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A Nanofluid Flow in a Non-Linear Stretching Surface Saturated in a Porous Medium with Yield Stress Effect

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Abstract: An analysis has been carried out to study a problem of the natural convective boundary-layer flow of a nanofluid past a non-linear stretching surface with convective boundary condition in the presence of yield stress in porous media. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The local similarity solutions are obtained by using an efficient numerical shooting technique with a fourth-order Runge–Kutta scheme (MATLAB package). The results corresponding to the dimensionless temperature profiles and the reduced Nusselt number, Sherwood number and skin friction coefficient are displayed graphically for various pertinent parameters. It was found that Nusselt number $(Re_x^{-1/2}Nu_x)$ and Sherwood number $(Re_x^{-1/2}Sh_x)$ is a decreasing function of the yield stress parameter Ω and the porous media parameter ξ , while the skin friction coefficient $(Re_x^{1/2}C_f)$is an increasing function of the yield stress parameter Ω and the porous media parameter ξ.

Keywords: Nanofluid; Non-Linear stretching surface; Porous media; Yield stress.

Nomenclature

a stretching coefficient
Bi Biot number (surface convection parameter)
D_B brownian diffusion coefficient
D_T thermophoretic diffusion coefficient
f dimensionless stream function
Gr_x local Grashof number
g gravitational acceleration vector
h convective heat transfer coefficient
K permeability of the porous medium
K_m thermal conductivity of the base fluid
Le Lewis number
m stretching index
N_b brownian motion parameter
N_t Thermophoresis parameter
Nu_x Local Nusselt number
Pr Prandtl number
Re_x local Reynolds number
Sh_x Local Sherwood number
T temperature of the nanofluid within the boundary layer
$T_\circ$ temperature of the fluid below the surface
T_w temperature at the surface of the sheet
T_∞ temperature of the ambient fluid
u, v velocity components along x- and y-directions, respectively
x, y cartesian coordinates along the plate and normal to it, respectively

Greek symbols

α thermal diffusivity of the nanofluid
$\alpha_\circ$ threshold gradient
β_T volumetric coefficient of thermal expansion
γ dimensionless rescaled nanoparticle volume fraction
λ thermal buoyancy parameter
λ^* nanoparticle buoyancy parameter
ρ_f fluid density
ρ_p nanoparticle mass density
$(\rho C_p)_f$ heat capacity of the fluid
$(\rho C_p)_p$ effective heat capacity of the nanoparticles material
μ fluid viscosity
ν kinematic coefficient of viscosity
Ψ stream function

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η similarity variable
θ dimensionless temperature
ϕ nanoparticles volume fraction
ϕ_w nanoparticle volume fraction at the surface of the sheet
ϕ_∞ ambient nanoparticle volume fraction attained as y tends to infinity
τ nanoparticle heat capacity ratio
$\tau_\circ$ yield stress

Subscripts

w surface conditions
∞ conditions far away from the surface

Superscript

$'$ differentiat ion with respect to η

1 Introduction

The flow over a stretching surface is an important problem in many engineering processes with applications in industries such as extrusion, melt-spinning, the hot rolling, wire drawing, glass–fiber production, manufacture of plastic and rubber sheets, cooling of a large metallic plate in a bath, which may be an electrolyte, etc. In industry, polymer sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a windup roller, which is located at a finite distance away. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through an ambient fluid [1]. Experiments show that the velocity of the stretching surface is approximately proportional to the distance from the orifice [2]. Crane [3] studied the steady two-dimensional incompressible boundary layer flow of a Newtonian fluid caused by the stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point due to the application of a uniform stress. This problem is particularly interesting since an exact solution of the two-dimensional Navier–Stokes equations has been obtained by Crane [3]. After this pioneering work, the flow field over a stretching surface has drawn considerable attention and a good amount of literature has been generated on this problem [4,5,6,7,8, 9]. Khan and Pop [10] analyze the development of the steady boundary layer flow, heat transfer and nanoparticle fraction over a stretching surface in a nanofluid. Rahman and Eltayeb [11] investigate the dynamics of the natural convection boundary layer flow of a viscous incompressible nanofluid considering Buongiorno's [12] nanofluid model over a nonlinear stretching sheet in the presence of an applied magnetic field with thermal radiation. Instead of the commonly used conditions of constant surface temperature or constant heat flux, a convective boundary condition is employed which makes this study unique and the results are more realistic and practically useful.

