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Superposition of Two Squeezed Displaced Fock States With Different Coherent Parameters

A.-S. F. Obada[1], *M. M. A. Ahmed*[1] *and Norhan H. Gerish*[2,*]

[1] Department of Mathematics, Faculty of Science, Al-Azhar University, Cairo, Egypt.
[2] Department of Mathematics, Faculty of Science, Suez Canal University, Ismailia, Egypt.

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Abstract: The s-parametrized characteristic function $C(\lambda,s)$ for a superposition of two squeezed displaced Fock states (**SDFSs**) is presented. The s-parametrized distribution functions for the superposition of **SDFSs** is investigated for different coherent parameters. The moments are obtained by using this characteristic function. The Gluaber second-order correlation function is calculated.The squeezing properties of this superposition are studied. Analytical and numerical results for the quadrature component distributions are presented. A generation scheme is discussed. The behavior of the above statistical aspectes differ on changing of the coherence parameters.

Keywords: SDFSs, Quasiprobability Function, correlation function, Squeezing

1 Introduction

The number (Fock) state $|\,n\rangle$ is the main block in builting the electromagnetic field state .It is the eigen state of the photon number operator $\hat{n} = a^{\dagger}a$ where $a(a^{\dagger})$ is the annihilation (creation) operator. There is an alernative important state that is the coherent state$|\,\alpha\rangle$ which is defined by applying a displacement operator $D(\alpha)$ on the vacuum state. It is the eigen state of the annihilation operator (a), and also it is defined to be linear superposition of all $|\,n\rangle$ states with coefficients selected such that the photon number distribution is Poissonian ([1]-[4]) .On the other hand the squeezed state is one of the non-classical states of the electromagnetic field ,such that certain observables reveal fluctuations less than for the vaccum state ([5]-[8]). It is known as the impact of the squeezed operator S(z) on the coherent state[5].Squeezed Displsed Fock states (SDFSs) have been studied and various aspects of these states such as squezzing and photon statistics have been investigated ([9]-[16]). Two-photon coherent states (squeezed coherent states), squeezed number states [17] and displaced Fock states ([18],[19]) are generated as special cases of squeezed displaced Fock states. Lately the creation of no-classical states of motion of a trapped ion as Fock states, coherent states, squeezed states and Schrodinger cat states have been reported experimentally ([20]-[23]).In the above experiments an ion with laser cooled in a Paul trap to the ground harmonic state .Then the ion is set into different quantum states of motion by applications of optical and electric fields. This moved the study of these states from the academic field to the world of experimentation, that encouraged researchers to study the superposition of these states.

Superposition of the quantum mechanical states of electromagnetic field have lately recevied more care in quantum optics([24]-[29]),because these states can reveal non-classical properties of light such as quadrature squeezing and sub Poissonian statistics [5].

The studies of some types of superposition of Glauber(ordinary) coherent states have shown quadrature squueezing and sub-Poissonian statistics([24]-[29]). Methods for production of superposition of coherent states in experiments have been made by many workers([25]-[29]). Superposition of two binomial states and two negative binomial states have been studied in [30]. Properties of the superposition of displaced Fock states and generation scheme have been discussed [31].

Let the class of quantum state $|\,\psi\rangle$ have the form

$$|\,\psi\rangle = A_N^{-1/2}\sum_{j=1}^{N} k_j\,|\,\alpha_j, z_j, m_j\rangle \tag{1}$$

* Corresponding author e-mail: norhangerish@gmail.com

withA_N as the normalization constant, and the SDFSs $|\alpha,z,m\rangle$ defined by

$$|\alpha,z,m\rangle = D(\alpha)S(z)\,|m\rangle \quad (2)$$

$$D(\alpha)=\exp(\alpha a^{\dagger}-\alpha^{*}a) \quad S(z)=\exp(\frac{1}{2}z^{*}a^{2}-\frac{1}{2}za^{\dagger 2}) \quad (3)$$

The operator $D(\alpha)$ is the displacement operator, $\alpha=|\alpha|exp(i\theta)$, and $S(z)$ is squeeze operator, $z=r exp(i\phi)$ [5]. Where $a(a^{\dagger})$ is the annihilation (creation) operator of the boson field.
In section 2 we discuss the construction and properties of superposition of two SDFSs for different coherent parameter and calculate the photon number distributions. In section 3 we study the s-parametrized quasiprobability distribution function. In section 4 we discusse some applications of the charactristic function and the quadrature component distribution for these states. In section 5 finally we present a generation scheme.

