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Analysis of Inverse Weibull Distribution based on Record Values

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Abstract: In this research paper, the estimates of the parameters of Inverse Weibull distribution (IWD) are obtained from lower record values by using Lloyd's Method and Gupta's Method. The comparisons between the performances of both the estimation methods, for location and scale parameters of IWD, have been made in terms of relative efficiency numerically and graphically. The study explored that relative efficiency of Gupta's estimators for location and scale parameters of IWD increases as the value of shape parameter increases.

Keywords: Record values, Inverse Weibull distribution, Location parameter, Scale parameter, Shape parameter and Best Linear Unbiased Estimates (BLUEs).

1 Introduction

Record values are identified in many situations of daily life as well as in many statistical applications involving data relating to sports, weather, economics and life testing studies. Often we are interested in observing new records and in recording them: for example, Olympic records, world records in sports, records of earthquakes, solar spots and solar activity etc. This importance of the record values provokes the necessity to construct mathematical model of record values. Soliman et al. [1] developed a Bayesian analysis on the basis of record values from the two-parametric Weibull distribution and derived the Maximum likelihood and Bayes estimates for the unknown parameters. The IWD is very flexible distribution as it approaches to different probability distributions and authors also presented theoretical analysis of the IWD mathematically and graphically [2]. By using the different estimation methods, the parameters of Inverse Weibull distribution based on record values, are estimated see [3]. Sultan [4] used the lower record values from Inverse Weibull distribution to estimate the parameters by using method of best linear unbiased estimate (BLUE) and method of Maximum likelihood. He computed the relative efficiency between the obtained estimates. On the basis of record values, the properties of Rayleigh and Inverse Rayleigh distribution are discussed and estimates of location and scale parameters using best linear unbiased estimate and an alternative linear estimate are also found from complete and censored data see [5]. Kundu and Howlader [6] described the Bayesian inference to estimate the unknown parameters of Inverse Weibull distribution under a squared error loss function for censored (type-II) data. Mubarak [7] proposed the estimation problem to estimate the parameters of the Frechet distribution using type II censored samples with random removals. Ahmad et al. [8] discussed the estimation of parameters of Rayleigh distribution on the basis of upper record values. Al-Saleh and Agarwal [9] suggested an extended form of Weibull distribution which has two shape parameters and finite mixture of distributions. Khan and King [10] introduced the generalized version of four parametric modified Inverse Weibull distribution and also provided comprehensive description of the mathematical properties of the modified Inverse Weibull distribution.

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2 Moments of Lower Record Values from IWD

A record is defined as an entry in a sequence of independent random variables that is larger or smaller than all previous entries. An entry that is smaller than all previous entries known as lower record value. Let $X_{L(1)}$, $X_{L(2)}$, ..., $X_{L(n)}$ be the first n lower record values from the Inverse Weibull distribution with probability density function

$$f(x) = cx^{-c-1}e^{-x^{-c}} \qquad x \geq 0, c > 0 \tag{1}$$

While the location-scale form of Inverse Weibull distribution has its density function given by

$$f(x) = \frac{c}{\lambda}\left(\frac{x-\mu}{\lambda}\right)^{-c-1} e^{-\left(\frac{x-\mu}{\lambda}\right)^{-c}} x \geq \mu, \lambda > 0, c > 0, \mu \geq 0 \tag{2}$$

where c is the shape parameter, μ is the location parameter and λ is the scale parameter.

The single and product moments of lower record values from Inverse Weibull distribution are given below

$$\mu_{(n)}^{j} = \frac{\Gamma\left(n-\frac{j}{c}\right)}{\Gamma(n)} \tag{3}$$

$$cov(X_{L(r)}, X_{L(s)}) = \frac{\Gamma\left(r-\frac{1}{c}\right)}{\Gamma(r)}\left[\frac{\Gamma\left(s-\frac{2}{c}\right)}{\Gamma\left(s-\frac{1}{c}\right)} - \frac{\Gamma\left(s-\frac{1}{c}\right)}{\Gamma(s)}\right] \qquad for \quad r < s \tag{4}$$

By using (3) and (4), we calculate the Means and variance covariance matrix of lower record values from Inverse Weibull distribution for different values of shape parameter c and are presented in the table 1 to table 8.