The study of convective heat transfer in nanofluids is gaining a lot of attention. The nanofluids have many applications in the industry since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties. For example, a small amount ($<1\%$ volume fraction) of Cu nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil is reported to increase the inherently poor thermal conductivity of the liquid by 40% and 150%, respectively as reported by Eastman et al. [13] and Choi et al. [14]. Conventional particle-liquid suspensions require high concentrations ($>10\%$) of particles to achieve such enhancement. However, problems of theology and stability are amplified at high concentrations, precluding the widespread use of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is orders of magnitude larger than predicted by well-established theories. Nanofluids are used in different engineering applications such as microelectronics, microfluidics, transportation, biomedical, solid-state lighting and manufacturing. The research on heat and mass transfer in nanofluids has been receiving increased attention worldwide. Many researchers have found unexpected thermal properties of nanofluids, and have proposed new mechanisms behind the enhanced thermal properties of nanofluids. Excellent reviews on convective transport in nanofluids have been made by Buongiorno [12] and Kakac and Pramuanjaroenkij [15]. Kuznetsov and Nield [16] studied analytically the natural convective boundary-layer flow of a nanofluid past a vertical plate. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. Also, it is interesting to note that the Brownian motion of nanoparticles at molecular and nanoscale levels is a key nanoscale mechanism governing their thermal behaviors. In nanofluid systems, due to the size of the nanoparticles, the Brownian motion takes place, which can affect the heat transfer properties. As the particle size scale approaches to the nanometer scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in the heat transfer.

Porous media heat transfer problems have several engineering applications such as geothermal energy recovery, crude oil extraction, ground water pollution, thermal energy storage and flow through filtering media. Cheng and Minkowycz [17] presented similarity solutions for free convective heat transfer from a vertical plate in a fluid saturated porous medium. Gorla and Zinolabedini [18] and Gorla and Tornabene [19] solved the nonsimilar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature or heat flux. Chen and Chen [20] and Mehta and Rao [21] presented similarity solutions for free convection of non-Newtonian fluids over horizontal surfaces in porous media.

Nakayama and Koyama [22] studied the natural convection over a non-isothermal body of arbitrary geometry placed in a porous medium. All these studies were concerned with Newtonian fluid flows. The boundary layer flows in nanofluids have been analyzed recently by Nield and Kuznetsov [16,23]. Hady et al. [24] reported the problem of non-Darcian free convection of a non-Newtonian fluid from a vertical sinusoidal wavy plate embedded in a porous medium. Hady and Ibrahim [25] studied the effect of the presence of an isotropic solid matrix on the forced convection heat transfer rate from a flat plate to power-law non-Newtonian fluid-saturated porous medium. Mahdy and Hady [26] studied the effects of thermophoretic particle deposition of the free convective flow over a flat plate embedded in non-Newtonian fluid-saturated porous medium in the presence of a magnetic field. The free convective heat transfer to the power-law non-Newtonian flow from a vertical plate in a porous medium saturated with nanofluid under laminar conditions is investigated by Hady et al. [27].Jumah and Mujumdar [28] studied the free convection heat and mass transfer of non-Newtonian power law fluids with yield stress over a vertical plate in saturated porous media subjected to constant wall temperature and concentration. Jumah and Mujumdar [29] also studied the natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress over a vertical plate in saturated porous media subjected to variable wall temperature and concentration. Hady et al. [30] study the effect of yield stress on free convection boundary-layer flow past a vertical flat plate embedded in a porous medium filled with a nanofluid, the basic fluid being a non-Newtonian fluid.

The objective of the present work is to analyze the development of the steady boundary layer flow, heat transfer and nanoparticle fraction past a non-linear stretching surface in a porous medium in a nanofluid flow under convective boundary condition. A similarity solution is presented. This solution depends on a Prandtl numberPr, a Lewis number Le, a Brownian motion numberN_b, a thermophoresis number N_t and yield stress parameter Ω. The dependency of the local Nusselt and local Sherwood numbers on these parameters is numerically investigated.

2 Analysis

We consider the steady two-dimensional boundary layer flow of a nanofluid moving over a heated vertical stretching sheet with the threshold gradient $\alpha_\circ = a\tau_\circ/\sqrt{k}$, where a is a constant, $\tau_\circ$ yield stress and k is the permeability for the porous medium. We consider a Cartesian coordinate system with the origin at the lower corner of the sheet. The $x-$axis is vertically upwards along the sheet and the $y-$axis is horizontal and perpendicular to the plane of the sheet. The flow being confined to $y > 0$. Two equal and opposite forces are introduced along the x axis so that the surface is stretched keeping the origin fixed. This continuous sheet is assumed to move with a velocity according to the power law form $u = ax^m$, where a is a dimensional constant known as the stretching rate and m is an arbitrary positive constant (i.e., not necessarily an integer) known as the stretching index. It is assumed that the left surface of the sheet is heated by convection from a hot fluid at temperature $T_\circ$ which provides a heat transfer coefficient h. We consider the nanofluid as a two-component mixture (i.e. base fluid plus nanoparticles) with the assumptions (1) incompressible flow, (2) no chemical reactions, (3) dilute mixture, (4) negligible viscous dissipation and (5) nanoparticles and base fluid locally in thermal equilibrium. Following these assumptions along with the usual boundary layer and Boussinesq approximations, the governing equations of the problem become (Buongiorno [12], Kuznetsov and Nield [16])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho_f \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + \left[\rho_{f\infty} g \left(1-\phi_\infty\right) \beta_T \left(T - T_\infty\right) - \left(\rho_P - \rho_{f\infty}\right) g \left(\phi - \phi_\infty\right) - \alpha_\circ \right] - \frac{\mu}{k} u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2 \right], \tag{3}$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u, v are the velocity components along x, y coordinates, respectively. Here ρ_f is the density, μ is the viscosity,β_T is the volume expansion coefficient of the base fluid, while ρ_P is the density of the particles. $\alpha = k_m/(\rho c)_f$ is the thermal diffusivity, k_m is the thermal conductivity and $(\rho c)_f$ is the heat capacity of the base fluid, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, $D_B$is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T is the temperature of the nanofluid in the boundary layer, T_∞ is the temperature of the ambient fluid outside the boundary layer, ϕ is the nanoparticle volume fraction while ϕ_∞ is its ambient value, and g is the acceleration due to gravity.

