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Chiral Spin Flipping Gate Implemented in IBM Quantum Experience

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Abstract: A chiral spin flipping gate is simulated by specially designed quantum circuits. We consider using ancillary qubits to accomplish the algorithm as well as quantum circuits based on different basic operations. Three methods are applied to build these algorithms, which result in three different quantum circuits that all realize the evolution process of the system. We also implement our quantum algorithms on the IBM Quantum Experience.

Keywords: Quantum computation

1 Introduction

We are now in the frontier of a new area of quantum simulation [1,2], since Feynman [3] proposed the idea of quantum computer and envisioned the possibility of efficiently simulating quantum systems, significant progress has been made. It will be fundamentally more efficiently to simulate the dynamics of quantum many-body systems [3,4] in condensed matter [5,6,7], quantum chemistry [8,9] and high-energy physics [10,11,12], which is intractable

on classical computers. Apart from simulation of the physical or chemical systems, it can also be used to other applications in high computational sciences, such as predicting rare natural phenomenons(volcanoes, earthquake, hurricane, etc) and simulating complex systems (social phenomenon, economic predictions, etc. [13,14,15]).

Recently, big companies (Google, IBM, US Defense) have invested towards conceiving and realizing quantum computer, but those quantum computers will be inaccessible to most of the people. However, the advent of cloud quantum computation changes this situation. IBM, in 2016, launched an interactive platform called the Quantum Experience (QE) [16,17], which released access to a universal five-qubit quantum computer based on superconducting transmon qubits and allowed for circuit design, simulation, testing, and actual execution of an algorithm on a physical device. We know that the quantum computers need not only new hardware components but also software languages and algorithms. With the help of IBM QE, scientists can conveniently implement their own algorithms, which will greatly contribute to the development of this area. Since it became available, a flurry of research results have been published derived from the platform hardware [18,19,20,21,22,23,24,25].

In this paper, we will introduce a chiral spin flipping gate and then try to realize this model by quantum algorithms. In Sec. 2, we illustrate the chiral spin flipping of three qubits. The quantum states of the three qubits transfer to their neighbors in different directions depending on the number of up spins. Later we will show how this operation can be realized in a quantum circuit that is composed by several basic quantum gates. In Sec. 3 the basic concept and operations of quantum circuits will be briefly introduced, and in Sec. 4 the procedure of three different methods to design the objective operation will be explained. In Sec. 5, we will show how to implement our algorithms on IBM Quantum Experience (QE).

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2 Physical Model

The Hamiltonian of the chiral spin rotation reads [26,27]

$$H = i\hbar\kappa \sum \sigma_{j+1}^{+}\sigma_{j}^{-} + h.c., \quad (1)$$

where the σ_j^+ and σ_j^- are the raising and lowering operators for the jth spin, κ is a real-number coupling constant. Since the Hamiltonian in Eq. (1) commutes with $\sum \sigma_j^z$, the number of the up spins is conserved. Thus, the time evolution of the system occurs in a subspace where the number of up spins remains the same. We can solve the dynamic equation separately in different subspaces with integer spins. First we investigate the subspace formed by the states $|\uparrow\downarrow\downarrow\rangle$, $|\downarrow\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\uparrow\rangle$, whose Hamiltonian will be

$$H_{sub} = i\hbar\kappa \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}. \quad (2)$$

We then solve the Schrödinger equation in this subspace $H_{sub}|\psi\rangle = i\hbar\frac{\partial}{\partial t}|\psi\rangle$. Given the initial state $|\psi(0)\rangle = |\uparrow\downarrow\downarrow\rangle$, we obtain its evolution,

$$|\psi(t)\rangle = \frac{1}{3}[c_0(t)|\uparrow\downarrow\downarrow\rangle + c_{-1}(t)|\downarrow\uparrow\downarrow\rangle + c_1(t)|\downarrow\downarrow\uparrow\rangle], \quad (3)$$

where

$$c_j(t) = 1 + 2\cos\left(\sqrt{3}\kappa t + j\frac{2\pi}{3}\right). \quad (4)$$

At time $t = T \equiv 2\pi/(3\sqrt{3}\kappa)$, $|\psi(T)\rangle = |\downarrow\uparrow\downarrow\rangle$, and at time $t = 2T$, $|\psi(2T)\rangle = |\downarrow\downarrow\uparrow\rangle$. One can see that the spin states move to the right during system evolution.

