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Propagation of Rayleigh Waves in Fiber-Reinforced Anisotropic Solid Thermo-Viscoelastic Media under Effect of Rotation

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Abstract: In this paper, we investigated the propagation of Rayleigh waves in a homogeneous, pre-stressed elastic layer of finite thickness over a homogeneous, pre-stressed elastic half - space subjected to the rotation. The dispersion equation has been derived for a layer over a half-space, when both media are considered as pre-stressed and the effect of initial rotation shown in earlier investigators, is in general not applicable to the case of pre-stressed media. The results indicate that the effect of the rotation on the Propagation of Rayleigh waves in Fiber- reinforced isotropic solid thermo-viscoelastic media are very pronounced.

Keywords: Rayleigh waves; anisotropic; rotation; Fiber- reinforced; thermo-viscoelastic

1 Introduction

During earthquakes, the Rayleigh waves play a more drastic role than other seismic waves in damages to human beings and buildings on the surface of the Earth. Therefore, it is of great importance to the seismologist to study the effect of initial stress on the propagation of Rayleigh waves. After the pioneering work of Rayleigh, many investigators have solved the Rayleigh problem for a half?space and one or more superficial layer situated over a half space inhomogeneous and non-homogeneous media. Schoenberg and Censor [1] were the first to study the propagation of plane harmonic waves in a rotating elastic medium where it is shown that the elastic medium becomes dispersive and anisotropic due to rotation. Later on, many researchers introduced rotation in different theories of thermoelasticity. Agarwal [2] studied thermo-elastic plane wave propagation in an infinite non-rotating medium.

The normal mode analysis was used to obtain the exact expression for the temperature distribution, the thermal stresses and the displacement components. The purpose of the present work is to show the thermal and rotational effects on the surface waves. Surface waves have been well recognized in the study of earthquake, seismology, geophysics and geodynamics. A good amount of literature for surface waves is available (in Refs. [3,4,5,6] Acharya and Singupta [7], Pal and Sengupta [8] and Sengupta and Nath [9] and his research collaborators have studied surface waves. These waves usually have greater amplitudes as compared with body waves and travel more slowly than body waves. There are many types of surface waves but we only discussed Stoneley, Love and Rayleigh waves. Earthquake radiate seismic energy as both body and surface waves. These are also used for detecting cracks and other defects in materials.

The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al. [10]. The superiority of fibre-reinforced composite materials over other structural materials attracted many authors to study different types of problems in this field. Fibre-reinforced composite structures are used due to their low weight and high strength. Two important components, namely concrete and steel of a reinforced medium are bound together as a single unit so that there can be no relative displacement between them i.e. they act together as a single anisotropic unit. The artificial

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structures on the surface of the earth are excited during an earthquake, which give rise to violent vibrations in some cases (Refs. [11] and [12]). Engineers and architects are in search of such reinforced elastic materials for the structures that resist the oscillatory vibration. The propagation of waves depends upon the ground vibration and the physical properties of the material structure. Surface wave propagation in fiber reinforced media was discussed by various authors ([13,14]). Abd-Alla et al. [15] investigated the transient coupled thermoelasticity of an annular fin The effects. of gravity field on surface waves in fiber-reinforced thermoelastic media was also discussed by Abd-Alla et al. [16]. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in (Refs. [17,18,19, 20]) Recently, Abd-Alla et al. [21] investigated the magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in ([22,23, 24,25,26]) The temperature-rate dependent theory of thermoelasticity, which takes into account two relaxation times, was developed by Green and Lindsay [27].

The aim of this paper is to investigate the propagation of thermoelastic Rayleigh waves in a rotating fibre-reinforced elastic anisotropic media. The Rayleigh wave speed is derived to study the effect of rotation and thermal on surface waves. The wave velocity equations have been obtained for Rayleigh waves, and are in good agreement with the corresponding classical result in the absence of temperature and rotation as well as homogeneity of the material medium. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. It is also observed that the corresponding classical results follow from this analysis, in elastic media, by neglecting reinforced parameters, rotational and thermal effects. Numerical results are given and illustrated graphically.