The boundary conditions suggested by the physics of the problem are

$$\begin{aligned} &u = ax^m, v = 0, -K_m \frac{\partial T}{\partial y} = h\left(T_\circ - T_w\right), \phi = \phi_w \ at \ \ y = 0, \\ &u \to 0, T \to T_\infty, \phi \to \phi_\infty \ as \ \ y \to \infty. \end{aligned} \tag{5}$$

where the subscripts w and ∞ refer to the wall and boundary layer edge, respectively.

We look for a similarity solution of Eqs. (2)-(4) with the boundary conditions (5) of the following form:

$$\eta = y\sqrt{\frac{a}{\nu}x^{m-1}}, \Psi = \sqrt{a\nu x^{m+1}}f(\eta), \theta = \frac{T-T_\infty}{T_w-T_\infty}, \gamma = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty}. \quad (6)$$

where the stream function Ψ is defined in the usually way as $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$.
Thus from Eq. (6) we have

$$u = ax^m f'(\eta), v = -\frac{\partial \Psi}{\partial x} = -\sqrt{a\nu x^{m-1}}\left[\frac{m+1}{2}f(\eta) + \frac{m-1}{2}\eta f'(\eta)\right] \quad (7)$$

Here f is a non-dimensional stream function and the prime denotes differentiation with respect to η.
Now substituting Eqs. (6) and (7) into Eqs. (2)–(4) we obtain the following ordinary differential equations:

$$f''' + \frac{m+1}{2}ff'' - mf'^2 + [\lambda\theta - \lambda^*\gamma - \Omega] - \xi f' = 0, \quad (8)$$

$$\theta'' + \frac{m+1}{2}f\Pr\theta' + N_b\Pr\theta'\gamma' + \Pr N_t\theta'^2 = 0, \quad (9)$$

$$\gamma'' + \frac{1}{2}Le(m+1)f\gamma' + (N_t/N_b)\theta'' = 0. \quad (10)$$

along with the boundary conditions

$$f = 0, f' = 1, \theta' = -Bi(1-\theta), \gamma = 1 \quad at \ \eta = 0, f' \to 0, \theta \to 0, \gamma \to 0 \quad as \ \eta \to \infty. \quad (11)$$

Where the parameters which govern the problem are defined by

$$\lambda = \frac{\rho_{f\infty}(1-\phi_\infty)}{\rho_f}\frac{Gr_x}{\mathrm{Re}_x^2}, Gr_x = \frac{g\beta_T(T_w-T_\infty)}{\nu^2}, \mathrm{Re}_x = \frac{ax^{m+1}}{\nu},$$
$$\lambda^* = \frac{(\rho_p-\rho_{f\infty})g(\phi_w-\phi_\infty)x}{\rho_f(ax^m)^2}, \Omega = \frac{x\alpha_\circ}{\rho_f(ax^m)^2},$$
$$\xi = \frac{x\nu}{kax^m}, \Pr = \frac{\nu}{\alpha}, N_b = \frac{\tau D_B(\phi_w-\phi_\infty)}{\nu}, N_t = \frac{\tau D_T(T_w-T_\infty)}{\nu},$$
$$Le = \frac{\nu}{D_B}, Bi = \frac{xh}{K_m}\mathrm{Re}_x^{-1/2}. \quad (12)$$

Here $\lambda, Gr_x, \mathrm{Re}_x, \lambda^*, \Omega, \xi, \Pr, N_b, N_t, Le$ and Bi denote a thermal buoyancy parameter, a local thermal Grashof number, a local Reynolds number, a nanoparticle buoyancy parameter, a yield stress parameter, porous media parameter, a Prandtl number for the base fluid, a Brownian motion parameter, a thermophoresis parameter, Lewis number and a surface convection parameter or so-called Biot number.