Similarly, we can calculate the time evolution in another subspace spanned by $|\downarrow\uparrow\uparrow\rangle$, $|\uparrow\downarrow\uparrow\rangle$ and $|\uparrow\uparrow\downarrow\rangle$, where the states have two up spins and will move to the left. It is surprising that the moving direction is opposite to the previous case. This evolution process can be called a chiral spin flipping gate, where spin states in different subspaces move in the opposite directions, namely

$$|\uparrow\downarrow\downarrow\rangle \to |\downarrow\uparrow\downarrow\rangle \to |\downarrow\downarrow\uparrow\rangle \to |\uparrow\downarrow\downarrow\rangle, \quad (5)$$

$$|\downarrow\uparrow\uparrow\rangle \to |\uparrow\uparrow\downarrow\rangle \to |\uparrow\downarrow\uparrow\rangle \to |\downarrow\uparrow\uparrow\rangle. \quad (6)$$

This is an example of a quantum system where spin states move in certain directions. Since states of a single spin span a two-dimensional space, we can regard the spin as a quantum bit (qubit). It motivates us to find a way to realize the spin rotation model in a quantum circuit with several qubits and a series of reversible operations.

3 Quantum Circuit

Before discussing how to design the chiral spin flipping gate in a quantum circuit, we first give a brief introduction to the concept of quantum circuit and some basic quantum operations.

3.1 What is quantum circuit?

The quantum circuit is a straightforward and efficient way to implement the quantum algorithms, that is, the quantum operations to qubits. It is widely used in quantum computation and is convenient and realistic for many other applications. The circuit consists of “wires” and “gates” [28]. Wires represent the qubits which store information, while gates are used to perform some computational operations. In some sense, the quantum circuit is very similar to the classical circuit. For example, we can see the classical NOT gate (Fig. 1(a)) and the quantum NOT gate (X gate, Fig. 1(b)) in the circuits. They have the same function of flipping the state of a bit/qubit.

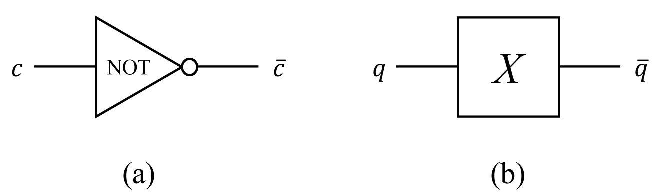


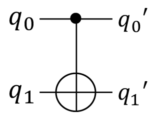
Fig. 1: NOT gates in classical and quantum circuits. (a) NOT gate in a classical circuit, which has input c and output $\overline{c}$ (the opposite state of c). (b) NOT gate in a quantum circuit (X gate), which has input q and output $\overline{q}$.

Nevertheless, unlike the normal classical circuit, the gates in quantum circuits must be reversible. For instance, the fan-in and fan-out are impossible in quantum circuit, since they will change the number of the qubits and, hence, are irreversible. Meanwhile, the quantum circuits allow no “loops”, which means feedback from one part of the circuit to another is forbidden.

3.2 Controlled gates

Conditional operations are essential in both classical and quantum computation. A controlled gate consists of two types of qubits, the control qubit and the target qubit (or qubits, if necessary). The dot on the control qubit means that the operation U is performed only if the control qubit is set to $|1\rangle$, and otherwise U is disabled (Fig. 2(a)). It is similar to the conditional statement “if the control qubit is $|1\rangle$, then do U” which allows us to do multi-qubit operation in the circuit.