2 Formulation of the problem and basic equations

Let us consider the problem of a thermo elastic half-space$(x \geq 0)$. The surface of the half space is subjected to a thermal shock which is a function of y and t. Thus, all quantities are independent of z and the third component of displacement vector vanishes. When all body forces are neglected the governing equations are:

(i) The constitutive equations for a fiber-reinforced linearly thermoplastic isotropic medium with respect to the reinforcement direction see [1]

$$\begin{aligned}\sigma_{ij} = {} & D_\lambda e_{\kappa k}\delta_{ij} + 2D\mu_T e_{ij} + D_\alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) \\ & + 2(D_{\mu L} - D_{\mu T})(a_i a_k e_{kj} + a_j a_k e_{ki}) + D_\beta a_k a_m e_{km} a_i a_j - \gamma(T - T_0)\delta_{ij}, \end{aligned} \quad (1)$$

where σ_{ij} are the components of stress, e_{ij} are the components of strain; $D_\lambda, D_{\mu T}$ are viscoelastic parameters, $D_\alpha, D_\beta, (D_{\mu L} - D_{\mu T})$ are reinforcement viscoelastic parameters, $\gamma = (3D_\lambda + 2D_\mu)\alpha_1$, α_1 is thermal expansion coefficient, δ_{ij} is the Kronecker delta, T is the temperature above reference temperature T_0, and $a = (a_1, a_2, a_3), a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fiber-direction as $a \equiv (1,0,0)$.

The strains can be expressed in terms of the displacementu_j as

$$e_{ij} = \frac{1}{2}(u_{i.j} + u_{j.i}) \quad (2)$$

The elastic medium is rotating uniformly with an angular velocity $\underline{\Omega} = \Omega_{\underline{n}}$ where $\underline{n}$ is a unit vector representing the direction of the axis of rotation .

The displacement equation of motion in the rotating frame has two additional term centripetal acceleration, $\overline{\Omega} \times (\overline{\Omega} \times \overline{u})$ is the centripetal acceleration due to time varying motion only and $2\overline{\Omega} \times \ \overline{u}$ is the Coriolis acceleration, and $\overline{\Omega} = (0,0,\Omega)$.

For plane strain deformation in the $x-y$ plane, displacement $\underline{u} = (u,v,0)$, $\partial/\partial z = 0$. Eq. (1) then yields

$$\sigma_{\times x} = A_{11}u_x + A_{12}v_y - \gamma(T - T_0), \quad (3)$$

$$\sigma_{yy} = A_{12}u_x + A_{22}v_y - \gamma(T - T_0), \quad (4)$$

$$\sigma_{zz} = A_{12}u_x + D_2 v_y - \gamma(T - T_0), \quad (5)$$

$$\sigma_{\times y} = D_{\mu L}(u_x + v_y), \sigma_{\times z} = \sigma_{yz} = 0, \quad (6)$$

where,

$A_{11} = \lambda_0 + 2\alpha_0 + 4\mu_{L0} - 2\mu_{T0} + \beta_0 + (\lambda_1 + 2\alpha_1 + 4\mu_{L1} - 2\mu_{T1} + \beta_1)\frac{\partial}{\partial t}$,

$A_{12} = \lambda_0 + \alpha_0 + (\lambda_1 + \alpha_1)\frac{\partial}{\partial t}$, $A_{22} = \lambda_0 + 2\mu_{T0} + (\lambda_1 + 2\mu_{T1})\frac{\partial}{\partial t}$,

$D_\lambda = \lambda_0 + \lambda_1\frac{\partial}{\partial t}$, $D_\alpha = \alpha_0 + \alpha_1\frac{\partial}{\partial t}$,

$D_\beta = \beta_0 + \beta_1\frac{\partial}{\partial t}$, $D_{\mu T} = \mu_{T0} + \mu_{T1}\frac{\partial}{\partial t}$, $D_{\mu L} = \mu_{L0} + \mu_{L1}\frac{\partial}{\partial t}$, $D_\mu = \mu_0 + \mu_1\frac{\partial}{\partial t}$,

$\gamma = 3\lambda_0 + 2\mu_0 + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}$;

where λ_0, μ_0 are elastic constant and λ_1, μ_1 are the parameters associated with 1th order viscoelasticity.

(ii) The equation of motion in the context of the Green-Naghdi theory is

$$\rho[\ddot{u}_1 + [\overline{\Omega} \times (\overline{\Omega} \times \overline{u}\}_j + (2\overline{\Omega} \times \dot{u})_j] = \sigma_{ij.j}, i,j = 1,2,3 \quad (7)$$

(iii) The heat conduction in the absence of heat sources under the G-N III theory is

$$KT_{ji} + K^* \dot{T}_{ji} = \rho C_E \ddot{T} + \gamma T_0 \ddot{u}_{i,j}, \quad (8)$$

where ρ is the mass density, C_E is the specific heat at constant strain, K^* and K are respectively the material constant characteristic of the theory and thermal conductivity. When, $K^* \to 0$ "equation (8)," reduces heat conduction equation of the G-N II theory. Eq.s (7), (8) and (1) constitute the complete system of generalized thermoelasticity under the G-N III theory.