Skin friction, Heat and Mass transfer coefficients

The primary objective of this study is to estimate the parameters of engineering interest in fluid flow, heat and mass transport problems are the skin friction coefficient C_f, the Nusselt number Nu_x and the Sherwood number Sh_x. These parameters characterize the surface drag, the wall heat and nanoparticle mass transfer, respectively. The shearing stress, local heat and local mass flux from the vertical plate can be obtained from

$$\tau_w = \mu\left[\frac{\partial u}{\partial y}\right]_{y=0}, q_w = -k_m\left[\frac{\partial T}{\partial y}\right]_{y=0}, q_m = -D_B\left[\frac{\partial \phi}{\partial y}\right]_{y=0}$$

The non-dimensional shear stress $C_f = \frac{2\tau_w}{\rho_f(ax^m)^2}$, the Nusselt number $Nu_x = \frac{q_w x}{k_m(T_w-T_\infty)}$ and the Sherwood number $Sh_x = \frac{q_m x}{D_B(\phi_w-\phi_\infty)}$, are given by $\mathrm{Re}_x^{1/2}C_f = f''(0)$, $\mathrm{Re}_x^{-1/2}Nu_x = -\theta'(0)$, $\mathrm{Re}_x^{-1/2}Sh_x = -\gamma'(0)$.

3 Numerical Results and discussion

The set of Eqs. (8)–(10) is highly nonlinear and coupled and cannot be solved analytically. The numerical solutions of Eqs. (8)–(10) subject to the boundary conditions (11) are obtained using an efficient numerical shooting technique with a fourth-order Runge–Kutta scheme (MATLAB package). For the purpose of discussing the results, the numerical calculations are presented graphically for nondimensional temperature profiles as a function of η, rate of heat transfer, rate of mass transfer and the rate of shear stress. In the calculations the values of the parameters, namely the thermal buoyancy parameter λ, nanoparticle buoyancy parameter λ^*, a yield stress parameter Ω, porous media parameter ξ, Brownian motion parameter N_b, thermophoresis parameter N_t, Biot number Bi, and stretching index m are varied keeping Prandtl number Pr and Lewis number Le as fixed. The accuracy of the aforementioned numerical method was validated by direct comparisons with the numerical results reported earlier by Khan and Pop [10] and Rahman and Eltayeb [11] for various Values of the reduced Nusselt number and the Sherwood number for different values of N_t and $Pr = 10, Le = 10$, in the limiting case ($\lambda = \lambda^* = \xi = \Omega = 0. Bi = \infty, m = 1$).This comparison is presented in table 1 ($N_b = 0.5$) and table 2 ($N_b = 0.1$) . It can be shown from this table that an excellent agreement between the results exists. Figs. 1,2 show the effect of yield stress parameter $\Omega = 0, 0.3, 0.5$ and porous media parameter $\xi = 0.5, 1, 1.5, 2$ on (a) velocity profiles (b) temperature function and (c) mass fraction function (rescaled nanoparticles volume fraction) with $m = 2$,$\lambda = 10$,$\lambda^* = 5$,$\Pr = 10$, $N_b = 0.2$, $N_t = 0.2$, $Le = 10$ and $Bi = \infty$. It is shown that the momentum boundary layer thickness decreases with Ω increase. On the other hand, the thermal and concentration boundary layer thicknesses increase as Ω increase as shown in Fig. 1 with $\xi = 1$. This means that higher values of heat and mass transfer rates are associated with small Ω. It is clear that the effect of ξ on velocity profiles ,temperature function and mass fraction function is similar to the effect of Ω which is discussed above. Figs. 3, show (a) the local rate of shear stress in terms of the skin friction coefficient $\mathrm{Re}_x^{1/2}C_f$,(b) the local rate of heat transfer in terms of Nusselt number $\mathrm{Re}_x^{-1/2}Nu_x$ from the heated surface to the nanofluid and (c) local rate mass transfer in terms of Sherwood number $\mathrm{Re}_x^{-1/2}Sh_x$ for different values of $\Omega = 0, 0.3, 0.5$ and $N_t = 0, 0.1, 0.2, 0.3, 0.4, 0.5$ at $m = 2$,$\lambda = 10$,$\lambda^* = 5$,$\xi = 1$,$\mathrm{Pr} = 10$, $N_b = 0.2$, $Le = 10$ and $Bi = \infty$. From these figures it is found that values of Nusselt number $\mathrm{Re}_x^{-1/2}Nu_x$ and Sherwood number $\mathrm{Re}_x^{-1/2}Sh_x$ decrease markedly with the increase of yield stress parameter Ω. On the other hand values of the skin friction coefficient $\mathrm{Re}_x^{1/2}C_f$ increase very rapidly with the increase of yield stress parameter Ω. The values of Nusselt number $\mathrm{Re}_x^{-1/2}Nu_x$ and the skin friction coefficient $\mathrm{Re}_x^{1/2}C_f$ decrease with the increase of Brownian motion parameter N_b as well as thermophoresis parameter N_t, while the Sherwood number $\mathrm{Re}_x^{-1/2}Sh_x$ increase with the increase of Brownian motion parameter N_b as well as thermophoresis parameter N_t as shown in Figs. 3,4. The variations of the skin friction coefficient $\mathrm{Re}_x^{1/2}C_f$,(b) the Nusselt number $\mathrm{Re}_x^{-1/2}Nu_x$ and (c) the Sherwood number $\mathrm{Re}_x^{-1/2}Sh_x$ for different values of porous media parameter are presented in Figs. 5,6. Keeping all other parameter values fixed as $m = 2$,$\lambda = 10$,$\lambda^* = 5$,$\mathrm{Pr} = 10, \Omega = 0.1$, $N_b = 0.2$, $Le = 10$ and $Bi = \infty$. It is found that values of the skin friction coefficient $\mathrm{Re}_x^{1/2}C_f$, increase very rapidly with the increase of porous media parameter ξ. but it can be seen that an increase in porous media parameter ξ, it led to an decrease in the Nusselt number $\mathrm{Re}_x^{-1/2}Nu_x$ and the Sherwood number $\mathrm{Re}_x^{-1/2}Sh_x$.