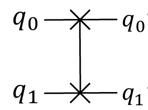
Notice that the state $|1\rangle$ should not be special, since $|0\rangle$ and $|1\rangle$ are in fact equivalent. We can use a small circle instead of the dot on the control qubit to represent the controlled gates performing operation of the control qubit is set to $|0\rangle$. Conventionally, however, conditional dynamics on the target qubit occurs when the control bits are set to $|1\rangle$. In practical circuits, we can use two X gates



CNOT

Input q_0	Input q_1	Output q_0'	Output q_1'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

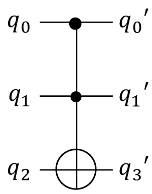
(a)



SWAP

Input q_0	Input q_1	Output q_0'	Output q_1'
0	0	0	0
0	1	1	0
1	0	0	1
1	1	1	1

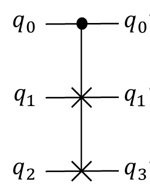
(c)



Toffoli

Input q_0	Input q_1	Input q_2	Output q_0'	Output q_1'	Output q_2'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

(b)

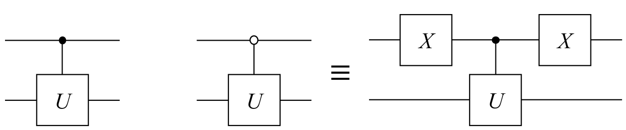


Fredkin

Input q_0	Input q_1	Input q_2	Output q_0'	Output q_1'	Output q_2'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

(d)

Fig. 3: The symbols in quantum circuits and truth tables of the four fundamental quantum gates.



(a) (b)

Fig. 2: The "controlled-U" gates. (a) Normal controlled gate, the operation U will be performed when the control qubit is set to $|1\rangle$. (b) $|0\rangle$-controlled gate. The operation U will be performed conditional on the control qubit being set to $|0\rangle$. It can be realized by adding two X gates to a normal $|1\rangle$-controlled gate.

(NOT gates) on the control qubit to change this condition (Fig. 2(b)).

3.3 CNOT and Toffoli gates

NOT gate is the most fundamental operation in both classical and quantum circuits, which simply flips the state. However, as the NOT gate is only a single-qubit operation, it yields no interactions between different qubits. By adding a control qubit to the NOT gate, we can obtain the "controlled-NOT" gate, or CNOT gate. CNOT gate is the simplest controlled gate where the target operation U is a NOT gate. As we have discussed, if the control qubit is $|0\rangle$, the target qubit will not be changed; if the control qubit is $|1\rangle$, then the NOT gate is performed, which flips the target qubit. Its representation and truth table are shown in Fig. 3(a). The function of CNOT gate is $|a,b\rangle \rightarrow |a,a\oplus b\rangle$ where a is the control qubit, and $\oplus$ is plus modulo 2. Moreover, if we add another control qubit to the CNOT gate, we can obtain the "controlled-controlled-NOT" gate, which is customarily called "Toffoli gate". The target qubit is flipped only if both control qubits are set to $|1\rangle$. The function of CNOT gate is $|a,b,c\rangle \rightarrow |a,c\oplus ab\rangle$, where a, b are the control qubits (Fig. 3(b)).

In fact, in a quantum circuit or a classical reversible circuit, Toffoli gate is a universal gate. A universal gate is a gate that can be used to construct all other gates. However, we seldom use only one type of gate to build the circuit. The circuit can be simplified by using different gates.

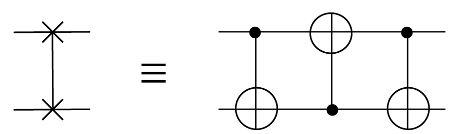


Fig. 4: Use three CNOT gates to accomplish swap gate.

3.4 Swap and Fredkin gates

Swap operation is another fundamental gate, which swaps (or exchanges) the states of two qubits in a circuit. Its function is $|a,b\rangle \rightarrow |b,a\rangle$. The swap operation can be realized in a circuit simply by using a sequence of three CNOT gates (Fig. 4). Its symbol in quantum circuits and truth table are shown in Fig. 3(c).

By adding a control qubit to the swap gate, we can obtain the "controlled-swap" gate, which is the so-called "Fredkin gate"(Fig. 3(d)). The two target qubits will be swapped when the control qubit is set to $|1\rangle$. Like the Toffoli gate, Fredkin gate is also a well-known universal gate for quantum and classical reversible computing.

Table 1: Truth table of chiral spin flipping gate. Note that the trivial input states $|000\rangle$ and $|111\rangle$ are not included here.