2 Superposition of Two SDFSs With Different Coherent Parameters

We use the quantum state (1) for the superposition of a pair of **SDFSs** with different coherent parameters but the squeezing parameters and number of photon are the same. The superposition state $|\psi\rangle$ is defined as

$$|\psi\rangle = N^{-1/2}[k_1\,|\alpha_1,z,m\rangle + k_2\,|\alpha_2,z,m\rangle] \quad (4)$$

with the normalization constant N given by

$$\begin{aligned} N=&|k_1|^2+|k_2|^2+[k_1k_2^{*}+k_1^{*}k_2] \\ &\exp(-\frac{|\bar{\alpha_1}-\bar{\alpha_2}|^2}{2})L_m(|\bar{\alpha_1}-\bar{\alpha_2}|^2) \end{aligned} \quad (5)$$

where $\bar{\alpha}=\mu\alpha+\nu\alpha^{*}$ with$\mu=\cosh r$, $\nu=\exp(i\phi)\sinh r$ and $r=|z|$,while $L_m^{\sigma}(x)$ is the associated Laguerre polynomial which is given by

$$L_m^{\sigma}(x)=\sum_{s=0}^{m}\binom{m+\sigma}{m-s}\frac{(-x)^s}{s!} \quad (6)$$

Now we try to obtain the photon statistics for the state of equation(4). First we set

$$\begin{aligned} |\alpha_j,z_j,m_j\rangle &= \sum_{n=0}^{\infty}\langle n\,|\,\alpha_j,z_j,m_j\rangle\,|n\rangle \\ &= \sum_{n=0}^{\infty}C_n^{j}(\alpha_j,z_j,m_j)\,|n\rangle,\ j=1,2 \end{aligned} \quad (7)$$

where $C_n(\alpha_j,z_j,m_j)$ defined [32] as

$$\begin{aligned} &C_n(\alpha_j,z_j,m_j)=\langle n\,|\,\alpha_j,z_j,m_j\rangle \\ &=(\frac{n!}{\mu_j m_j!})^{1/2}(\frac{\nu_j}{2\mu_j})^{n/2}\exp(-\frac{|\bar{\alpha}_j|^2}{2}+\frac{\nu_j^{*}}{2\mu_j}\bar{\alpha}_j^{2}) \\ &\sum_{i=0}^{min(n,m_j)}\binom{m_j}{i}\frac{(2/\mu_j\nu_j)^{i/2}}{(n-i)!}(\frac{-\nu_j^{*}}{2\mu_j})^{(m_j-i)/2} \\ &\times H_{n-i}(\frac{\bar{\alpha}_j}{(2\mu_j\nu_j)^{1/2}})H_{m_j-i}(\frac{-\alpha_j^{*}}{(-2\mu_j\nu_j^{*})^{1/2}})\ j=1,2 \end{aligned} \quad (8)$$

while $H_n(x)$ is the Hermite polynomial given by

$$H_n(x)=\sum_{m=0}^{[n/2]}\frac{n!(-1)^m(2x)^{n-2m}}{m!(n-2m)!} \quad (9)$$

Then we can cast the state (4) as

$$|\psi\rangle = N^{-1/2}\sum_{n=0}^{\infty}[k_1C_n(\alpha_1,z,m)+k_2C_n(\alpha_2,z,m)]\,|n\rangle \quad (10)$$

and the photon number distribution P(n) is given by

$$P(n)=|\langle n\,|\,\psi\rangle|^2=N^{-1}\,|k_1C_n(\alpha_1,z,m)+k_2C_n(\alpha_2,z,m)|^2 \quad (11)$$

Where $C_n(\alpha_j,z,m)$ are defined in equation(8). With probability distribution function is obtained therefore some statistical aspects can be calculated and discussed.

3 S-Parametrized Quasiprobability Function

Quasiprobability distribution function such as Glaubers $P(\beta)$ function [2] , the Wigner $W(\beta)$ [[33],[34]] function and Husimi $Q(\beta)$ C[[35],[36]] function have proved to be very useful theoreticaly tools in performing quantum optical calculations [32]. The s-parametrized characteristic function $C(\lambda,s)$ plays an important role in the fundamental exposition of the quasiprobability functions. It is defined as the trace of the product of the density operator with the displacement operator as follows:

$$C(\lambda,s)=Tr[\rho D(\lambda)]\exp(\frac{s}{2}\,|\lambda|^2) \quad (12)$$

with $D(\lambda)$ defined in equation(3). The s-parametrized quasiprobability distribution function can be defined as a Fourier transformation of the s-parametrized characteristic function,and it is given by

$$F(\beta,s)=\frac{1}{\pi^2}\int C(\lambda,s)\exp(\lambda^{*}\beta-\lambda\beta^{*})d^2\lambda \quad (13)$$