Using the summation convention, from Equations (3) - (6) we note that the third equation of motion in Eq. (7) identically satisfied and the first two equations become

$$\rho\left[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u-2\Omega\frac{\partial v}{\partial t}\right]=A_{11}\frac{\partial^2 u}{\partial x^2}+B_2\frac{\partial^2 v}{\partial x\partial y}+B_1\frac{\partial^2 u}{\partial y^2}-\gamma\frac{\partial T}{\partial x}, \tag{9}$$

$$\rho\left[\frac{\partial^2 v}{\partial t^2}-v\Omega^2+2\Omega\frac{\partial u}{\partial t}\right]=A_{22}\frac{\partial^2 v}{\partial y^2}+B_2\frac{\partial^2 u}{\partial x\partial y}+B_1\frac{\partial^2 v}{\partial x^2}-\gamma\frac{\partial T}{\partial y}, \tag{10}$$

where,

$$B_1=D_{\mu L}=\mu_{L0}+\mu_{L1}\frac{\partial}{\partial t}, B_2=A_{12}+D_{\mu L}=\lambda_0+\alpha_0+\mu_{L0}+(\lambda_1+\alpha_1+\mu_{L1})\frac{\partial}{\partial t},$$

For convenience, the following non-dimensional variables are used:

$$x'=c_1\eta x, y'=c_1\eta y, u'=c_1\eta y, u, v'=c_1\eta v, t'=c_1^2\eta t,$$

$$\theta=\gamma(T-T_0)/\rho c_1^2, \sigma'_{ij}=\sigma_{ij}/D_{\mu j}, \Omega'=\Omega/c_1^2\eta, \tag{11}$$

where,

$$\eta=\rho C_{E/K}, c_1^2=K/\rho, \qquad i,j=1,2,3,$$

In terms of non-dimensional quantities defined in equation (11), the above governing equations reduce to dropping the prime for convenience

$$\frac{\partial^2 u}{\partial t^2}-(\Omega^2 u+2\Omega\frac{\partial v}{\partial t})=h_{11}\frac{\partial^2 u}{\partial x^2}+h_2\frac{\partial^2 v}{\partial x\partial y}+h_1\frac{\partial^2 u}{\partial y^2}-\frac{\partial\theta}{\partial x}, \tag{12}$$

$$\frac{\partial^2 v}{\partial t^2}-(v\Omega^2-2\Omega\frac{\partial u}{\partial t})=h_{22}\frac{\partial^2 v}{\partial y^2}+h_2\frac{\partial^2 u}{\partial x\partial y}+h_1\frac{\partial^2 v}{\partial x^2}-\frac{\partial\theta}{\partial y}, \tag{13}$$

$$\varepsilon_2\theta_{ji}+\varepsilon_3\,\dot{\theta}_{ji}-\ddot{\theta}=\varepsilon_1\,\ddot{e}, \tag{14}$$

where,

$$(h_{11},h_{22},h_1,h_2)=(A_{11},A_{22},B_1,B_2)/\rho c_1^2,$$

$$\varepsilon_1=\gamma^2 T_0/\rho^2 C_E c_1^2, \varepsilon_2=K/\rho^2 C_E c_1^2, \varepsilon_3=\eta K^*/\rho C_E,$$

$$D_{\mu T}\sigma_{xx}=A_{11}u_x+A_{12}v_y-\rho c_1^2\theta, \tag{15}$$

$$D_{\mu T}\sigma_{yy}=A_{12}u_x+A_{22}v_y-\rho c_1^2\theta, \tag{16}$$

$$D_{\mu T}\sigma_{zz}=A_{12}u_x+D_\lambda v_y-\rho c_1^2\theta, \tag{17}$$

$$D_{\mu T}\sigma_{xy}=D_{\mu L}(u_x+v_y), \sigma_{xz}=\sigma_{yz}=0, \tag{18}$$

where ε_1 is usually the thermoelastic coupling factor, ε_2 is the characteristic parameter of the G-N theory of type II and is the characteristic parameter of the G-N theory of type III.

3 Solution of the problem

The normal mode analysis gives exact solutions without any assumed restrictions on temperature, displacement and stress distributions. It is applied to a wide range of problems in different branches; see [19,20,21,22,23] It can be applied to boundary-layer problems, which are described by the linearized Navier - Stokes equations in electro hydrodynamics, see [24,25]. The normal mode analysis is, in fact, to look for the solution in the Fourier transformed domain. Assume that all the field quantities are sufficiently smooth on the real line such that normal mode analysis of these functions exists.

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[u,v,\theta,\sigma_{ij}](x.y,t)=[u^*(x),v^*(x),\theta^*(x),\sigma^*_{ij}(x)]\exp\text{''}(\omega t+iby), \tag{19}$$

whereb ω is a complex time constant, $i=\sqrt{-1}$, b is the wave number in the $y-$ direction, $u^*(x), v^*(x), \theta^*(x)$ and $\sigma^*_{ij}(x)$ are the amplitudes of the field quantities.

