or current response called *CU* indicating being a single chromosome of the method. Next, the secondary or candidate response (*CA*) will be created according to the neighbor and the cities which can satisfy limitations of the problem. At this point, gravitational force is calculated between these two masses and will be considered as a solution to the problem. Then, speed and mass will be updated. Speed makes the modified GELS searches only the best and the most efficient solutions each time and attracts them. This will cause an increase in object mass in gravitational force which is the reason of the modified GELS superiority. Finally, as it is known in gravity law, an object with higher weight will have more gravitational force and therefore it will attract the best response toward itself. The modified GELS is repeated so that the object will have more weight by absorbing optimal solution each time and thus, it will return the most optimal solution.

5 SIMULATION RESULTS

The modified GELS, was coded in C# language on a Ci3, 2.2 GHz CPU with a 1024 MB RAM. Simulation results are compared to the other existing optimization algorithms in the literature. Standard issues of the mTSP are also included in the modified GELS simulations, which are derived from the TSPLIB library and articles in [45,46]. These results are illustrated in Tables 1 to 5. Comparison parameters in these tables are time and travelled distance in a journey.

Table 2 compares the modified GELS with the ACO and SW+ ASelite to solve the mTSP. As it can be seen, the average of the total distance computed by the modified GELS has 13.7 percent improvement over the ACO and 10.2 percent improvement over the SW+ ASelite. Hence, the modified GELS shows better performance compared to the ACO and SW+ algorithms. Furthermore, the GELS solves the mTSP faster with the

```
Begin
1 Parameters are Distance, Velocity, Time, CU, CA, R, F, V, K, N, M, Best Fit CU & Best Fit CA.
2 Generate a feasible solution using the Sweep Algorithm.
3 Set Record= objective function value of the current solution, Set Best Record= Record,
        Set Deviation= 0.01 × Record, Set itr= 0.
4       Generate a feasible solution using the GELS Algorithm.
5 Distance = Create Matrix Distance (), Velocity = Create Matrix Velocity (), Time = Create  Matrix  Time (),
  CU= Create Parent (), CA= Create Child (), Best Fit CU= Fitness (CU), Best Fit CA= Fitness (CA).
6 While (Number Of City's Region)
7         if Best Fit CA < Best Fit CU then
            R=Calculate (CA, Distance).
            F= 6.672*(Best Fit CU-Best Fit CA) / (R^2).
            V= Matrix Velocity (CA.City_Current, CA.City_Old).
            V= V + F.
            Velocity (CA.City_Current, CA.City_Old) = V.
   Time (CA.City_Current, CA.City_Old) = (Distance (CA.City_Current, CA.City_Old)*60) / V.
            Swap (CU, CA).
              CA= Create Child (), CA.Fit = Fitness (CA).
        Else Empty (CA).
            CA= Create Child ().
8     end if
9  end while
10 while itr ≤M do
11              Set count = 0.
12              while count ≤K do
13              for i= 1 to I do
14               One Point Move with GELS travel In One Path, One-Point Move In between Routs,
Two Point Move, with GELS travel between routes and Two-opt Move with GELS travel. Feasibility must
be maintained.
15                      if no feasible GELS travel move is made then go to line20
16                      end if
17              if a new record is produced then update Record and Deviation, CU=CA, CA=Create New CU Of CA &Set
count = 0.
18                      end if
19              end for
20              For the current solution, apply One Point Move (within and between routes)
                Two Point Move (between routes), Two-opt Move (within and between routes)
                        if a new record is produced then update Record and Deviation, CU=CA, CA= Create New CU Of
                        CA & Set count = 0.
21                      end if
22                      Try to insert each route between each pair of consecutive nodes of another route.
                        Feasibility must be maintained. If it is possible, we make the insertion even if the
                        total distance traveled increases since we want to use as few vehicles as possible.
23                      count = count + 1.
24              end while
```

Fig. 4: The modified GELS pseudo codes

minimum travelled distance. The mean values of Table 1 results are illustrated in Fig. 6.

Table 3 demonstrates comparison results of the modified GELS and the enhanced GA [10] to solve the mTSP. As it is shown, average of the total distance computed by the modified GELS has 18.8 percent improvement over the enhanced GA. Therefore, the modified GELS has again significant superiority in achieving more optimal and acceptable solutions even in highly complex scenarios. Fig. 7 illustrates the mean values of Table 2 results.