Table 1. Means of Lower Record Values from IWD for $c = 3\, and\, 1 \leq n \leq 8$

n	1	2	3	4	5	6	7	8
Means	1.354120	0.902745	0.752288	0.668700	0.612975	0.572110	0.540326	0.514596

Table 2. Var-Cov matrix of Lower Record Values from IWD for $c = 3\, and\, r \leq s$

s/r	1	2	3	4	5	6	7	8
1	0.845303							
2	0.117046	0.078030						
3	0.052889	0.035259	0.029383					
4	0.032129	0.021419	0.017849	0.015866				
5	0.022348	0.014899	0.012416	0.011036	0.010117			
6	0.016799	0.011199	0.009333	0.008296	0.007605	0.007098		
7	0.013280	0.008853	0.007378	0.006558	0.006012	0.005611	0.005299	
8	0.010874	0.007249	0.006041	0.005369	0.004922	0.004594	0.004339	0.004132

Table 3. Means of Lower Record Values from IWD for $c = 6\, and\, 1 \leq n \leq 8$

n	1	2	3	4	5	6	7	8
Means	1.128787	0.940656	0.862268	0.814364	0.780432	0.754418	0.733462	0.715998

Table 4. Var-Cov matrix of Lower Record Values from IWD for $c = 6\, and\, r \leq s$

s/r	1	2	3	4	5	6	7	8
1	0.079958							
2	0.021494	0.017912						
3	0.011496	0.009580	0.008782					
4	0.007639	0.006366	0.005836	0.005511				
5	0.005642	0.004702	0.004309	0.004070	0.003901			
6	0.004435	0.003696	0.003388	0.003199	0.003066	0.002964		
7	0.003632	0.003027	0.002774	0.002620	0.002511	0.002427	0.002360	
8	0.003063	0.002552	0.002339	0.002209	0.002117	0.002047	0.001990	0.001943

Table 5. Means of Lower Record Values from IWD for $c = 9\, and\, 1 \leq n \leq 8$

n	1	2	3	4	5	6	7	8
Means	1.077760	0.958008	0.904785	0.871275	0.847073	0.828249	0.812911	0.800007

Table 6. Var-Cov matrix of Lower Record Values from IWD for $c = 9\, and\, r \leq s$

s/r	1	2	3	4	5	6	7	8
1	0.028587							
2	0.008881	0.007894						
3	0.004984	0.004430	0.004184					
4	0.003403	0.003025	0.002857	0.002751				
5	0.002561	0.002276	0.002149	0.002070	0.002013			
6	0.002042	0.001815	0.001714	0.001650	0.001605	0.001569		
7	0.001691	0.001503	0.001419	0.001367	0.001329	0.001299	0.001276	
8	0.001439	0.001279	0.001208	0.001164	0.001132	0.001106	0.001086	0.001068

Table 7. Means of Lower Record Values from IWD for $c = 15\, and\, 1 \leq n \leq 8$

n	1	2	3	4	5	6	7	8
Means	1.043170	0.973622	0.941168	0.920253	0.904916	0.892850	0.882930	0.874521

Table 8. Var-Cov matrix of Lower Record Values from IWD for $c = 15\, and\, r \leq s$

s/r	1	2	3	4	5	6	7	8
1	0.008858							
2	0.003043	0.002841						
3	0.001771	0.001653	0.001598					
4	0.001235	0.001153	0.001114	0.001089				
5	0.000943	0.000880	0.000850	0.000832	0.000818			
6	0.000760	0.000709	0.000685	0.000671	0.000659	0.000651		
7	0.000635	0.000593	0.000573	0.000560	0.000551	0.000544	0.000538	
8	0.000545	0.000508	0.000492	0.000481	0.000473	0.000466	0.000461	0.000457

3 Lloyd's Method for Location and Scale parameters

Let $X_{L(1)}$, $X_{L(2)}$, ..., $X_{L(n)}$ be the first n lower record values from the Inverse Weibull distribution with probability density function in (2) with cdf

$$F(x) = e^{-\left(\frac{x-\mu}{\lambda}\right)^{-c}} \tag{5}$$

and let $Y_{L(i)} = \frac{X_{L(i)}-\mu}{\lambda}$, i=1, 2, 3, ... , n be the corresponding record values from Inverse Weibull distribution

$$f(y) = cy^{-c-1}e^{-y^{-c}} \quad 0 \leq y \leq \infty \tag{6}$$

The distribution of first n lower record values is

$$f_n(y) = \frac{1}{\Gamma(n)} cy^{-nc-1}e^{-y^{-c}} \quad 0 \leq y \leq \infty \tag{7}$$

Let us denote
$E\left(X_{L(i)}\right) = \alpha_i, \quad V\left(X_{L(i)}\right) = \beta_{ii}, \quad Cov\left(X_{L(i)}, \quad (X_{L(j)}\right) = \beta_{ij} \quad \underline{x} = \left(x_{L(1)}, x_{L(2)}, ..., x_{L(n)}\right)^t$
$\underline{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_n)^t \quad \underline{1} = (1, 1, ..., 1)^t$
and B=$((\beta_{ij}))$, $1 \leq i, j \leq n$. Then the BLUEs of location parameter μ and scale parameter λ are given by David [11]

$$\hat{\mu}_L = \sum_{i=1}^{n} a_i x_{L(i)} \tag{8}$$

$$\hat{\lambda}_L = \sum_{i=1}^{n} b_i x_{L(i)} \tag{9}$$

where

$$a = \frac{\alpha' B^{-1}\alpha 1' B^{-1} - \alpha' B^{-1} 1 \alpha' B^{-1}}{(\alpha' B^{-1}\alpha)(1' B^{-1} 1) - (\alpha' B^{-1} 1)^2} \tag{10}$$

$$b = \frac{1' B^{-1} 1 \alpha' B^{-1} - 1' B^{-1} \alpha 1' B^{-1}}{(\alpha' B^{-1}\alpha)(1' B^{-1} 1) - (\alpha' B^{-1} 1)^2} \tag{11}$$