It can be cast into the form[37] as

$$F(\beta,s)=\frac{2}{\pi}\sum_{k=0}(-1)^k\frac{(1+s)^k}{(1-s)^{k+1}}\langle\beta,k\,|\,\rho\,|\,\beta,k\rangle \quad (14)$$

The density operator corresponding to the state (4) is given by

$$\rho = N^{-1}[(k_1 \mid \alpha_1, z, m\rangle + k_2 \mid \alpha_2, z, m\rangle) (k_1^*\langle\alpha_1, z, m \mid + k_2^*\langle\alpha_2, z, m \mid)] \tag{15}$$

Then the characteristic function can be written in the following form

$$C(\lambda, s) = (\frac{\exp(\frac{s}{2} \mid \lambda \mid^2)}{N})[\exp(-\frac{\mid \bar{\lambda} \mid^2}{2})L_m(\mid \bar{\lambda} \mid)^2 [\mid k_1 \mid^2 \exp(\alpha_1^*\lambda - \alpha_1\lambda^*) + \mid k_2 \mid^2 \exp(\alpha_2^*\lambda - \alpha_2\lambda^*)] + k_1^*k_2 \exp(-\frac{1}{2} \mid \bar{\lambda} + \bar{\alpha}_2 - \bar{\alpha}_1 \mid^2)L_m(\mid \bar{\lambda} + \bar{\alpha}_2 - \bar{\alpha}_1 \mid^2) + k_1k_2^* \exp(-\frac{1}{2} \mid \bar{\lambda} + \bar{\alpha}_1 - \bar{\alpha}_2 \mid^2)L_m(\mid \bar{\lambda} + \bar{\alpha}_1 - \bar{\alpha}_2 \mid^2)], \tag{16}$$

Where $\bar{\lambda} = \mu\lambda + \nu\lambda^*$, by using equations (14),(15) and (8) we can obtain the s-parametrized distribution function as follows

$$F(\beta, s) = \frac{2}{\pi}\sum_k -1^k(\frac{(1+s)^k}{(1-s)^{k+1}}) [\mid k_1 \mid^2 \mid C_k^1(\gamma, z, m) \mid^2 + \mid k_2 \mid^2 \mid C_k^2(\delta, z, m) \mid^2 + k_1k_2^*C_k^1(\gamma, z, m)C_k^{2^*}(\delta, z, m) + k_1^*k_2C_k^2(\delta, z, m)C_k^{1^*}(\gamma, z, m)] \tag{17}$$

and also $F(\beta, s)$ in a mathematics form is:

$$F(\beta, s) = \frac{2}{\pi}\sum_k A_K \sum_{i=j=0}^{min(n,k)} \binom{n}{i}\binom{n}{j} \frac{(2/\mu\nu)^{i/2}(2/\mu\nu^*)^{j/2}}{(k-i)!(k-j)!} [\exp(-\frac{\mid \gamma \mid^2}{2} + \frac{\nu^*}{2\mu}\gamma^2)H_{(k-i)}(\frac{\bar{\gamma}}{(2\nu\mu)^{1/2}}) H_{(n-i)}(\frac{-\gamma^*}{(-2\nu^*\mu)^{1/2}})[\mid k_1 \mid^2 \exp(-\frac{\mid \gamma \mid^2}{2} + (\frac{\nu}{2\mu})\gamma^{*^2}) H_{(k-j)}(\frac{-\gamma}{(-2\nu\mu)^{1/2}})H_{(n-j)}(\frac{\bar{\gamma}^*}{(2\nu^*\mu)^{1/2}}) + k_1k_2^* \exp(-\frac{\mid \delta \mid^2}{2} + (\frac{\nu}{2\mu})\delta^{*^2})H_{(k-j)}(\frac{\bar{\delta}^*}{(2\nu^*\mu)^{1/2}}) H_{(n-j)}(\frac{-\delta}{(-2\nu\mu)^{1/2}})] + \exp(-\frac{\mid \delta \mid^2}{2} + (\frac{\nu^*}{2\mu})\delta^2) H_{(k-i)}(\frac{\bar{\delta}}{(2\nu\mu)^{1/2}})H_{(n-i)}(\frac{-\delta^*}{(-2\nu^*\mu)^{1/2}}) [\mid k_2 \mid^2 \exp(-\frac{\mid \delta \mid^2}{2} + (\frac{\nu}{2\mu})\delta^{*^2})H_{(k-j)}(\frac{-\delta}{(-2\nu\mu)^{1/2}}) H_{(n-j)}(\frac{\bar{\delta}^*}{(2\nu^*\mu)^{1/2}}) + k_1^*k_2 \exp(-\frac{\mid \gamma \mid^2}{2} + (\frac{\nu}{2\mu})\gamma^{*^2}) H_{(k-j)}(\frac{\bar{\gamma}^*}{(2\nu^*\mu)^{1/2}})H_{(n-j)}(\frac{-\gamma}{(-2\nu\mu)^{1/2}})]], \tag{18}$$