4 Conclusions

We have examined the influence of the stretching plate parameter on non-linear stretching surface in a porous medium in a nanofluid flow under convective boundary condition in the presence of yield stress effect . Using similarity transformations the governing equations of the problem are transformed into nonlinear ordinary differential equations and solved for local similar solutions by using an efficient numerical shooting technique with a fourth-order Runge–Kutta scheme (MATLAB package). From the present study the following conclusions can be drawn:
The local rate of shear stress in terms of the skin friction coefficient $\mathrm{Re}_x^{1/2}C_f$, increases with an increase of the yield stress parameter Ω and the porous media parameter ξ but it decreases with the increase of the Brownian motion parameter N_b and the thermophoresis parameter N_t.
The local rate of heat transfer in terms of Nusselt number $\mathrm{Re}_x^{-1/2}Nu_x$ from the surface of the sheet to the fluid decreases with an increase of each of the yield stress parameter Ω, the porous media parameter ξ, the Brownian motion parameter N_b and the thermophoresis parameter N_t.

Table 1: Comparison test results. Values of the reduced Nusselt number and the Sherwood number for different values of N_t and $\mathrm{Pr} = 10, Le = 10, N_b = 0.5$ in the limiting case ($\lambda = \lambda^* = \xi = \Omega = 0. Bi = \infty, m = 1$)

N_t	$-\theta'(0)$		$-\gamma'(0)$	
	Ref. [10]	Present results	Ref. [10]	Present results
0.1	0.0543	0.05425	2.3836	2.38357
0.2	0.0390	0.03904	2.4468	2.44681
0.3	0.0291	0.02914	2.4984	2.49837
0.4	0.0225	0.02250	2.5399	2.53986
0.5	0.0179	0.01792	2.5731	2.57310

Table 2: Comparison test results. Values of the reduced Nusselt number and the Sherwood number for different values of N_t and $\mathrm{Pr} = 10, Le = 10, N_b = 0.5$ in the limiting case ($\lambda = \lambda^* = \xi = \Omega = 0. Bi = \infty, m = 1$)

N_t		$-\theta'(0)$			$-\gamma'(0)$	
	Ref.[11]	Ref.[10]	Present results	Ref.[11]	Ref.[10]	Present results
0.1	0.952376	0.9524	0.952327	2.129393	2.1294	2.129534
0.2	0.693174	0.6932	0.693110	2.274020	2.2740	2.274280
0.3	0.520079	0.5201	0.520078	2.528636	2.5286	2.528644
0.4	0.402581	0.4026	0.402579	2.795167	2.7952	2.795179
0.5	0.321054	0.3211	0.321053	3.035139	3.0351	3.035134

The local rate mass transfer in terms of Sherwood number $\mathrm{Re}_x^{-1/2}Sh_x$ decreases with an increase of the yield stress parameter Ω and the porous media parameter ξ but it increases with the increase of the Brownian motion parameter N_b and the thermophoresis parameter N_t.