Input $q_0\ q_1\ q_2$	*Output* $q_0'q_1'q_2'$
$\|001\rangle$	$\|010\rangle$
$\|010\rangle$	$\|100\rangle$
$\|011\rangle$	$\|101\rangle$
$\|100\rangle$	$\|001\rangle$
$\|101\rangle$	$\|110\rangle$
$\|110\rangle$	$\|011\rangle$

4 Gate Designing Procedure

In order to design a gate in a circuit, first of all we have to code different states. Let us denote the up state with $|0\rangle$ and the down state with $|1\rangle$, then we can obtain the truth table of the chiral spin flipping gate (Table 1) according to Eqs. (5, 6). Then our task is to design some quantum circuits that perform the function described in the truth table.

4.1 Semi-classical method

In classical digital circuits, once we have the truth table, it is very easy to design a circuit accordingly [29,30]. We only need to treat the output bit by bit, list different output states according to the input in tables, then combine the minterms to represent the result in the most concise form by Karnaugh maps [31], where the cells are ordered in Gray code [32,33]. After knowing the relations between the input and each output bits, we can apply classical AND, OR and NOT gates to accomplish this relations in the digital circuit.

If we have a 3-bit input $q_1q_2q_3$, after the gate the output will be $q_0'q_1'q_2'$. According to the truth table (Table 1), we can build the Karnaugh map for each output bit, and find the relations (Fig. 5). From the tables we can see

$$q_0' = \overline{q_0}q_1 + q_1q_2, \tag{7}$$

$$q_1' = \overline{q_1}q_2 + q_0q_1, \tag{8}$$

$$q_2' = q_0\overline{q_2} + q_1q_2, \tag{9}$$

where $\overline{q_k}$ means to flip the state of q_k (use NOT gate), the multiplication sign (omitted) means AND operation and the plus sign ("+") means OR operation.

However, in quantum circuit we cannot use this method directly, since the classical AND and OR gates are irreversible. For the AND and OR gates, we have only one output bit with two input bits, which makes them different from quantum gates. In order to realize the AND gate in quantum circuits, we introduce the concept of ancillary qubit. We conserve the values of input by two qubits, which will not be changed during the operation, and add an ancillary qubit to show the output. Then we obtain a three-qubit "AND gate". It is not difficult to find that this gate is actually the Toffoli gate. When we use control qubits for input and initially set the target qubit to $|0\rangle$, then only if both control qubits (input qubits) are $|1\rangle$ will the output of target qubit be flipped to $|1\rangle$, exactly what AND gate does.

By using ancillary qubits, we can apply the techniques in classical circuits to quantum circuits. The chiral spin flipping gate operates on a three-qubit system. We need three control qubits to be input qubits and another three for the output. First we need to initialize the ancillary qubits to $|000\rangle$. Then we use Toffoli gates to perform the AND gate. The objective quantum circuit is shown in Fig. 6.

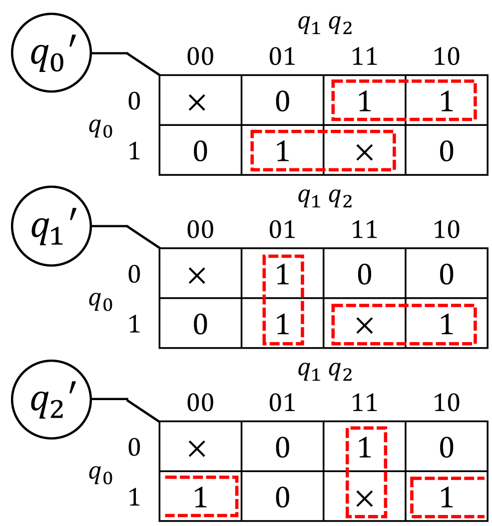


Fig. 5: Karnaugh maps for three qubits. Each input state is represented by a cell in the tables, where the output state determined by the input will be shown. The "×" in the cell means the output can be any state. In fact, every cell having "1" is a minterm for the output qubit. Then we combine the different minterms to obtain the most concise form, as shown by the red dashed boxes.

This circuit is straightforward since it can be derived from the classical method directly, but we need to double the number of qubits since we use ancillary qubits.