where

$$A_k = (-1)^k(\frac{(1+s)^k}{(1-s)^{k+1}})(\frac{k!}{\mu\, n!})(\frac{\nu}{2\mu})^{k/2}(\frac{\nu^*}{2\mu})^{k/2} \tag{19}$$

Since we use $\gamma = \alpha_1 - \beta$, $\delta = \alpha_2 - \beta$. The general represtation function $F(\beta, s)$ reduce to the weight functions $Q(\beta)$, $W(\beta)$ and $P(\beta)$ when the order parameter take the values $s = -1, zero$ and $+1$. respectively from this characteristic function we can calculate any expectation value for the field operators.

4 Some Statistical Properties

After the characteristic function is calculated, we bocus on some statistical quantities namely: correlation function, squeezing and quadrature distribution.

4.1 The auto-correlation function

We present the moments of the photon operators for the superposition of two **SDFSs** in order to calculate different statistical quantities. The s-parametrized averge value of a and $a^\dagger$ are presented as

$$\langle[a^{\dagger^k}a^l]_s\rangle = Tr[\rho\{a^{\dagger^k}a^l\}_s] = \frac{\partial^k}{\partial\lambda^k}\frac{\partial^l}{\partial(-\lambda^*)^l}C(\lambda, s)\mid_{\lambda=\lambda^*=0} \tag{20}$$

i.e. the average values of power of creation and annihilation operators are derived by differentiating the characteristic function with respect to λ and $-\lambda^*$ as shown above. Also by using equation (8) we can calculatethe averge values for any different α_j ,m_j and z_j as

$$\langle[a^{\dagger^k}a^l]_s\rangle = \sum_{s=0}\sqrt{\frac{s!(s+l-k)!}{(s-k)!}}C_s^*(\alpha_j, z_j, m_j) C_{s+l-k}(\alpha_j, z_j, m_j)\ s \geq k \tag{21}$$

therefor, by equation (20) we clculate;

$$\langle a^\dagger\rangle = N^{-1}[\mid k_1 \mid^2 \alpha_1^* + \mid k_2 \mid^2 \alpha_2^* + k_1k_2^* \exp(-\frac{1}{2} \mid \bar{\alpha_1} - \bar{\alpha_2} \mid^2)(A_1 - B_1)[-\frac{1}{2}L_m(\mid \bar{\alpha_1} - \bar{\alpha_2} \mid^2) + L_{m-1}^1(\mid \bar{\alpha_1} - \bar{\alpha_2} \mid^2)] + k_1^*k_2 \exp(-\frac{1}{2} \mid \bar{\alpha_2} - \bar{\alpha_1} \mid^2) (A_2 - B_2)[-\frac{1}{2}L_m(\mid \bar{\alpha_2} - \bar{\alpha_1} \mid^2) + L_{m-1}^1(\mid \bar{\alpha_2} - \bar{\alpha_1} \mid^2)]] = \langle a\rangle^* \tag{22}$$

and

$$\begin{aligned}\langle a^{\dagger 2}\rangle = {} & N^{-1}[\mid k_1\mid^2 \alpha_1^{*^2} + \mid k_2\mid^2 \alpha_2^{*^2} - (\mid k_1\mid^2 + \mid k_2\mid^2) \\ & [\mu\nu^* + 2\mu\nu^* m] + k_1 k_2^* \exp(-\frac{1}{2}\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2) \\ & [[(-\mu\nu - \frac{1}{4}(A_1 - B_1)^2)]L_m(\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2) \\ & -[(A_1 - B_1))^2 - 2\mu\nu]L^1_{m-1}(\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2) \\ & +(A_1 - B_1)^2 L^2_{m-2}(\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2)] \\ & +k_1^* k_2 \exp(-\frac{1}{2}\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2)[[(-\mu\nu - \frac{1}{4}(A_2 - B_2)^2)] \\ & L_m(\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2) - [(A_2 - B_2)^2 - 2\mu\nu] \\ & L^1_{m-1}(\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2) + (A_2 - B_2)^2 L^2_{m-2}(\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2)]] \\ & = \langle a^2\rangle^* \end{aligned} \tag{23}$$