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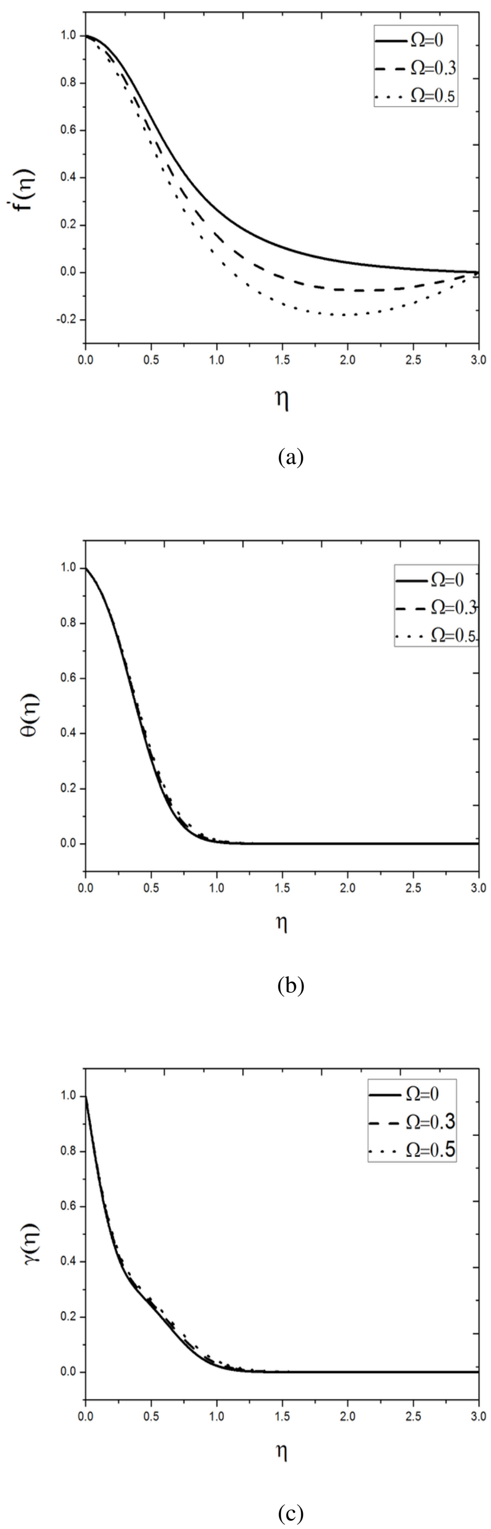


Fig. 1: Effects of yield stress parameter Ω on (a) velocity profiles (b) temperature function (c) mass fraction function with $m=2$, $\lambda=10$, $\lambda^{*}=5$, $\Pr=10$, $N_b=0.2$, $\xi=1$, $N_t=0.2$, $Le=10$ and $Bi=\infty$

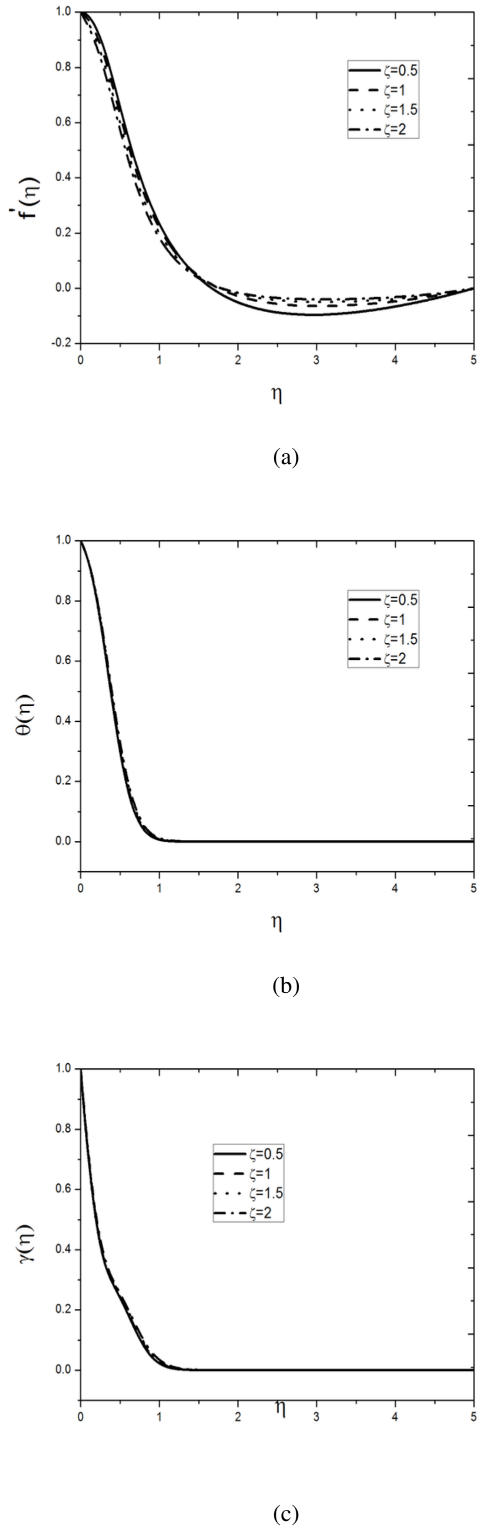
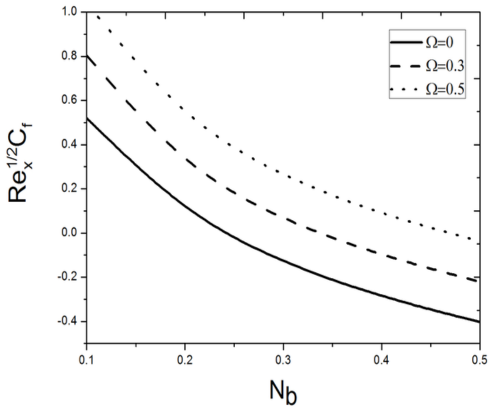
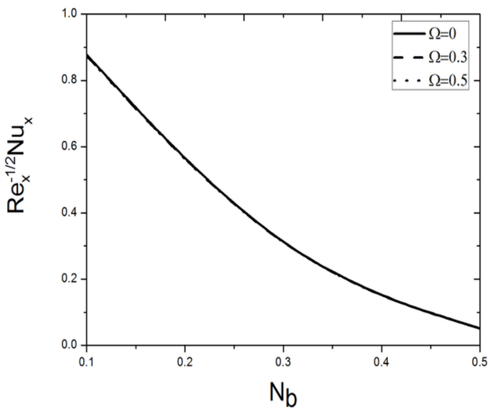