By using Eq.(19) , then Eqs.(12)-(18) take the from

$$[h^*_{11}D^2-A_1]u^*+[ibh^*_2D+2\omega\Omega]v^*=D\theta^*, \tag{20}$$

$$[-2\Omega\omega+ibh^*_2D]u^*+[h^*_1D^2-A_2]v^*=ib\theta^*, \tag{21}$$

$$A_3Du^*+ibA_3v^*=(\varepsilon D^2-A_4)\theta^*, \tag{22}$$

$$D^*_{\mu T}\sigma^*_{xx}=A^*_{11}Du^*+ibA^*_{12}v^*-\rho c_1^2\theta^*, \tag{23}$$

$$D^*_{\mu T}\sigma^*_{yy}=A^*_{12}Du^*+ibA^*_{22}v^*-\rho c_1^2\theta^*, \tag{24}$$

$$D^*_{\mu T}\sigma^*_{zz}=A^*_{12}Du^*+ibD^*_\lambda v^*-\rho c_1^2\theta^*, \tag{25}$$

$$D^*_{\mu T}\sigma^*_{xy}=D^*_{\mu L}(ibu^*+Dv^*), \sigma_{xz}=\sigma_{yz}=0, \tag{26}$$

where,

$$(h^*_{11},h^*_{22},h^*_1,h^*_2)=(A^*_{11},A^*_{22},B^*_1,B^*_2)/\rho c_1^2,$$

$A^*_{11}=\lambda_0+2\alpha_0+4\mu_{L0}-2\mu_{T0}+\beta_0+(\lambda_1+2\alpha_1+4\mu_{L1}-2\mu_{T1}+\beta_1)\omega$,
$A^*_{12}=\lambda_0+\alpha_0+(\lambda_1+\alpha_1)\omega, D^*_{\mu T}=\mu_{T0}+\mu_{T1}\omega$,
$A^*_{22}=(\lambda_0+2\mu_{T0})+(\lambda_1+2\mu_{T1})\omega, B^*_1=\mu_{L0}+\mu_{L1}\omega, D^*_{\mu L}=\mu_{L0}+\mu_{L1}\omega$,

$B_2^* = \alpha_0 + \lambda_0 + \mu_{L0} + (\alpha_1 + \lambda_1 + \mu_{L1})\omega$,
$A_1 = \omega^2 + h_1^* b^2 - \Omega^2, A_2 = \omega^2 + h_{22}^* b^2 - \Omega^2, A_3 = \omega^2 \varepsilon_1$,
$A_4 = \varepsilon b^2 + \omega^2, \varepsilon = \varepsilon_2 + \varepsilon_3 \omega, D = \frac{d}{dx}$,

Eliminating $\theta^*(x)$ and $v^*(x)$ between equations (20)-(22) we obtain the ordinary differential equation satisfied with $u^*(t)$

$$[D^6 - AD^4 + BD^2 - C]u^*(x) = 0, \tag{27}$$

where,

$$A = \frac{1}{h_1^* h_{11}^* \varepsilon}[h_1^* A_1 \varepsilon + h_1^* h_{11}^* A_4 + A_2 h_{11}^* \varepsilon + h_1^* A_3 - b^2 h_2^{*2} \varepsilon], \tag{28}$$

$$B = \frac{1}{h_1^* h_{11}^* \varepsilon}[h_1^* A_4 A_1 + h_{11}^* A_2 A_4 + A_1 A_2 \varepsilon + A_2 A_3 + b^2 A_3 h_{11}^* - b^2 h_2^{*2} A_4 - 2h_2^* A_3 b^2 - 4\Omega^2 \omega^2 \varepsilon], \tag{29}$$

$$C = \frac{1}{h_1^* h_{11}^* \varepsilon}[A_1 A_2 A_4 + A_1 A_3 b^2 + 4\Omega^2 \omega^2 A_4], \tag{30}$$

In the similar manner, we can show that satisfy the equation

$$[D^6 - AD^4 + BD^2 - C]v^*(x) = 0. \tag{31}$$

Eq.(27) can be factored as

$$(D^2 - \overline{k}_1^2)(D^2 - k_2^2)(D^2 - k_3^2)u^*(x) = 0. \tag{32}$$

Eq.(31) represent the initial integral equation of six orders, has six roots, i.e.,

$$[k^6 - Ak^4 + Bk^2 - C] = 0 \tag{33}$$

We have six roots three positive and three negative the positive roots have given an unbounded solution for $u^*(t)$ since $x \geq 0$ hence we should suppress the positive exponentials hence the solution of equation (27) has the form:

$$u^*(x) = \sum_{n=1}^{3} M_n e^{-k_n x} \tag{34}$$

And we can get $v^*(x)$ from the relation between $u^*(x)$ and $v^*(x)$:

$$h_{11}^* D^2 - A_1 + \frac{2\Omega\omega}{ib} D - h_2^* D^2]u^*(x) - [\frac{h_1^*}{ib} D^2 - \frac{A_2}{ib} D - ibh_2^* D - 2\omega\Omega]v^*(x), \tag{35}$$