While the average number of photons can be calculated analogously as follows

$$\begin{aligned}\langle a^\dagger a\rangle = {} & N^{-1}[\mid k_1\mid^2\mid \alpha_1\mid^2 + \mid k_2\mid^2\mid \alpha_2\mid^2 + (\mid k_1\mid^2 + \mid k_2\mid^2) \\ & [m(\mid \mu\mid^2 + \mid \nu\mid^2) - C] + k_1 k_2^* \exp(-\frac{1}{2}\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2) \\ & [\frac{-1}{4}(A_1 - B_1)^2) - [((\mid \mu\mid^2 + \mid \nu\mid^2) - C) - (A_1 - B_1)^2] \\ & L^1_{m-1}(\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2) - (A_1 - B_1)B_1 L^2_{m-2}(\mid \bar{\alpha_1} - \bar{\alpha_2}\mid^2)] \\ & +k_1^* k_2 \exp(-\frac{1}{2}\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2)[\frac{-1}{4}(A_2 - B_2)^2) \\ & -[((\mid \mu\mid^2 + \mid \nu\mid^2) - C) - (A_2 - B_2)^2] \\ & L^1_{m-1}(\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2) - (A_2 - B_2)B_2 L^2_{m-2}(\mid \bar{\alpha_2} - \bar{\alpha_1}\mid^2)], \end{aligned} \tag{24}$$

where

$$\begin{aligned} A_1 = -A_2 = \nu^*(\bar{\alpha_1} - \bar{\alpha_2}), \quad B_1 & = -B_2 = \mu(\bar{\alpha_1}^* - \bar{\alpha_2}^*) \\ C = \frac{1}{2}[s - (\mid \mu\mid^2 + \mid \nu\mid^2)] \end{aligned} \tag{25}$$

The Gluaber second-order correlation function is defined by

$$g^{(2)} = \frac{\langle a^{\dagger^2} a^2\rangle}{\langle a^\dagger a\rangle^2} \tag{26}$$

The light with $g^{(2)} < 1$ has a sub-Possionian distribution, the light with $1 < g^{(2)} < 2$ has a super-Possianion distribution, and the light with $g^{(2)} > 2$ has a super-thermal distribution. Coherent light has $g^{(1)} = 1$ while thermal light has $g^{(2)} = 2$.

In figure 1 and 2 we plot the correlation function $g^{(2)}$ against the squeeze parameter r, and we take the values $m = 0, 1, 2, 3$, with different value of the displacement parametersα_1, α_2 and the direction of squeezing is zero $\phi = 0$.

We plot figure 1 for $\alpha_1 = 1, \alpha_2 = 2$, and $k_1 = 1, k_2 = 1$. There is thermal distribution for $m = 0$,sub-Possionian is observed for $m = 1$ in the range $0.5 < r < 1$, and for $m = 2, 3$ there is super-Possionian distribution $\forall r$.

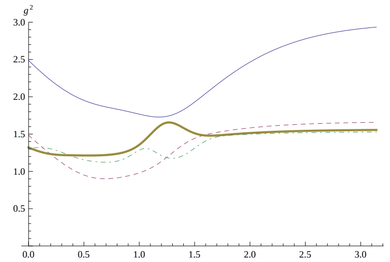

Fig. 1: Correlation function$g^{(2)}$against the squeezed parameter r,with $\alpha_1 = 1, \alpha_2 = 2$ for $k_1 = 1, k_2 = 1$. The squeeze parameter is assumed to be real and runs from 0 to 3. The photon number has the value $m = 0$(line),$m = 1$(dashed),$m = 2$(thick)and $m = 3$(dot dashed).

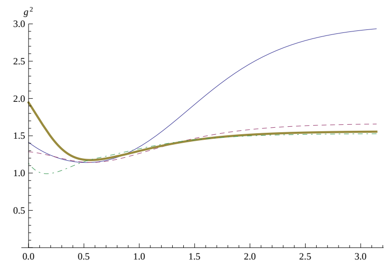

Fig. 2: Correlation function$g^{(2)}$against the squeezed parameter r, with $\alpha_1 = 1, \alpha_2 = -2$ for $k_1 = k_2 = 1$. The squeeze parameter is assumed to be real and runs from 0 to 3. The photon number has the value $m = 0$ (line), $m = 1$(dashed), $m = 2$(thick) and $m = 3$(dot dashed).