Table 4 shows simulation results of the modified GELS and the ACO [11]. The goal is on minimizing the total distance travelled by salesmen and time required to solve the mTSP. As it is indicated, the average of the total distance computed by the modified GELS has 13.95 percent improvement over the ACO. It can also be seen that the modified GELS performs close to the ACO in three basic problems in this table. Fig. 8 shows the mean values of Table 3 results.

Table 5 demonstrates simulation results of the modified GELS and the TCX [5] minimizing the total travelled distance and time by a salesman. As it is shown, the average of the total distance computed by the modified GELS has 26.8 percent improvement over the TCX. Fig. 9 shows the mean values of the Table 4 results. These results insure that the modified GELS has ability to solve various problems.

In all the aforementioned tables, Time is given in seconds and calculated by a Ci3, 2.2 GHz CPU with a 1024 MB RAM. Avg. is the average distance for each problem and %Improvement is equal to ((avg. value of a

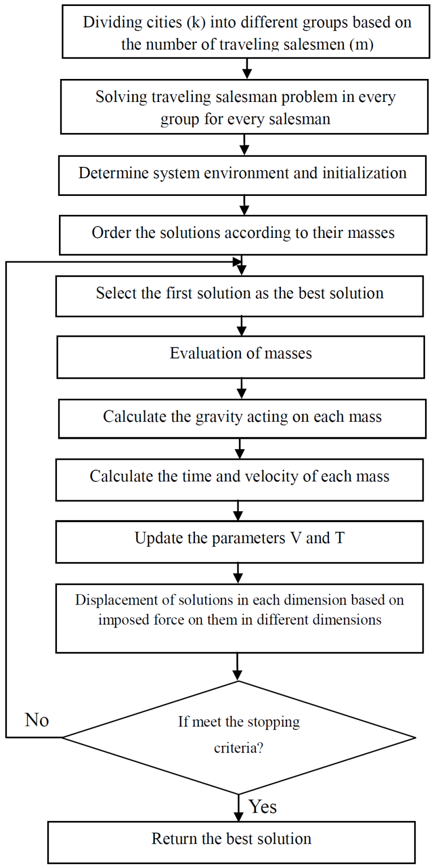

Fig. 5: The modified GELS flowchart

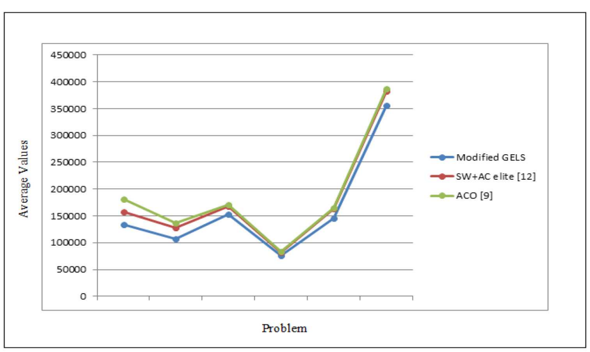

Fig. 6: Mean values of the results in Table 1

compared algorithm - avg. value of the modified GELS)/avg. value of a compared algorithm) 100.

In overall, as the modified GELS finds the shortest path between cities travelled by salesmen and solves large-scale problems with a minimum travelled distance and time, these two parameters are important criteria comparing performance of the modified GELS with the other algorithms in solving the mTSP. Hence, if the modified GELS provides a shortest travelled distance and terminates faster, it is superior over the other existing optimization algorithms.

The GELS also has some random elements within itself, but it does not merely go forward presumptively. Although it uses local search neighborhood for finding a solution, but it will not move always between them in one form. Although, it has specific behavior of the greedy algorithm, it does not always find the best way to search. Based on the simulation results, improvement of the total travelled distances computed by the modified GELS to solve the mTSP is 10% at least and it will be increased to 26% even in some cases.