One get:

$$v^*(x) = \sum_{n=1}^{3} H_{1n} M_n e^{-k_n x} \tag{36}$$

And similarly for equation (20)

$$\theta^*(x) = \sum_{n=1}^{3} H_{2n} M_n e^{-k_n x} \tag{37}$$

where M_n are parameters, $k_n^2 (n = 1,2,3)$ are the roots of the characteristic equation (32) and

$$H_{1n} = \frac{ibh_{11}^* k_n^2 - ibA_1 - ibh_2^* k_n^2 + 2\Omega\omega k_n}{-h_1^* k_n^2 + A_2 k_n - b^2 h_2^* k_n - 2i\omega b\Omega}, \tag{38}$$

$$H_{2n} = \frac{A_1 - h_{11}^* k_n^2 + ibh_2^* k_n H_{1n} - 2\omega\Omega H_{1n}}{k_n}, \tag{39}$$

Using equations (33),(35),(36) into equations (23)-(26) we get the following relations:

$$\sigma_{xx}^* = \sum_{n=1}^{3} H_{3n} M_n e^{-k_n x} \tag{40}$$

$$\sigma_{yy}^* = \sum_{n=1}^{3} H_{4n} M_n e^{-k_n x} \tag{41}$$

$$\sigma_{zz}^* = \sum_{n=1}^{3} H_{5n} M_n e^{-k_n x} \tag{42}$$

$$\sigma_{xy}^* = \sum_{n=1}^{3} H_{6n} M_n e^{-k_n x} \tag{43}$$

where,

$$H_{3n} = \frac{(-k_n A_{11}^* + ibA_{12}^* H_{1n} - \rho C_1^2 H_{2n})}{D_{\mu T}^*}, \tag{44}$$

$$H_{4n} = \frac{(-k_n A_{12}^* + ibA_{22}^* H_{1n} - \rho C_1^2 H_{2n})}{D_{\mu T}^*}, \tag{45}$$

$$H_{5n} = \frac{(-k_n A_{12}^* + ibD_2^* H_{1n} - \rho C_1^2 H_{2n})}{D_{\mu T}^*}, \tag{46}$$

$$H_{6n} = \frac{(ib - k_n H_{1n}) D_{\mu L}^*}{D_{\mu T}^*}, \tag{47}$$

4 The Boundary conditions of the problem

The parameter has to be chosen such that the boundary conditions on the surface at $x = 0$ take the form:

1- A thermal boundary conditions that the surface of the half-space is

$$\theta(0,y,t) = f(0,y,t) = 0 \tag{48}$$

2- A mechanical boundary condition that the surface of the half-space is traction free

$$\sigma_{xx}(0,y,t) = \sigma_{xy}(0,y,t) = 0 \tag{49}$$

Using the expressions of the variables considered into the above boundary conditions (4.4.1) and (4.4.2), we get

$$\sum_{n=1}^{3} H_{2n} M_n = 0 \tag{50}$$

$$\sum_{n=1}^{3} H_{3n} M_n = 0 \tag{51}$$

$$\sum_{n=1}^{3} H_{6n} M_n = 0 \tag{52}$$

To determine the constants $M_n, n = 1,2,3$, it's necessary that the determinant of the constant coefficients must be vanish, i.e.,

$$\begin{vmatrix} H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{61} & H_{62} & H_{63} \end{vmatrix} \tag{53}$$

where,

$$\left.\begin{aligned} H_{1n} &= \frac{ibh_{11}^* k_n^2 - ibA_1 - ibh_2^* k_n^2 + 2\Omega\omega k_n}{-h_1^* k_n^2 + A_2 k_n - b^2 h_2^* k_n - 2i\omega b\Omega}, \\ H_{2n} &= \frac{A_1 - h_{11}^* k_n^2 + ibh_2^* k_n H_{1n} - 2\omega\Omega H_{1n}}{k_n}, n = 1,2,3, \\ H_{3n} &= \frac{(-k_n A_{11}^* + ibA_{12}^* H_{1n} - \rho C_1^2 H_{2n})}{D_{\mu T}^*}, \\ H_{6n} &= \frac{(ib - k_n H_{1n}) D_{\mu L}^*}{D_{\mu T}^*}, \end{aligned}\right\} \tag{54}$$

Equation (53) determines the Rayleigh surface waves under the influences of the viscosity and rotation in Fiber-reinforced isotropic solid thermo-viscoelastic media, from determining this equation has complex roots. The real part (Re) gives the velocity of Rayleigh waves and the imaginary part (Im) gives the attenuation coefficient. We discuss this case and special cases in Green-Naghdi Theory II and III.