In figure 2 we plot the displacement parameters $\alpha_1 = 1$ and $\alpha_2 = -2$ and $k_1 = 1, k_2 = 1$. We note that for $m = 0, 1, 2, 3$ there is super-Possionian distribution , super thermal is observed for $m = 0$, $r > 2$. While a small amount of sub-possionian is noted for $m = 3$ for a very short period of r $0.1 < r < 0.3$ The results are different from earlier studies ($\alpha_1 = 1$, $\alpha_2 = -1$) see [32].

4.2 Squeezing

We discuss the squeezing for the superposition of **SDFSs** (4). The quadrature operators of the one mode field are defined by

$$X_1 = \frac{1}{2}(a + a^\dagger) \quad X_2 = \frac{1}{2i}(a - a^\dagger) \quad [X_1, X_2] = i/2 \tag{27}$$

which satisfy the uncertainty relation ,$\langle(\Delta X_1)^2\rangle\langle(\Delta X_2)^2\rangle \geq \frac{1}{16}$ with the variance

$$\langle(\Delta X_j)^2\rangle = \langle X_j^2\rangle - \langle X_j\rangle^2 \quad j = 1, 2 \tag{28}$$

The field is said to be squeezed if $(\Delta X_j)^2 \leq \frac{1}{4}$ for j=1,2. The average values $\langle X_1\rangle$and $\langle X_2\rangle$ of the quadrature field operators are directly computed, the variances $\langle(\Delta X_1)^2\rangle$ and $\langle(\Delta X_2)^2\rangle$ of the quadrature field operators are presented from equations (22),(23)and(24).

The squeezing is best parametrized by the q parameter defined as

$$q_j = \frac{\langle(\Delta X_j)^2\rangle - 0.25}{0.25}, \; j = 1,2 \tag{29}$$

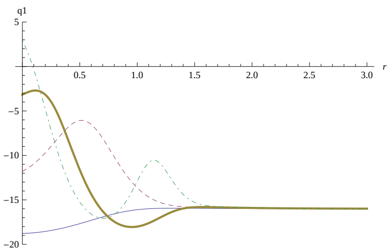

Fig. 3: Squeezing parameter q_1 with the squeeze parameter r for $k_1 = k_2 = 1$ and $\alpha_1 = 1, \alpha_2 = 2$ with different values for $m = 0$(line), $m = 1$(dashed), $m = 2$(thick)and $m = 3$(dot dashed)

In figure 3 we plot q_1 with the squeeze parameter r for $k_1 = k_2 = 1$, and $\alpha_1 = 1, \alpha_2 = 2$ with different values for $m = 0,1,2,3$. We observe that the squeezing exists $-20 < q_j < 0$ in all r,i.e., the squeezing condition reads $q_j < 0$ and it increases with r. The results are different from the earlier studies ($\alpha_1 = 1, \alpha_2 = -1$) see[32].

4.3 Quadrature Distributions

The quadrature component distribution for the superposition state (4) is defined as

$$P(x,\varphi) = \mid \langle x,\varphi \mid \psi_m\rangle \mid^2 \tag{30}$$

We first expand the eigenstate of quadrature component

$$\mathbf{x}(\varphi) = \frac{1}{2^{1/2}}[\exp(-i\varphi)a + \exp(i\varphi)a^\dagger] \tag{31}$$

in the photon number basis as

$$\mid x,\varphi\rangle = \frac{1}{\pi^{1/4}}\exp(-\frac{x^2}{2})\sum_{j=0}^{\infty}\frac{\exp(i\varphi j)}{(2^j j!)^{1/2}}H_j(x)\mid j\rangle \tag{32}$$

Then by using equations (29) the quadrature component distribution becomes

$$\begin{aligned} P(x,\varphi) = {} & \frac{1}{N\pi^{1/2}}\exp(-\frac{x^2}{2})\sum_{j,l=0}^{\infty}\frac{\exp(i\varphi(j-l)}{(2^{j+l}l!j!)^{1/2}} \\ & [k_1C_j^s(\alpha_1,z,m) + k_2C_j^s(\alpha_2,z,m)] \\ & [k_1^*C_l^s(\alpha_1,z,m) + k_2^*C_l^s(\alpha_2,z,m)]H_j(x)H_l(x) \end{aligned} \tag{33}$$