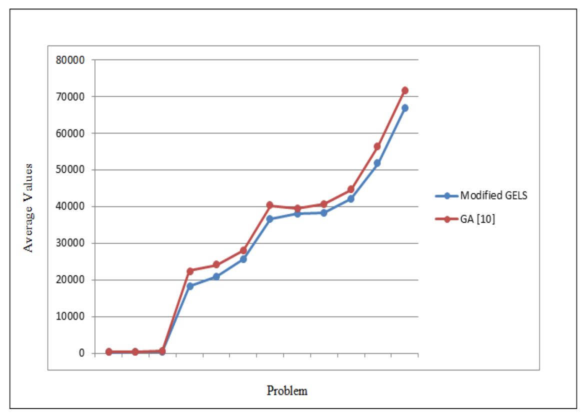

Fig. 7: Mean values of the results in Table 2

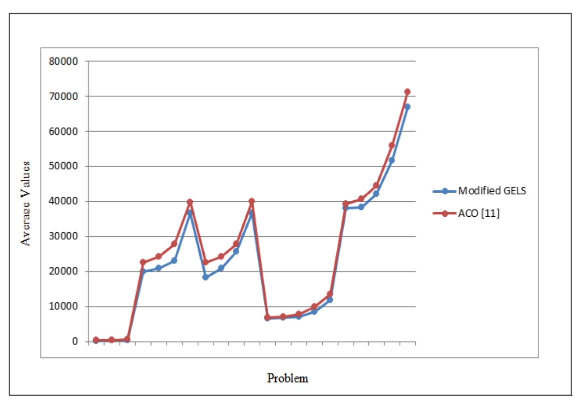

Fig. 8: Mean values of the results in Table 3

Table 2: Comparison results of the modified GELS, ACO, and SW+ ASelite in order to minimize the total travelled distance and time by a salesman

Problem				ACO [9]		SW+AS$_{elite}$ [12]		Modified GELS		% Improvement of Modified GELS	
Name	N	m	I	Avg.	Time (s)	Avg.	Time (s)	Avg.	Time (s)	ACO	SW+AS$_{elite}$
Pr76	76	5	20	180690	51	157562	19	147734.36	1.14	22	6
Pr152	152	5	40	136341	128	128004	41	119205.97	3.23	14	7
Pr226	226	5	50	170877	143	168156	62	16035.21	6.86	6.7	5
Pr299	299	5	70	83845	288	82195	65	76654.17	8.04	8	6
Pr439	439	5	100	165035	563	162657	95	155523.97	14.03	6.1	4.5
Pr1002	1002	5	220	387205	2620	381654	186	356341.15	22.31	7	6
									Minimum	6.1	4.5
									Average	10.6	5.7
									Maximum	22	7

Table 3: Comparison results of the modified GELS and the Enhanced GA in order to minimize the total travelled distance and time by a salesman

Problem		Enhanced GA [10]				Modified GELS				% Improvement of the Modified GELS over
Name	m	Best	Avg.	Worst	Time (s)	Best	Avg.	Worst	Time (s)	Enhnced GA
MTSP-51	3	447.42	448.5	449.62	7.10	393.11	405.55	418	0.97	10.4
	5	476.11	478.41	482.41	8.78	410	419.9	429.81	1.21	13.8
	10	583.57	587.39	589.86	11.20	528	531	335	1.45	10
MTSP-100-II	3	22366.57	22466.41	22611.24	17.27	21792	21814	21837	1.53	3
	5	23895.38	24040.57	24095.96	20.18	22.74.26	22129.165	22184.33	1.84	8.6
	10	27675.42	28033.53	28216.64	26.52	25020.74	25586.99	26153.24	2.27	8
	20	39993.83	40274.58	40582.55	34.89	37774	38271.5	38769	2.58	5.2
MTSP-150-II	3	39179.41	39361.04	39557.43	28.04	37744.60	37955.86	38167.12	2.79	3
	5	40437.18	40663.31	40803.15	34.02	38156.18	38248.66	38341.14	3.30	5
	10	44088.29	44546.77	44782.05	44.32	42113.63	43285.10	42665.58	5.12	3
	20	55959.70	56417.86	56572.87	57.13	51593.08	51693.36	51793.65	6.17	8
	30	71605.25	71808.99	71923.98	67.47	66574.13	66823.13	67072.14	9.51	6
									Minimum	3
									Average	6.7
									Maximum	13.8