The variances and covariance of the above estimators are given by David [11]

$$Var(\hat{\mu}_L) = \left[\frac{\alpha' B^{-1}\alpha}{(\alpha' B^{-1}\alpha)(1' B^{-1}1) - (\alpha' B^{-1}1)^2}\right]\lambda^2 \tag{12}$$

$$Var(\hat{\lambda}_L) = \left[\frac{1' B^{-1}1}{(\alpha' B^{-1}\alpha)(1' B^{-1}1) - (\alpha' B^{-1}1)^2}\right]\lambda^2 \tag{13}$$

$$Cov(\hat{\mu}_L, \hat{\lambda}_L) = \left[\frac{\alpha' B^{-1}1}{(\alpha' B^{-1}\alpha)(1' B^{-1}1) - (\alpha' B^{-1}1)^2}\right]\lambda^2 \tag{14}$$

We are focusing on the study that what will be the effect of changing the value of shape parameter on estimators of the location and scale parameters which are obtained by Lloyd [12]. For this study, we consider the following four cases:

- Case I: Estimation of μ and λ for c=3
- Case II: Estimation of μ and λ for c=6
- Case III:Estimation of μ and λ for c=9
- Case IV: Estimation of μ and λ for c=15

3.1 Case I: Estimation of μ and λ for c=3

By using the values of means, variances and covariance of the probability density function, given in table 1 and table 2, we estimated the coefficients of BLUEs of μ and λ in (10) and (11), based on first n lower record values, which are presented in table 9 and table 10. The values of $Var(\hat{\mu}_L)$, $Var(\hat{\lambda}_L)$, and $Cov(\hat{\mu}_L, \hat{\lambda}_L)$ in (12), (13) and (14) respectively, are computed and presented in table 11.

Table 9. Coefficients of the BLUE of Location Parameter μ for c=3

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
2	-1.999989	2.999989						
3	-0.500003	-2.999994	4.499997					
4	-0.235294	-1.411754	-2.117659	4.764708				
5	-0.140001	-0.839989	-1.259992	-1.620046	4.860028			
6	-0.094214	-0.565263	-0.847946	-1.090190	-1.308104	4.905716		
7	-0.068401	-0.410397	-0.615621	-0.791491	-0.949707	-1.096009	4.931626	
8	-0.052285	-0.313706	-0.470571	-0.605021	-0.725971	-0.837733	-0.942392	4.947680

Table 10. Coefficients of the BLUE of Scale Parameter λ for c=3

n	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
2	2.215453	-2.215453						
3	0.664644	3.987825	-4.652469					
4	0.351868	2.111189	3.166837	-5.629894				
5	0.228396	1.370346	2.055546	2.642910	-6.297199			
6	0.164677	0.988033	1.482137	1.905555	2.286460	-6.826863		
7	0.126592	0.759535	1.139353	1.464839	1.757662	2.028412	-7.276393	
8	0.101604	0.609616	0.914451	1.175715	1.410757	1.627953	1.831339	-7.671434

Table 11. Variances and Covariances of the BLUEs of μ and λ for c=3

n	$Var(\hat{\mu}_L)$	$Var(\hat{\lambda}_L)$	$Covr(\hat{\mu}_L, \hat{\lambda}_L)$
2	2.678904	3.382756	-2.967509
3	0.669736	1.235330	-0.890266
4	0.315168	0.740304	-0.471314
5	0.187526	0.526009	-0.305927
6	0.126195	0.407237	-0.220578
7	0.091620	0.331969	-0.169564
8	0.070034	0.280074	-0.136095

3.2 Case II: Estimation of μ and λ for c=6

By using the values of means, variances and covariance of the probability density function, given in Table 3 and Table 4, we estimated the coefficients of BLUEs of μ and λ in (10) and (11), based on first n lower record values, which are presented in Table 12 and Table 13. The values of $Var(\hat{\mu}_L)$, $Var(\hat{\lambda}_L)$, and $Covr(\hat{\mu}_L, \hat{\lambda}_L)$ in (12), (13) and (14) respectively, are computed and presented in table 14.