Fig. 2: Effects of porous media parameter ξ on (a)velocity profiles (b)temperature function (c) mass fraction function with $m=2$, $\lambda=10$, $\lambda^{*}=5$, $\Pr=10$, $N_b=0.2$, $\Omega=0.1$, $N_t=0.2$, $Le=10$ and $Bi=\infty$



(a)



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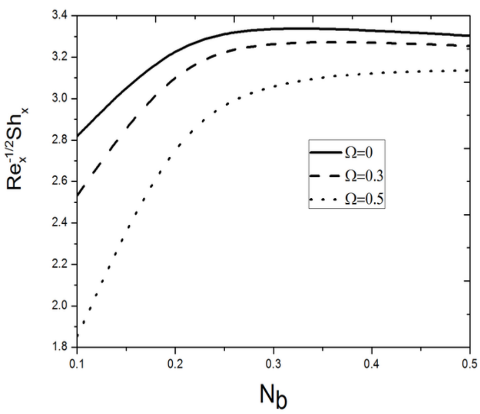
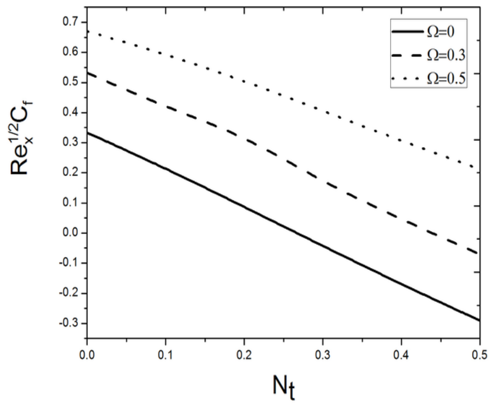
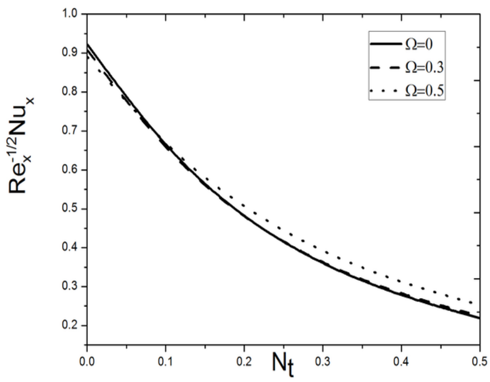


(c)

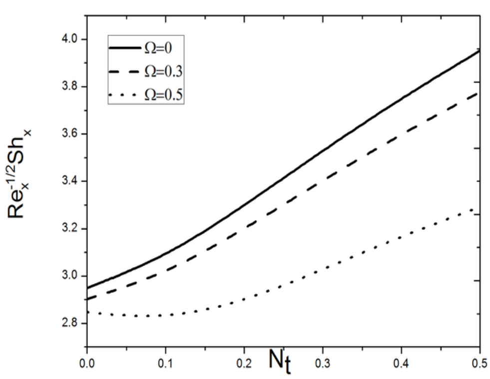
Fig. 3: Effects of yield stress parameter Ω on (a) $\mathrm{Re}_x^{1/2}C_f$ (b) $\mathrm{Re}_x^{-1/2}Nu_x$ (c) $\mathrm{Re}_x^{-1/2}Sh_x$ with $m=2$, $\lambda=10$, $\lambda^*=5$, $\mathrm{Pr}=10$, $\xi=1$, $N_t=0.2$, $Le=10$ and $Bi=\infty$



(a)

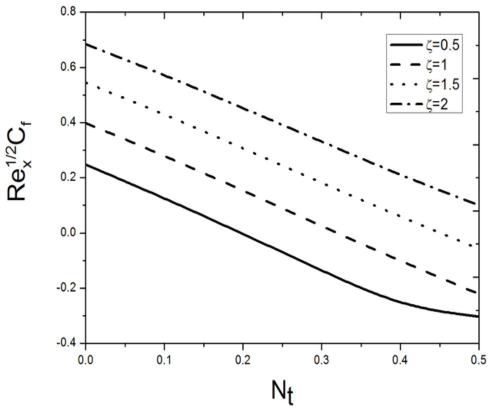


(b)