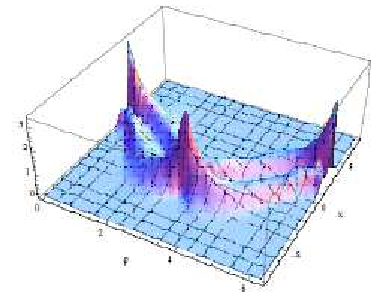

Fig. 4: Quadrature distribution $P(x,\varphi)$ of the state $\mid \psi\rangle$ consisting of the superposition ofSDFSs, with $m = 0, \alpha_1 = 1, \alpha_2 = 2, r = 1$, and $k_1 = k_2 = 1$

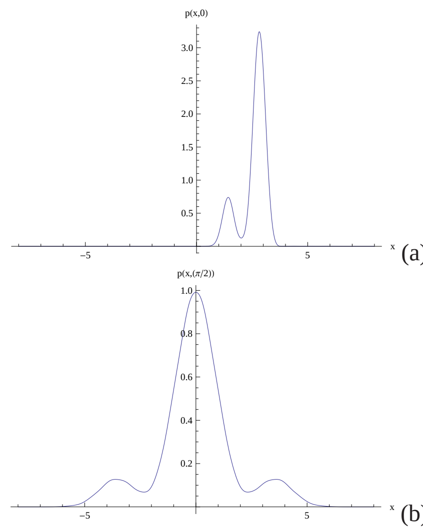

Fig. 5: Quadrature distribution $P(x,\varphi)$ against x (a) at $\varphi = 0$,(b) at $\varphi = \pi/2$

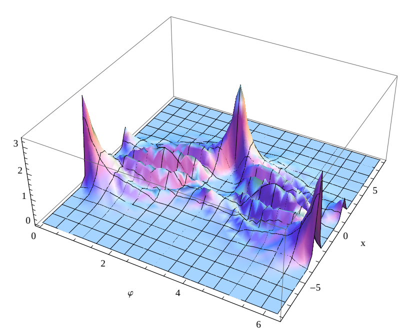

Fig. 6: Quadrature distribution $P(x,\varphi)$ of the state $\mid \psi\rangle$ consisting of the superposition ofSDFSs,with $m = 0, \alpha_1 = 1, \alpha_2 = -2, r = 1, \phi = 0$ and $k_1 = k_2 = 1$

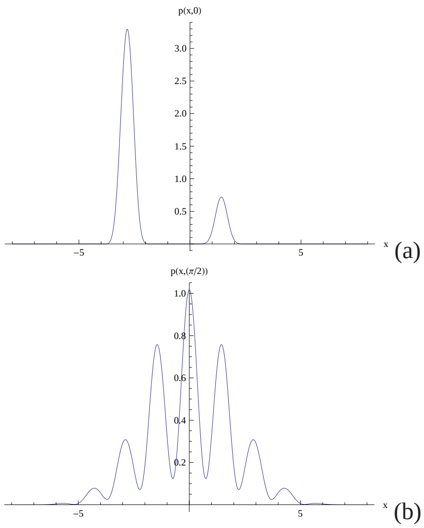

Fig. 7: Quadrature distribution $P(x,\varphi)$ against x (a)at $\varphi=0$,(b)at $\varphi=\pi/2$

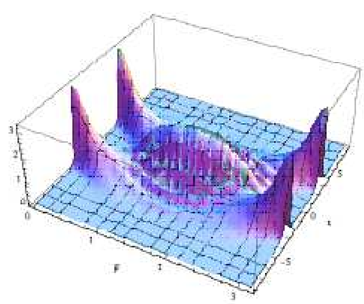

Fig. 8: Quadrature distribution $P(x,\varphi)$ of the state $\mid\psi\rangle$ consisting of the superposition of SDFSs, with $m=0,\alpha_1=2=-\alpha_2,r=1,\phi=0$ and $k_1=k_2=1$

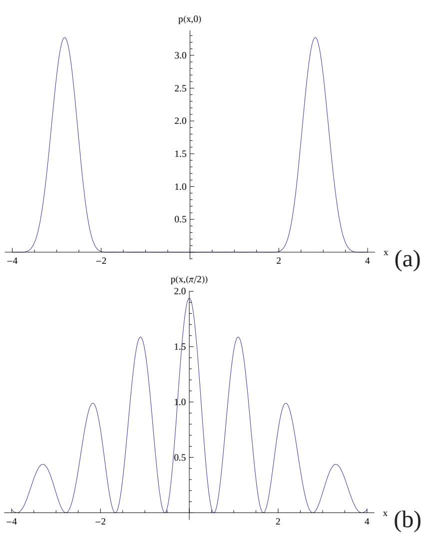

Fig. 9: Quadrature distribution $P(x,\varphi)$ against x (a) at $\varphi=0$,(b) at $\varphi=\pi/2$