Table 12. Coefficients of the BLUE of Location Parameter μ for c=6

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
2	-5.000005	6.000005						
3	-1.999996	-4.199941	7.199937					
4	-1.162636	-2.441051	-2.933277	7.536965				
5	-0.790600	-1.661874	-1.989159	-2.238686	7.680319			
6	-0.585878	-1.230169	-1.479049	-1.656790	-1.812261	7.764147		
7	-0.458748	-0.964186	-1.154498	-1.301038	-1.423780	-1.502895	7.805146	
8	-0.373222	-0.783497	-0.939905	-1.058303	-1.147519	-1.241502	-1.299722	7.843668

Table 13. Coefficients of the BLUE of Scale Parameter λ for c=6

n	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
2	5.315445	-5.315445						
3	2.319460	4.870806	-7.190267					
4	1.427682	2.997611	3.601479	-8.026771				
5	1.013022	2.129164	2.549193	2.868877	-8.560255			
6	0.776598	1.630610	1.960093	2.196876	2.402229	-8.966407		
7	0.625458	1.314392	1.574245	1.773935	1.940378	2.050867	-9.279276	
8	0.521221	1.094176	1.312709	1.478100	1.603684	1.732293	1.817316	-9.559499

Table 14. Variances and Covariances of the BLUEs of μ and λ for c=6

n	$Var(\hat{\mu}_L)$	$Var(\hat{\lambda}_L)$	$Covr(\hat{\mu}_L, \hat{\lambda}_L)$
2	1.354109	1.550605	-1.439541
3	0.541637	0.740302	-0.628154
4	0.314873	0.483106	-0.579661
5	0.214123	0.357949	-0.274361
6	0.158669	0.283992	-0.210320
7	0.124248	0.235339	-0.169397
8	0.101044	0.200874	-0.141118

3.3 Case III:Estimation of μ and λ for c=9

By using the values of means, variances and covariance of the probability density function, given in table 5 and table 6, we estimated the coefficients of BLUEs of μ and λ in (10) and (11), based on first n lower record values, and are presented in table 15 and table 16. The values of $Var(\hat{\mu}_L)$, $Var(\hat{\lambda}_L)$, and $Covr(\hat{\mu}_L, \hat{\lambda}_L)$ in (12), (13) and (14) respectively, are computed and presented in table 17.

Table 15. Coefficients of the BLUE of Location Parameter μ for c=9

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
2	-7.999933	8.999933						
3	-3.499982	-5.624928	10.124911					
4	-2.143617	-3.445097	-3.874861	10.463575				
5	-1.512070	-2.431281	-2.723350	-2.949091	10.615793			
6	-1.151175	-1.850229	-2.085196	-2.229061	-2.413623	10.729285		
7	-0.922576	-1.484002	-1.660117	-1.811505	-1.922987	-1.910413	10.711601	
8	-0.763184	-1.225645	-1.373575	-1.493963	-1.608923	-1.593549	-1.802115	10.860954

Table 16. Coefficients of the BLUE of Scale Parameter λ for c=9

n	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
2	8.350591	-8.350591						
3	3.868302	6.216869	-10.08517					
4	2.460340	3.954116	4.447158	-10.86162				
5	1.784969	2.869951	3.215744	3.481763	-11.35243			
6	1.389886	2.233854	2.517135	2.693522	2.911313	-11.74571		
7	1.134829	1.825239	2.042857	2.227637	2.363890	2.356923	-11.95138	
8	0.954009	1.532151	1.717794	1.867407	2.007606	1.997462	2.244584	-12.32101

Table 17. Variances and Covariances of the BLUEs of μ and λ for c=9

n	$Var(\hat{\mu}_L)$	$Var(\hat{\lambda}_L)$	$Covr(\hat{\mu}_L, \hat{\lambda}_L)$
2	1.190133	1.305354	-1.242299
3	0.520684	0.641149	-0.575478
4	0.318911	0.423734	-0.366029
5	0.224902	0.316225	-0.265497
6	0.171254	0.251932	-0.206767
7	0.137247	0.209596	-0.168823
8	0.113534	0.179080	-0.141924

3.4 Case IV: Estimation of μ and λ for c=15

By using the values of means, variances and covariance of the probability density function, given in Table 7 and Table 8, we estimated the coefficients of BLUEs of μ and λ in (10) and (11), based on first n lower record values, which are presented in table 18 and table 19. The values of $Var(\hat{\mu}_L)$, $Var(\hat{\lambda}_L)$, and $Covr(\hat{\mu}_L,\hat{\lambda}_L)$ in (12), (13) and (14) respectively, are computed andpresented in table 20.