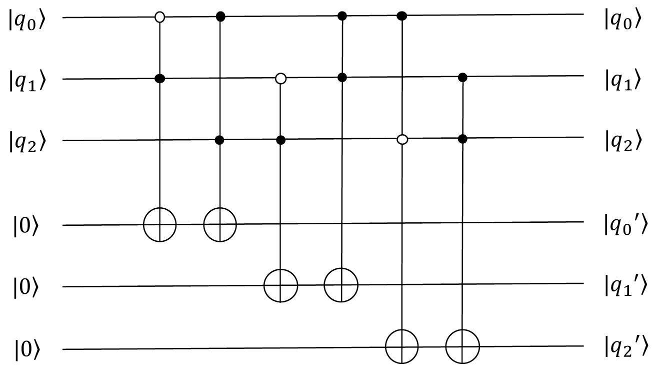


Fig. 6: The quantum circuit for chiral spin flipping gate with ancillary qubits. In this circuit we directly use the result of classical method and apply Toffoli gates with different control conditions to accomplish the function of chiral spin flipping gate. Notice that the input qubits remain unchanged during the algorithm, while the ancillary qubits show the output.

Knowing that nowadays the number of qubits in a real-world device for universal quantum computation are restricted to only approximately ten qubits [34,17], this requirement seems not scalable. It is even more difficult to apply a series of gates if we want to see the evolution process of the system, because in this form the number of qubits required will grow simultaneously. In the following sections, we will try to find better ways to realize the function of chiral spin flipping gate without this disadvantage.

4.2 Quantum circuit based on CNOT-Toffoli gates

The Toffoli gate is a universal gate which can build an entire space of reversible operations. Nevertheless, the circuit would be complicated if we use Toffoli gate alone. To some extent, CNOT gate is similar to the Toffoli gate, because Toffoli gate has only one more control qubit than CNOT gate. In this section we try to use CNOT and Toffoli gates as the bases to build the quantum circuit of chiral spin flipping gate.

Note that these two gates are all based on the NOT operation, the function of which is simply flipping the qubit, no matter it is $|0\rangle$ or $|1\rangle$. Unlike the classical method where we classify the input states by the output being $|0\rangle$ or $|1\rangle$, in order to apply NOT gates we should pay attention to whether the qubits are flipped after chiral spin flipping gate. Let us call the input states "flip states" of a qubit if when input these states the qubit will be flipped after the chiral spin flipping gate. In Table (2-a) we list the flip states for all of the three qubits (the trivial input states $|000\rangle$ and $|111\rangle$ are not included here) according to the truth table (Table 1). For example, after the chiral spin flipping gate the $|001\rangle$ state becomes $|010\rangle$, where q_1 and q_2 are flipped while q_0 remains unchanged. The state $|001\rangle$ is the flip state of both q_1 and q_2.

For all the flip states of q_0, we find that only the valid input states $|x01\rangle$ ("x" means this qubit is either $|0\rangle$ or $|1\rangle$) are not included, we know that once q_1 is $|1\rangle$ or q_2 is $|0\rangle$, q_0 should be flipped. In order to obtain the right output of q_0, the q_0', we should apply a CNOT gate and a Toffoli gate to the circuit to flip q_0 (the target qubit of the gates), as shown in Fig. 7(a). We cannot simply use two CNOT gate because for the $|x10\rangle$ state the target qubit will be flipped twice. After the two-gate operation the 1st qubit is exactly what we want to have in the output, and the remaining two qubits are unchanged. We should deal with them in the following steps. As the current state is now different from the original input because q_0 has been changed, the flip states of the remaining two qubits should be updated accordingly, as showed in the Table (2-b). Similarly, now we find that when either q_0 or q_2 is $|1\rangle$, q_1 will be flipped. We apply another two gates to change q_1 (Fig. 7(b)). After that q_1 is already what we want in the output. Then we perform the same procedure on q_2 (another two gates whose target qubit is q_2) and obtain the final circuit (Fig. 7).