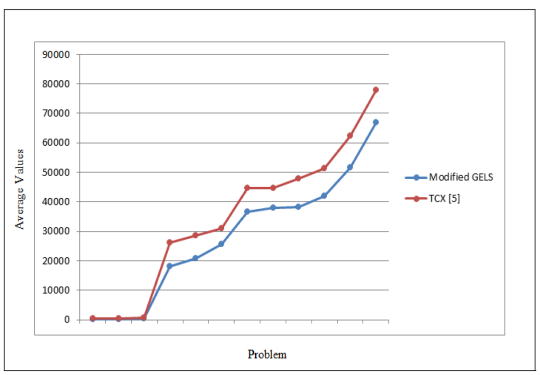

Fig. 9: Mean values of the results in Table 4

6 Conclusion

In this paper, the GELS is modified to solve the symmetric mTSP. The GELS is a local search algorithm based on the Newtons law of gravity. The goal is on finding the shortest distance travelled by all salesmen and obtaining minimum time required for the salesmen to visit each city once. The advantages of the modified GELS are high-speed convergence, simplicity, low running time, and very low assessment values. Simulation results show performance of the modified GELS compared to other existing optimization algorithms in the literature. Efficiency and superiority of the modified GELS were compared based on the computed total travelled distance and time required to solve the mTSP. As simulation results show, the average of the total distance computed by the modified GELS has 13.7 percent improvement over the ACO, 10.2 percent improvement over the SW+ ASelite, 18.8 percent improvement over the enhanced GA, and 26.8 percent improvement over the TCX. Hence, the modified GELS has superiority over the well-known optimization algorithms for solving the mTSP. For future work, effective hybrid algorithms can be achieved combining the modified GELS with other algorithms in the literature.

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Table 4: Comparison results of the modified GELS and the ACO in more complex problems to minimize the total travelled distance and time by a salesman

Problem		ACO [11]				Modified GELS				% Improvement of the Modified GELS over
Name	m	Best	Avg.	Worst	Time (s)	Best	Avg.	Worst	Time (s)	ACO
MTSP-51	3	447	449.5	452	2.48	393.11	405.55	418	0.97	10.4
	5	472	478.41	480	4.66	410	419.9	429.81	1.21	13.8
	10	583	584	585	11.31	528	531	335	1.45	10
MTSP-100-I	3	22417	22544	22696	8.79	19511.41	19966.66	20421.92	1.53	11
	5	23802	24092	24333	15.96	20734.92	20783.33	20832.45	1.84	13
	10	27552	27828	28121	36.76	22902.26	23107.51	23312.76	2.27	16
	20	39208	39813	40298	95.4	36291.84	36517.02	36742.21	2.58	8
MTSP-100-II	3	22366.57	22466.41	22611.24	17.27	21792	21814	21837	1.53	3
	5	23895.38	24040.57	24095.96	20.18	22074.26	22129.165	22184.33	1.84	8.6
	10	27675.42	28033.53	28216.64	26.52	25020.74	25586.99	26153.24	2.27	8
	20	39993.83	40274.58	40582.55	34.89	37774	38271.5	38769	2.58	5.2
MTSP-150-I	3	6728	6824	6855	19.44	6631.42	6677.76	6724.10	2.79	2
	5	6935	6994	7038	34.046	6573.22	6724.72	6876	3.30	3
	10	7588	7674	7732	77.92	6946.18	6987.65	7029.13	5.12	8
	20	9787	9924	10017	188.83	8789.12	8868.13	8947.14	6.17	12
	30	13275	13459	13371	351.43	12753	12842	12931	9.51	4
MTSP-150-II	3	38811	39247	39681	19.1	37744.60	37955.86	38167.12	2.79	3
	5	40301	40900	40647	33.66	38156.18	38248.66	38341.14	3.30	5
	10	44133	44436	44713	73.51	42113.63	43285.10	42665.58	5.12	2.6
	20	55550	55980	56361	169.87	51593.08	51693.36	51793.65	6.17	7
	30	70854	71266	71572	315.07	66574.13	66823.13	67072.14	9.51	6
									Minimum	2
									Average	7.8
									Maximum	13.8

Table 5: Comparison results of the modified GELS and the TCX to minimize the total travelled distance and time by a salesman

Problem		TCX [5]	Modified GELS	% Improvement of the Modified GELS over
Name	m	Average distance	Average distance	Average distance
MTSP-51	3	492	405	21.4
	5	519	429.81	20.7
	10	670	531	26
MTSP-100	3	26130	21814	19.7
	5	28612	22129.6	29.2
	10	30988	25587	21.1
	20	44686	38271.5	16.7
MTSP-150	3	44676	37955.86	17.7
	5	47811	38248	25
	10	51326	43285.10	18.5
	20	62400	51693	20.7
	30	78023	66823.13	16.7
			Minimum	16.7
			Average	21.11
			Maximum	29.2

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