Table 18. Coefficients of the BLUE of Location Parameter μ for c=15

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
2	-13.99928	14.999281						
3	-6.500053	-8.570578	16.070632					
4	-4.129755	-5.445212	-5.834583	16.409552				
5	-2.989115	-3.941321	-4.223110	-4.418737	16.572284			
6	-2.324786	-3.065341	-3.284559	-3.436647	-3.556821	16.668155		
7	-1.892689	-2.495622	-2.674004	-2.797958	-2.895808	-2.972619	16.728702	
8	-1.590341	-2.096933	-2.246872	-2.350962	-2.433246	-2.497750	-2.557033	16.773138

Table 19. Coefficients of the BLUE of Scale Parameter λ for c=15

n	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
2	14.378559	-14.37856						
3	6.906369	9.106322	-16.01269					
4	4.787629	5.917084	6.340195	-16.74491				
5	3.303198	4.355454	4.666853	4.883035	-17.20854			
6	2.603782	3.433209	3.678733	3.849077	3.983676	-17.54848		
7	2.143646	2.826522	3.028559	3.168944	3.279770	3.366768	-17.81421	
8	1.818528	2.397809	2.569259	2.688284	2.782373	2.856137	2.923922	-18.03631

Table 20 Variances and Covariances of the BLUEs of μ and λ for c=15

n	$Var(\hat{\mu}_L)$	$Var(\hat{\lambda}_L)$	$Covr(\hat{\mu}_L,\hat{\lambda}_L)$
2	1.096942	1.160181	-1.126661
3	0.509323	0.576792	-0.541160
4	0.323594	0.383394	-0.351635
5	0.234218	0.287024	-0.258829
6	0.182163	0.229325	-0.204024
7	0.148305	0.190931	-0.167969
8	0.124614	0.163537	-0.142494

4 Gupta's Method for Location and Scale parameters

The exact estimates given by Lloyd [12] become cumbersome and certainly impracticable when the sample size is large and moreover the exact variance covariance matrix is not available for certain distributions for large sample size. The coefficients of Gupta's method are obtained by assuming the variance covariance matrix to be a unit matrix, see [13]. In this method, the linear estimates are obtained as

$$\hat{\mu}_G = \sum_{i=1}^{n} d_i x_{L(i)} \tag{15}$$

$$\hat{\lambda}_G = \sum_{i=1}^{n} e_i x_{L(i)} \tag{16}$$

$$d_i = \frac{1}{n} - \frac{\bar{\mu}_k(\mu_i - \bar{\mu}_k)}{\sum_{i=1}^{n}(\mu_i - \bar{\mu}_k)^2} \tag{17}$$

$$e_i = \frac{(\mu_i - \bar{\mu}_k)}{\sum_{i=1}^{n}(\mu_i - \bar{\mu}_k)^2} \tag{18}$$

where

$$\bar{\mu}_k = \frac{1}{n}\sum_{j=1}^{n}\mu_j$$

$$Var(\hat{\mu}_G) = (d'Bd)\lambda^2 \tag{19}$$

$$Var(\hat{\lambda}_G) = (e'Be)\lambda^2 \qquad (20)$$

We are concentrating on the study that what will be the result of changing the value of shape parameter on estimators of the location and scale parameters which are obtained by Gupta [13]. For this study, we consider the four cases:

- Case I: Estimation of μ and λ for c=3
- Case II: Estimation of μ and λ for c=6
- Case III:Estimation of μ and λ for c=9
- Case IV: Estimation of μ and λ for c=15

4.1 Case I: Estimation of μ and λ for c=3

By using Table 1 and Table 2, we obtained the coefficients d and e in (17) and (18) from Gupta's method and presented in Table 21 and Table 22. The values of $Var(\hat{\mu}_G)$ and $Var(\hat{\lambda}_G)$ in (19) and (20) are also computed and presented in table 23.

Table 21. Coefficients of Location Parameter μ by Gupta's Method for c=3

n	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
2	-1.999989	2.999989						
3	-1.461534	0.846156	1.615379					
4	-1.177142	0.304892	0.798900	1.073351				
5	-0.998284	0.092291	0.455813	0.657771	0.792409			
6	-0.874038	-0.009943	0.278086	0.438103	0.544781	0.623011		
7	-0.781992	-0.064995	0.174002	0.306779	0.395297	0.460210	0.510698	
8	-0.710657	-0.096721	0.107923	0.221615	0.297409	0.352991	0.396222	0.431219

Table 22. Coefficients of Scale Parameter by Gupta's Method λ for c=3

n	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
2	2.215453	-2.215453						
3	1.789408	-0.511262	-1.278146					
4	1.552147	-0.059700	-0.596978	-0.895469				
5	1.396332	0.125511	-0.298092	-0.533430	-0.690320			
6	1.284044	0.217905	-0.137471	-0.334904	-0.466526	-0.563048		
7	1.198154	0.269275	-0.040349	-0.212363	-0.327039	-0.411135	-0.476543	
8	1.129674	0.299731	0.023086	-0.130607	-0.233069	-0.308208	-0.366649	-0.413958