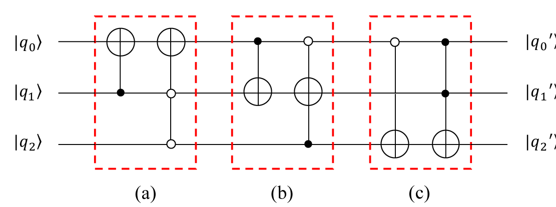


Fig. 7: Quantum circuit for chiral spin flipping gate based on CNOT and Toffoli gates. The three dashed boxes show different sub-operations for each qubit.

4.3 Quantum circuit based on swap-Fredkin gates

As we have mentioned, the Fredkin gate, or the controlled-swap gate, is another well-known universal gate. Here we try to use swap gate and Fredkin gate as the basic gates, which are all based on swap operation.

Now we should focus on the swap operation. Note that the most notable part of the chiral spin flipping gate is that the states in single spin-up subspace and single spin-down subspace move in different directions. Imagine that we swap q_0 and q_1 first then swap q_1 and q_2, the final result will be every qubit moving to the left. Oppositely, if we change the steps, swap q_1 and q_2 first then swap q_0 and q_1, the result will be every qubit moving to the right, exactly what the chiral spin-flipping gate does (Fig. 8).

Table 2: The "flip state" of each qubit. (a) The table of flip states according to the truth table(Table 1). (b) After applying two gates to q_0, the updated flip states for q_1 and q_2. (c) After changing q_0 and q_1, the current flip states for q_2.

(a)

Qubits	q_0	q_1	q_2
Flip states	$\|010\rangle$	$\|001\rangle$	$\|001\rangle$
	$\|011\rangle$	$\|010\rangle$	$\|100\rangle$
	$\|100\rangle$	$\|011\rangle$	$\|101\rangle$
	$\|110\rangle$	$\|101\rangle$	$\|110\rangle$
The rest states	$\|0\boxed{01}\rangle$	$\|100\rangle$	$\|010\rangle$
	$\|1\boxed{01}\rangle$	$\|110\rangle$	$\|011\rangle$

(b)

Qubits	q_1	q_2
Flip states	$\|001\rangle$	$\|001\rangle$
	$\|110\rangle$	$\|000\rangle$
	$\|111\rangle$	$\|101\rangle$
	$\|101\rangle$	$\|010\rangle$
The rest states	$\|\boxed{0}0\boxed{0}\rangle$	$\|110\rangle$
	$\|\boxed{0}1\boxed{0}\rangle$	$\|111\rangle$

(c)

Qubits	q_2
Flip states	$\|011\rangle$
	$\|000\rangle$
	$\|111\rangle$
	$\|010\rangle$
The rest states	$\|\boxed{10}0\rangle$
	$\|\boxed{10}1\rangle$

$$\begin{array}{llll} qubits: & q_0\ q_1\ q_2 & q_0\ q_1\ q_2 & q_0\ q_1\ q_2 \\ states: & |a_0 a_1 a_2\rangle \xRightarrow{swap\ q_0,q_1} & |a_1 a_0 a_2\rangle \xRightarrow{swap\ q_1,q_2} & |a_1 a_2 a_0\rangle \\ result: & \multicolumn{3}{l}{|a_0 a_1 a_2\rangle \rightarrow |a_1 a_2 a_0\rangle, every\ state\ moves\ to\ the\ left.} \end{array}$$

$$\begin{array}{llll} qubits: & q_0\ q_1\ q_2 & q_0\ q_1\ q_2 & q_0\ q_1\ q_2 \\ states: & |a_0 a_1 a_2\rangle \xRightarrow{swap\ q_1,q_2} & |a_0 a_2 a_1\rangle \xRightarrow{swap\ q_0,q_1} & |a_2 a_0 a_1\rangle \\ result: & \multicolumn{3}{l}{|a_0 a_1 a_2\rangle \rightarrow |a_2 a_0 a_1\rangle, every\ state\ moves\ to\ the\ right.} \end{array}$$

Fig. 8: Different swap orders to a three-qubit state. We can see that if we swap the first two qubits first and then swap the last two qubit, the final states will be each qubit state moving to the left. If we change the order of the two swap operations, the moving direction will be opposite accordingly.