5 Special case

If the rotation is neglected:

6 Numerical results and discussion

To illustrate the theoretical results obtained in the preceding section, to compare both types II and III of the G-N theory of thermoelasticity and to study the effect of the time on wave propagation in a conducting fiber-reinforced and in the absence and presence of reinforcement. We now present some numerical results for the physical constants, as discussed in [5].

$\rho = 7800 Kg/m^3$, $\lambda_0 = 5.65 \times 10^{10} Nm^{-2}$, $\lambda_1 = 2.25 \times 10^4$,

$\mu_0 = 2.345 \times 10^{10}$, $\mu_1 = 0.563 \times 10^{10}$, $\mu_{T0} = 2.46 \times 10^{10} Nm^{-2}$,

$\mu_{L0} = 5.66 \times 10^9 Nm^{-2}$, $\mu_{T1} = 2.46 \times 10^{10} Nm^{-2}$,

$\mu_{L1} = 5.66 \times 10^{10} Nm^{-2}$, $B_0 = 220.90 \times 10^9$, $B_1 = 220.90 \times 10^{10}$,

$\alpha_0 = -1.28 \times 10^9 Nm^{-2}$, $\alpha_1 = -1.28 \times 10^{10}$, $C_E = 50 \times 10^5 J.kg^{-1}.K^{-1}$,

$K = 10^7 w.m^{-1} K^{-1}$, $K^* = 5 \times 10^{10} w.m^{-1}.K^{-1}$, $T_0 = 200K$,

$\omega_0 = -0.1$, $\xi = 0.45$, $\Omega = 0.5 \times 10^5$, $b = 1.2$.

The numerical technique, outlined above, used study propagation of Rayleigh waves in Fiber reinforced isotropic solid thermo-viscoelastic media under the effect of rotation and specific heat C_E, viscoelastic parameters and 'reinforced viscoelastic parameters .

i) G-N Theory III

From Fig.(1a) and Fig. (2a) describe the effect of rotation Ω, we find that the velocity of Rayleigh waves, fixed, then it decreases with increasing of the rotation, while it increases with increasing of wave number b.

From Fig.(1b) and Fig. (2b) that clarify the effect of specific heat C_E, we find that the velocity of Rayleigh waves decreases with increasing of the specific heat, while it increases with increasing of wave number b.

Fig.(1c) and Fig. (2c) that clarify the effect of viscoelastic parameters and reinforced viscoelastic parameters we find that the velocity of Rayleigh waves increasing with increasing of the viscoelastic parameters and reinforced viscoelastic parameters, while it decreases with increasing of wave number b.

ii) G-N theory II , i.e $K^* \to 0$

From Fig.(1a) and Fig. (2a) describe the effect of rotation Ω, we find that the velocity of Rayleigh waves increases with increasing of the rotation value, while it decreases with increasing of wave number b.

From Fig.(1b) and Fig. (2b) show that the effect of specific heat C_E, we find that the velocity of Rayleigh waves increases with increasing of the specific heat, while it decreases with increasing of wave number b.

Fig.(1c) and Fig. (2c) show that the effect of viscoelastic parameters and reinforced viscoelastic parameters we find that the velocity of Rayleigh waves decreases with increasing of the viscoelastic parameters and reinforced viscoelastic parameters, while it decreases with increasing of wave number b.

Special cases

(i) If the rotation Ω is neglected in the case of G-N Theory III:

From Fig. (3) and Fig. (4) show that the effect of specific heat C_E and viscoelastic parameters and reinforcement viscoelastic parameters , we find that the velocity of Rayleigh waves increases with increasing of the specific heat and viscoelastic parameters and reinforcement viscoelastic parameter value in (G-N Theory III).