Table 23: Variances of $\hat{\mu}_G$ and $\hat{\lambda}_G$ by Gupta's Method for c=3

n	$Var(\hat{\mu}_G)$	$Var(\hat{\lambda}_G)$
2	2.678904	3.382756
3	1.495324	2.365041
4	1.012715	1.874811
5	0.754062	1.578171
6	0.594607	1.376565
7	0.487315	1.229293
8	0.410629	1.116265

4.2 Case II: Estimation of μ and λ for c=6

By using Table 3 and Table 4, we obtained the coefficients d and e in (17) and (18) from Gupta's method and presented in Table 24 and Table 25. The values of $Var(\hat{\mu}_G)$ and $Var(\hat{\lambda}_G)$ in (19) and (20) are also computed and presented in Table 26.

Table 24. Coefficients of Location Parameter μ by Gupta's Method for c=6

n	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
2	-4.999926	5.999926						
3	-3.613500	1.286030	3.327470					
4	-2.885913	0.182534	1.461035	2.242344				
5	-2.430628	-0.216144	0.706544	1.270411	1.669817			
6	-2.115682	-0.388706	0.330857	0.770592	1.082071	1.320867		
7	-1.883192	-0.469316	0.119790	0.479800	0.734808	0.930310	1.087800	
8	-1.703574	-0.506958	-0.008375	0.296315	0.512138	0.677599	0.810888	0.921967

Table 25. Coefficients of Scale Parameter λ by Gupta's Method for c=6

n	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
2	5.315360	-5.315360						
3	4.038764	-0.974887	-3.063877					
4	3.348476	0.072039	-1.293123	-2.127392				
5	2.905802	0.459674	-0.559530	-1.182380	-1.623566			
6	2.593122	0.630994	-0.186548	-0.686159	-1.040050	-1.311361		
7	2.358068	0.712494	0.026848	-0.392160	-0.688957	-0.916497	-1.099795	
8	2.173514	0.751170	0.158535	-0.203632	-0.460167	-0.656840	-0.815273	-0.947306

Table 26: Variances of $\hat{\mu}_G$ and $\hat{\lambda}_G$ by Gupta's Method for c=6

n	$Var(\hat{\mu}_G)$	$Var(\hat{\lambda}_G)$
2	1.354109	1.550605
3	0.776659	1.007156
4	0.543082	0.766582
5	0.415598	0.626281
6	0.335367	0.533142
7	0.280316	0.466297
8	0.240262	0.415729

4.3 Case III: Estimation of μ and λ for $c = 9$

By using Table 5 and Table 6, we obtained the coefficients d and e in (17) and (18) from Gupta's method and presented in Table 27 and Table 28. The values of $Var(\hat{\mu}_G)$ and $Var(\hat{\lambda}_G)$ in (19) and (20) are also computed and presented in Table 29.

Table 27. Coefficients of Location Parameter μ by Gupta's Method for c=9

n	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
2	-7.999933	8.999933						
3	-5.759353	1.718038	5.041316					
4	-4.585850	0.054285	2.116562	3.415003				
5	-3.852718	-0.528141	0.949446	1.879756	2.551657			
6	-3.346260	-0.769117	0.376278	1.097436	1.618279	2.023384		
7	-2.972851	-0.873782	0.059136	0.646515	1.070739	1.400695	1.669547	
8	-2.684676	-0.916152	-0.130143	0.364740	0.722160	1.000157	1.226672	1.417241

Table 28. Coefficients of Scale Parameter λ by Gupta's Method for c=9

n	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
2	8.350591	-8.350591						
3	6.215858	-1.412698	-4.803160					
4	5.074573	0.205377	-1.958706	-3.321245				
5	4.349436	0.781451	-0.804316	-1.802739	-2.523832			
6	3.841259	1.023245	-0.229202	-1.017762	-1.587286	-2.030253		
7	3.461865	1.129588	0.093022	-0.559615	-1.030970	-1.397584	-1.696306	
8	3.165802	1.173118	0.287482	-0.270127	-0.672850	-0.986083	-1.241309	-1.456033

Table 29: Variances of $\hat{\mu}_G$ and $\hat{\lambda}_G$ by Gupta's Method for c=9

n	$Var(\hat{\mu}_G)$	$Var(\hat{\lambda}_G)$
2	1.190133	1.305354
3	0.689447	0.823344
4	0.487714	0.617087
5	0.377049	0.498772
6	0.306944	0.421063
7	0.258531	0.365739
8	0.223096	0.324162

4.4 Case IV: Estimation of μ and λ for c=15

By using Table 7 and Table 8, we obtained the coefficients d and e in (17) and (18) from Gupta's method and presented in Table 30 and Table 31. The values of $Var(\hat{\mu}_G)$ and $Var(\hat{\lambda}_G)$ in (19) and (20) are also computed and presented in Table 32.