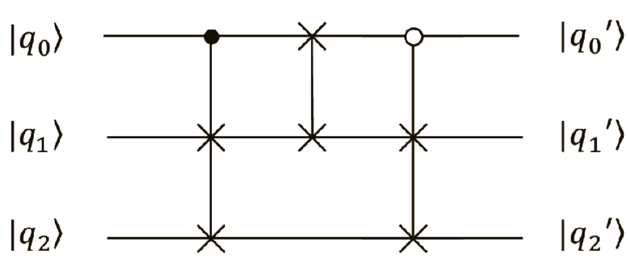


Fig. 9: The quantum circuit of chiral spin flipping gate using swap and Fredkin gates. The first Fredkin gate only swaps the last two qubits of the states in one-$|0\rangle$ subspace, and for the last Fredkin gate that performs swap operation conditional on the first qubit being $|0\rangle$, only makes effect on the states in one-$|1\rangle$ subspace. So in this way the states in different subspace will be swapped in different order, result in the different rotating directions.

We should manage to apply the swap operations in different orders in two different subspaces, where we can use controlled-swap gate, the Fredkin gate. It is obvious that when the two qubits are equivalent, the swap operation can do nothing to them. So if we apply the Fredkin gate to the three qubits, for instance, q_0 being the control qubit (q_1 and q_2 are swapped if q_0 is $|1\rangle$), only the states in single spin-up subspace can be changed. That is because for the states in single spin-down subspace, they have only one $|1\rangle$ qubit, and only the state $|100\rangle$ will make the swap gate work. In this case the rest two qubits q_1 and q_2 are equal, which means it does not make any effect. As for the state $|011\rangle$, it is true that the swap gate is disabled by the only $|0\rangle$ qubit, but likewise the other two qubits are equivalent, so it makes no difference. Similarly, if we apply a different Fredkin gate that the swap gate is performed conditioned by the control qubit being $|0\rangle$, only the states in single spin-down subspace can be swapped. By using two Fredkin gates which have different control states and a swap gate, we can obtain a new quantum circuit for chiral spin flipping gate, as shown in Fig. 9.

5 IBM QE Implementation

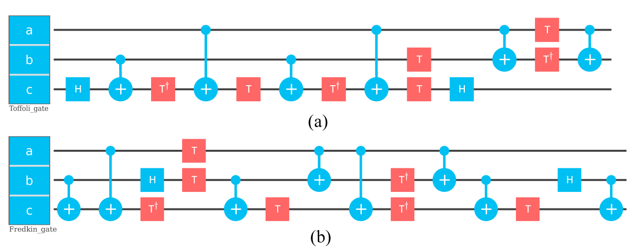


Fig. 10: Subroutines of the gates. (a) The "Toffoli_gate" subroutine for Toffoli gate. *a* and *b* are control qubits and *c* is the target. (b) The the "Fredkin_gate" subroutine for Fredkin gate. *a* is the control qubit and swaps *b* and *c* if set to $|1\rangle$.

In order to demonstrate our designed gates intuitively, we implement our algorithms on IBM QE [16]. It is an commercially available universal quantum computer system where we can either simulate classically or run the quantum algorithms on the real device. The circuits on IBM QE can be built by the "quantum composer", where