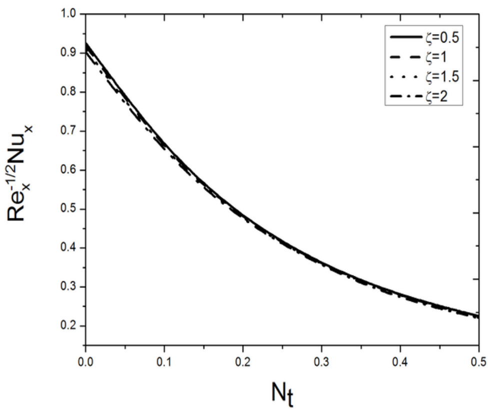


(c)

Fig. 4: Effects of yield stress parameter Ω on (a) $\mathrm{Re}_x^{1/2}C_f$ (b) $\mathrm{Re}_x^{-1/2}Nu_x$ (c) $\mathrm{Re}_x^{-1/2}Sh_x$ with $m=2$, $\lambda=10$, $\lambda^*=5$, $\mathrm{Pr}=10$, $\xi=1$, $N_b=0.2$, $Le=10$ and $Bi=\infty$.



(a)



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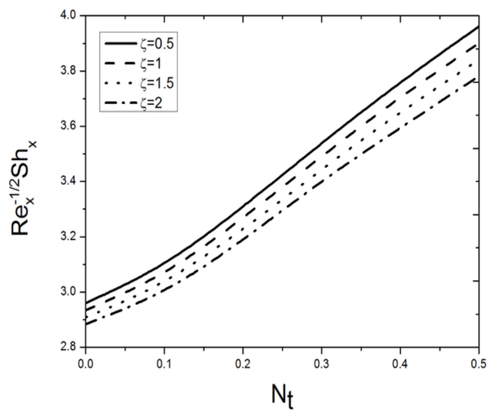
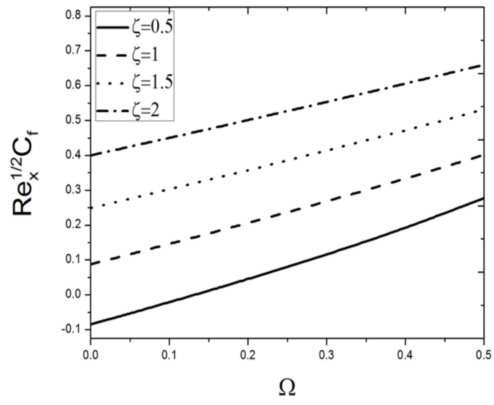
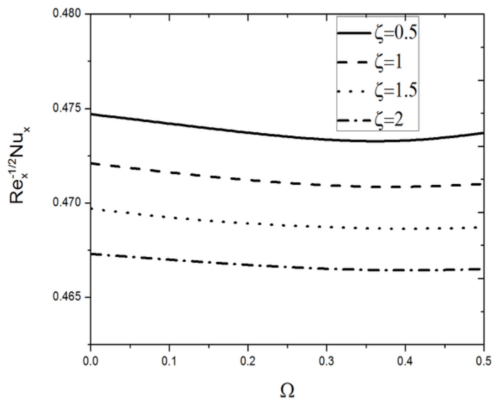


(c)

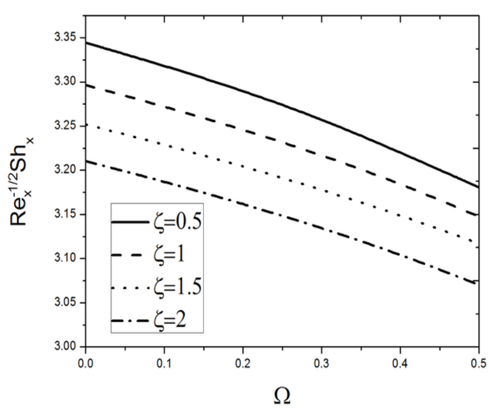
Fig. 5: Effects of porous media parameter ξ on (a) $\mathrm{Re}_x^{1/2} C_f$ (b) $\mathrm{Re}_x^{-1/2} Nu_x$(c) $\mathrm{Re}_x^{-1/2} Sh_x$ with $m = 2$, $\lambda = 10$, $\lambda^* = 5$, $\mathrm{Pr} = 10$, $\Omega = 0.1$, $N_b = 0.2$, $Le = 10$ and $Bi = \infty$



(a)



(b)



(c)

Fig. 6: Effects of yield stress parameter Ω and porous media parameter ξ on (a) $\mathrm{Re}_x^{1/2} C_f$ (b) $\mathrm{Re}_x^{-1/2} Nu_x$ (c) $\mathrm{Re}_x^{-1/2} Sh_x$ with $m = 2$, $\lambda = 10$, $\lambda^* = 5$, $\mathrm{Pr} = 10$, $N_t = 0.2$, $N_b = 0.2$, $Le = 10$ and $Bi = \infty$

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