Table 30. Coefficients of Location Parameter μ by Gupta's Method for c=15

n	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
2	-13.99928	14.999281						
3	-10.04717	2.577893	8.469272					
4	-7.980347	-0.204886	3.423469	5.761764				
5	-6.690819	-1.153271	1.430780	3.096074	4.317236			
6	-5.800968	-1.529875	0.463195	1.747631	2.689509	3.430508		
7	-5.145550	-1.681736	-0.065376	0.976288	1.740142	2.341085	2.835147	
8	-4.640191	-1.732867	-0.376188	0.498124	1.139259	1.643655	2.058342	2.409865

Table 31. Coefficients of Scale Parameter λ by Gupta's Method for c=15

n	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
2	14.378559	-14.37856						
3	10.528031	-2.276460	-8.251571					
4	8.488803	0.469171	-3.273125	-5.684849				
5	7.203255	1.414629	-1.286585	-3.027384	-4.303915			
6	6.308305	1.793391	-0.313456	-1.671216	-2.666862	-3.450162		
7	5.644056	1.947298	0.222237	-0.889480	-1.704703	-2.346060	-2.873348	
8	5.128390	1.999472	0.539388	-0.401563	-1.091565	-1.634406	-2.080700	-2.459015

Table 32: Variances of $\hat{\mu}_G$ and $\hat{\lambda}_G$ by Gupta's Method for c=15

n	$Var(\hat{\mu}_G)$	$Var(\hat{\lambda}_G)$
2	1.096942	1.160181
3	0.640789	0.713842
4	0.457672	0.528131
5	0.356875	0.423126
6	0.292699	0.354799
7	0.248151	0.306487
8	0.215380	0.270374

5 Relative Efficincy

Relative efficiency of Lloyd's estimates to Gupta's estimates of location parameter μ and scale parameter λ when c=3, 6, 9, and 15 is presented as follows

Table 33: R.E of Lloyd's estimate to Gupta's estimate of location parameter μ when c=3, 6, 9, and 15

n	c=3 RE($\hat{\mu}_L, \hat{\mu}_G$)	c=6 RE($\hat{\mu}_L, \hat{\mu}_G$)	c=9 RE($\hat{\mu}_L, \hat{\mu}_G$)	c=15 RE($\hat{\mu}_L, \hat{\mu}_G$)
2	100.00	100.00	100.00	100.00
3	44.79	69.74	75.52	79.48
4	31.12	57.98	65.39	70.70
5	24.87	51.52	59.65	65.63
6	21.22	47.31	55.79	62.24
7	18.80	44.32	53.09	59.76
8	17.06	42.06	50.89	57.86

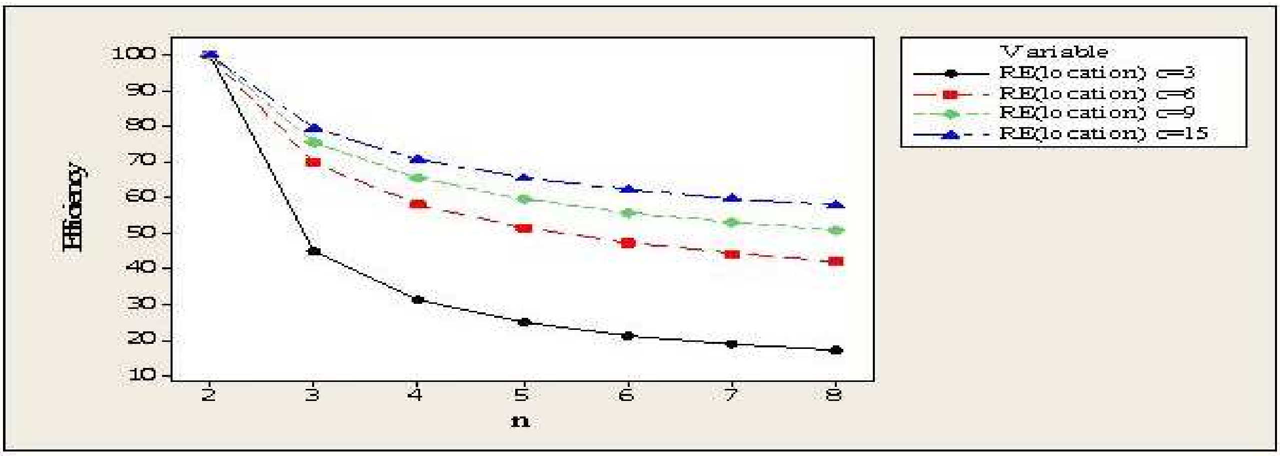

Fig. 1: R.E. of Lloyd's to Gupta's Methods for $\hat{\mu}$ when c=3, 6, 9, and 15

Table 33 and Figure 1 are showing that relative efficiency of Lloyd's to Gupta's methods for $\hat{\mu}$ increases as the value of shape parameter c increases.