(ii) If the rotation Ω is neglected in the case of G-N Theory II, i.e:

From Fig. (3) and Fig. (4) that clarify the effect of specific heat and viscoelastic parameters and reinforced

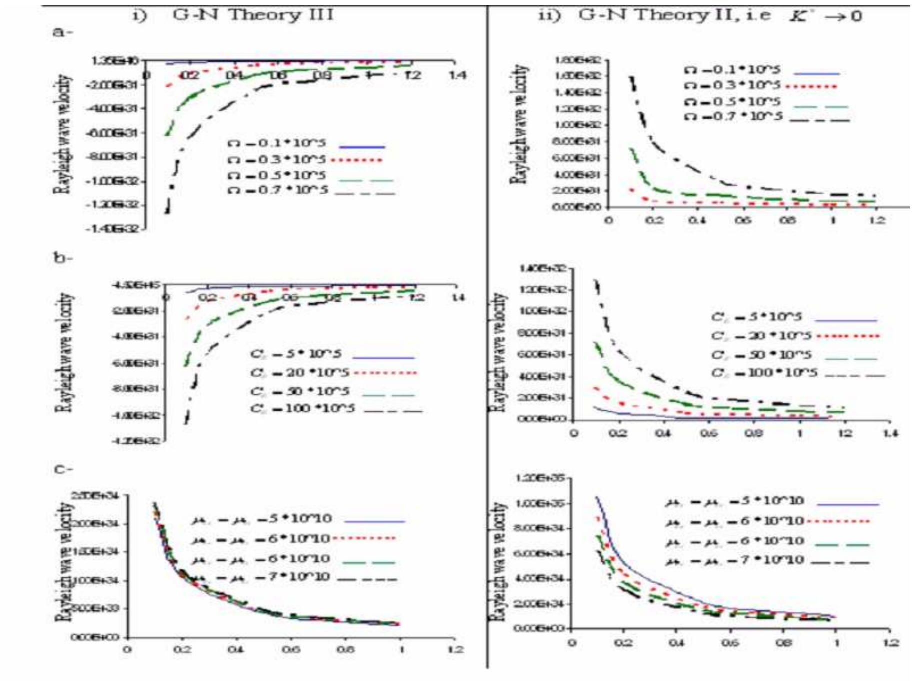

Fig. 1: Effects of Ω, C_E and $\mu_{Li}, \mu_{Ti}, i = 0,1$ on the Rayleigh wave velocity with respect wave number.

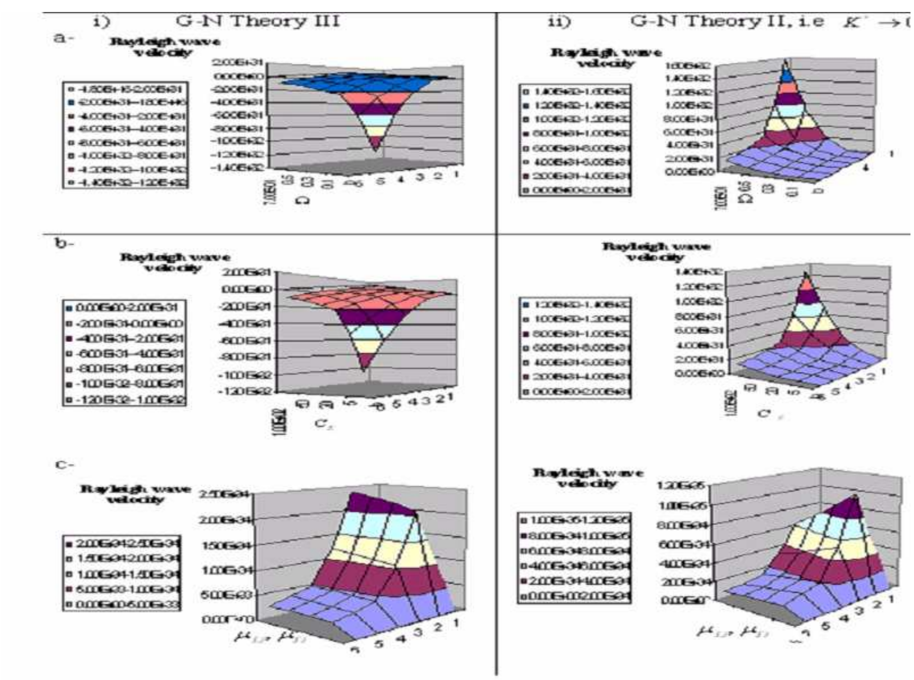

Fig. 2: Effects of Ω, C_E and $\mu_{Li}, \mu_{Ti}, i = 0,1$ on the Rayleigh wave velocity with respect wave number.