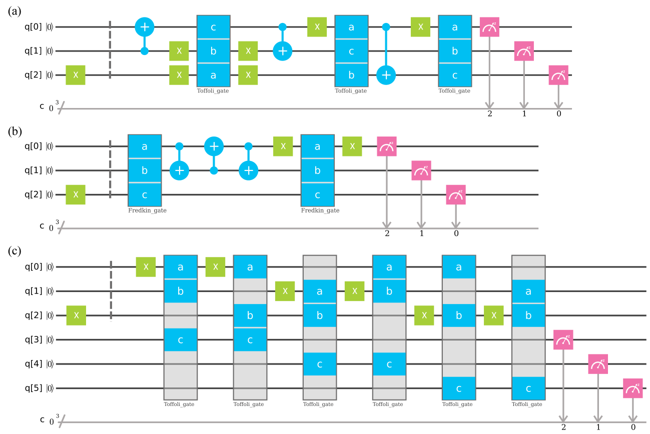


Fig. 11: Quantum scores of chiral spin flipping gate on IBM QE. On the right side of the barriers (dashed lines) are the algorithm parts, while on the left are the input parts. The qubits are automatically initialize to $|0\rangle$, we use a X gate to set the input state to $|1\rangle$. The example inputs in the circuits are all $|001\rangle$. And the X gates in circuits work for making the $|0\rangle$-controlled control qubits(Fig. 2(b)). (a) Circuit of chiral spin flipping gate based on CNOT and Toffoli gates. (b) Circuit of chiral spin flipping gate based on swap and Fredkin gates. (c) Circuit of chiral spin flipping gate using ancillary qubits for output. Note that we input the states by $q_0 \sim q_2$ and have the output in $q_3 \sim q_5$.

a circuit can be called “quantum score”, which are all analogous to the musical terminology. Because of the imperfection and some restrictions of the real device, it is sufficient to implement our gate in simulation, which will yield the prospective output without device-based errors. Meanwhile, we can set the topology of the quantum circuit freely without the limits in the real device.

In a real quantum circuit it is comparatively difficult to implement three-qubit gate like Toffoli and Fredkin gates. First of all, we need to decompose the two gates into a series of single-qubit and two-qubit gates. There are two fundamental single-qubit gates named “Hadamard gate” and “$\pi/8$ gate”. The Hadamard gate, denoted by H, will change the base of the qubits, i.e., $|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. While the so-called $\pi/8$ gate, denoted by T, will add a $\pi/4$ phase to the state $|1\rangle$ but do nothing to the state $|0\rangle$, i.e., $|0\rangle \rightarrow |0\rangle$, $|1\rangle \rightarrow e^{i\pi/4}|1\rangle$. With these gates as well as the CNOT gate, we can design the circuits for the three-qubit gates in Fig. (10).

Now the Toffoli and Fredkin gates are available for us to implement. We can build our “quantum scores” for the circuits of chiral spin flipping gate on IBM QE and test the results (Fig. 11). The algorithm parts are on the right side of the “barriers” (the dashed lines). To the very right there are three pink symbols meaning that we measure the qubits and project the results to the classical bits (the line in the bottom). Notice that q_0 is projected to c_2 while q_2 is projected to c_0, because the on the platform the output reads $c_2c_1c_0$ rather than $c_0c_1c_2$. On the left side of the barrier is the input. Since the qubits will all be initialized as $|0\rangle$, if we want to input $|1\rangle$ we need to use a X gate (NOT gate) to flip the $|0\rangle$ to $|1\rangle$. For example, in the circuits in Fig. 11 we only put the X gate in q_2, so the inputs are all $|001\rangle$. We can change the number and position of the X gates to change the input state. Note that a $|0\rangle$-controlled control qubit, which is denoted by a small circle on the control qubits, can be realized by adding two X gates to a normal $|1\rangle$-controlled control qubit (Fig. 2(b)).

According to the circuit shown in Fig. 7 and Fig. 9, we can get the quantum scores of the three-qubit circuits based on CNOT-Toffoli gates (Fig. 11(a)) and swap-Fredkin gates (Fig. 11(b)) on IBM QE. If we want to implement the “semi-classical” circuits(Fig. 6) on IBM QE, we need a circuit consisting of six qubits, as shown in Fig. 11(c). This circuit is more complicated compared to the previous designs.

6 Conclusion

In this work, three quantum algorithms have been designed with different methods to simulate the evolution process of chiral spin flipping system, which we can regard as a quantum operation, the chiral spin flipping gate, to the qubits. In order to build the circuits for the chiral spin-flipping gate, first we emulate the classical method in digital circuits and apply it to quantum circuits by adding some ancillary qubits. This method is intuitive but needs extra qubits, which is not scalable in real devices. Then we try to use some basic gates to build the circuits directly. For CNOT and Toffoli gates which are based on the NOT operation, we considered the flip behavior of the states, while for swap and Fredkin gates we focused on the swap operation. Then designed the algorithms accordingly.

This work may inspire other scientists to conceive quantum circuits to simulate some physical processes. It shows that we can design quantum algorithms to accomplish many different operations.

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