Table 34: R.E of Lloyd's estimate to Gupta's estimate of scale parameter λ when c=3, 6, 9, and 15

n	c=3 RE($\hat{\lambda}_L, \hat{\lambda}_G$)	c=6 RE($\hat{\lambda}_L, \hat{\lambda}_G$)	c=9 RE($\hat{\lambda}_L, \hat{\lambda}_G$)	c=15 RE($\hat{\lambda}_L, \hat{\lambda}_G$)
2	100.00	100.00	100.00	100.00
3	52.23	73.50	77.87	80.80
4	39.49	63.02	68.67	72.59
5	33.33	57.15	63.40	67.83
6	29.58	53.27	59.83	64.64
7	27.00	50.47	57.31	62.30
8	25.09	48.32	57.00	60.49

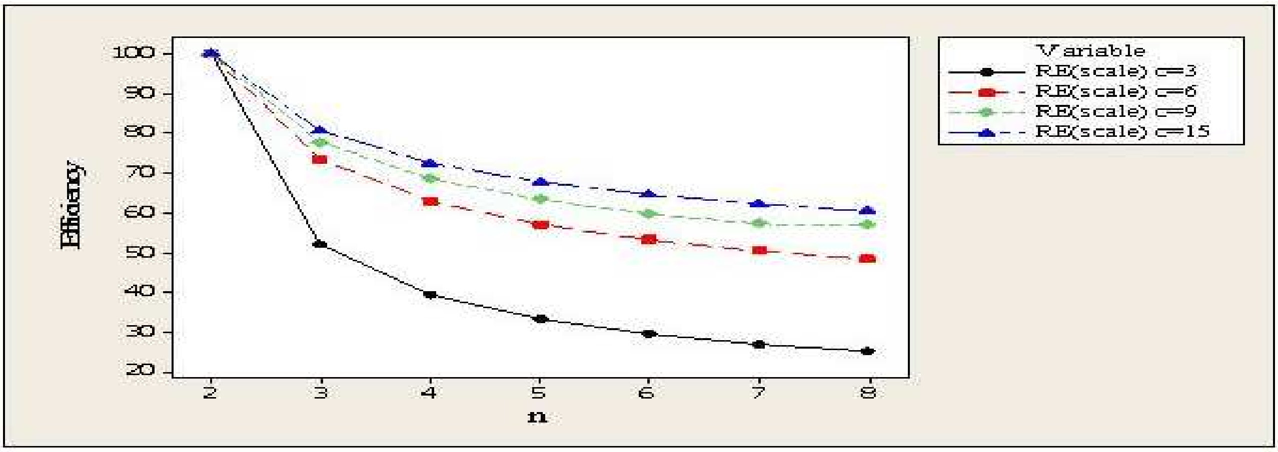

Fig. 2: R.E. of Lloyd's to Gupta's Methods for $\hat{\lambda}$ when c=3, 6, 9, and 15

Table 34 and Figure 2 are presenting that relative efficiency of Lloyd's to Gupta's methods for $\hat{\lambda}$ increases as the value of shape parameter c increases.

6 Conclusion

Purpose of this paper is to analyze the effect of changing the value of shape parameter on estimators of the location and scale parameters which are obtained by Lloyd's and Gupta's methods. For this study, we considered four cases of shape parameter. After this analysis we determined the following points:

– From figure 1 and figure 2, we can see that relative efficiency of Gupta's estimates of location and scale parameters for $c = 3$ decreases exponentially as sample size increases. While for larger choice of c, the decrease in the efficiency of Gupta's estimates become slower.
– Estimators of both methods are remained equal efficient for changing the value of shape parameter at $n = 2$.
– From table 33, we can observe that the second maximum relative efficiencies of Gupta's estimate of location parameter at n=3 are 44.79, 69.74, 75.52 and 79.48. The efficiency of Gupta's estimator at c=6 is increased by 55.70%, it is also increased by 69.07% and 77.45% for $c = 9$ and $c = 15$ respectively as compared to efficiency of the Gupta's estimate of location parameter at $c = 3$.
– From table 34, we can observe that the second maximum relative efficiencies of Gupta's estimate of scale parameter at $n = 3$ are 52.23, 73.50, 77.87 and 80.80. The efficiency of Gupta's estimator at $c = 6$ is increased by 40.72%, it is also increased by 49.09% and 54.70% for $c = 9$ and $c = 15$ respectively as compared to efficiency of the Guptas estimate of scale parameter at $c = 3$.
– Tables 33-34 are showing that the efficiencies of Gupta's estimates are increasing with increase in true value of the parameter c.

In general, we can say that relative efficiency of Gupta's estimators for location and scale parameters of IWD increases as the value of shape parameter increases.

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