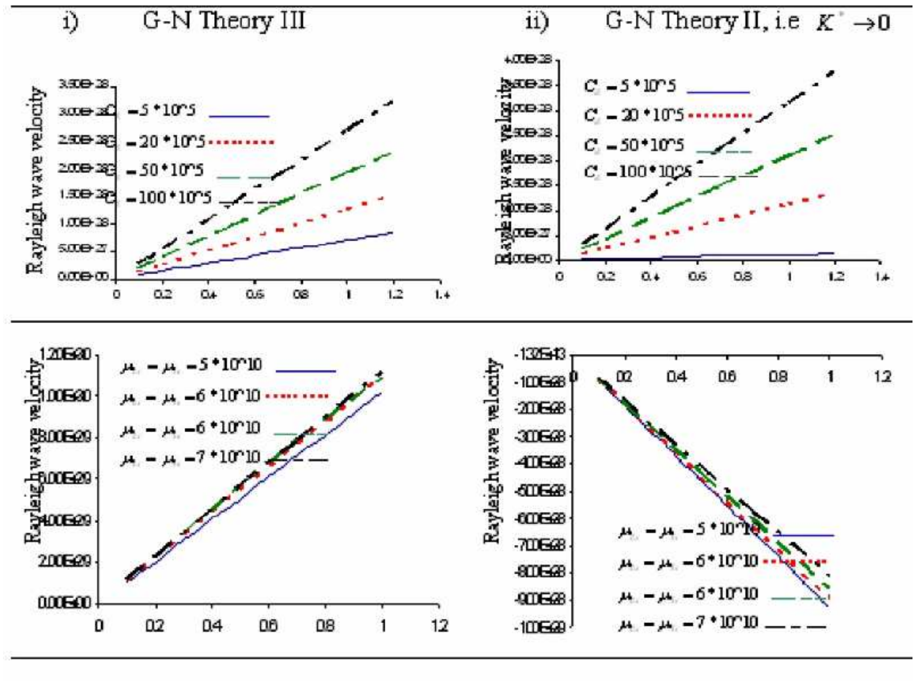

Fig. 3: Effects of C_E an $\mu_{Li}, \mu_{Ti}, i = 0, 1$ on the Rayleigh wave velocity with respect wave number.

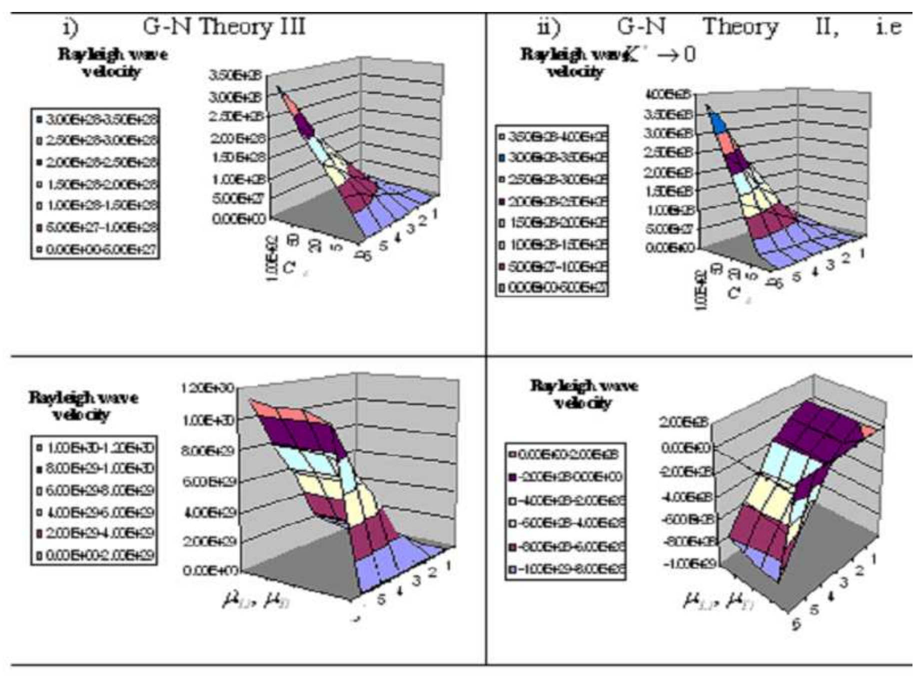

Fig. 4: Effects of C_E an $\mu_{Li}, \mu_{Ti}, i = 0, 1$ on the Rayleigh wave velocity with respect wave number.

viscoelastic parameters , we find that the velocity of Rayleigh waves increases with increasing of the specific heat and viscoelastic parameters and reinforced viscoelastic parameter value, while it increases with increasing of wave number in (G-N Theory II, i.e $K^* \to 0$).

7 Conclusion

Due to the complicated nature of the governing equations of the thermoelasticity fiber-reinforced theory, the work done in this field is unfortunately limited in number. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the fibre-reinforced anisotropic general viscoelastic media without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. Important phenomena are observed in all these computations:

– It was found that the solutions obtained in the context of the thermoelasticity fiber-reinforced theory, however, exhibit the behaviour of speeds of Rayleigh wave.
– The results presented in this paper should prove useful for researchers in material science, designers of new materials.
– Study of the phenomenon of rotation is also used to improve the conditions of oil extractions.

Finally, if the rotation is neglected, the relevant results obtained are deduced to the results obtained by Pal and Sengupta [8].

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