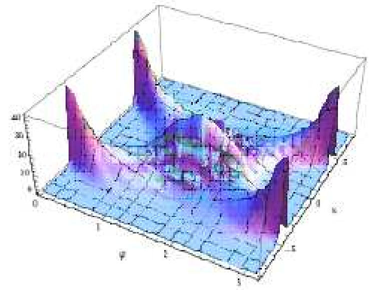

Fig. 10: Quadrature distribution $P(x,\varphi)$ of the state $\mid\psi\rangle$ consisting of the superposition of SDFSs, with $m=0,\alpha_1=3=-\alpha_2,r=1,\phi=0$ and $k_1=k_2=1$

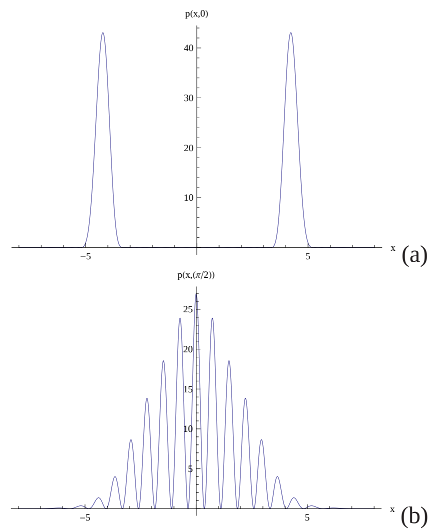

Fig. 11: Quadrature distribution $P(x,\varphi)$ against x (a)at $\varphi=0$,(b)at $\varphi=\pi/2$

In figures 4, 6, 8 and 10 we plot the phase-parametrized field strength distribution(quadrature component) $P(x,\varphi)$ with $m=0$,$r=1$, and different value of α_1,α_2.

In figure 4 in case $\alpha_1=1,\alpha_2=2$, $k_1=k_2=1$ we note that two symmetric peaks at $\varphi=0,2\pi$, the small peak is at $x\approx 0.7$ and the large peak at $x\approx 3$. As φ increase they becomes three peaks with small heights compared to the original ones. The middle is larger while the side peaks are small but as φ reaches to π ,they combine into 2 peaks ,and then they are swing the small peak is around $x\approx -0.7$ and the large peak is around $x\approx -3$ see figure 5.The oscillation is repeated with a period 2π and also we find that it is bounded. Incidentally we have also the same results if we take $k_1=-k_2=1$.

In figure 6 we take $\alpha_1=1,\alpha_2=-2$, $k_1=k_2=1$ we note that the motion of the peak in the (x,φ) plan is the same as figure 4 but in opposite directions as may be seen from figure 7.

In figure 8 we take $\alpha_1 = 2, \alpha_2 = -2$ we found that the figure is symmetric around $x = 0$ and $\varphi = \pi/2$. It is observed that there are two peaks only for $\varphi = 0, \pi$ and with height $x \approx 3$. Also we observe that the height of peak is increasing with increasing the value of α as shown in figure 10 where $\alpha_1 = 3, \alpha_2 = -3$ where the height reche ≈ 40 . Thus behavior is different from the case $\alpha_1 = 1 = -\alpha_2$ see[32].
In figure 5,7,9 and 11 we plot Quadrature distribution $P(x,\varphi)$ against x with different values of φ , We find another structures and oscillation between the peaks and the number of them increase with increasing α as show in the case $\varphi = \pi/2$ see figures 5(b),7(b),9(b)and 11(b).

5 Generation Scheme

After the discussion of the properties of the superposition of the **SDFSs**, we wish to consider the production of such state. We learn Let a two-level ion of mass **M** move in a harmonic potential of frequency $\omega_x$in the x-direction. Let $a(a^\dagger)$ stand for the annihilation (creation) operator of the vibrational quanta in the x-direction. Then the position operator is given by $x = \Delta x_0(a + a^\dagger)$ with $\Delta x_0 = (2\omega_x \mathbf{M})^{-1/2}$ the width of the harmonic ground state. In this scheme, four beams of laser applied along the x-axis are required to manipulate the motion of the atom; they are detuned by $\pm\omega_x$ and $\pm 2\omega_x$. In the rotating-wave approximation the Hamiltonian for this system is given by

$$H = \omega_x a a^\dagger + (\frac{\omega_0}{2})\sigma_z - [\mu E^-(x,t)\sigma_+ + hc] \tag{34}$$

The first two terms describe the external and internal free motions of the ion and the last term stands for the atom-field interaction. μ is the dipole matrix element and ω_0 the transition frequency of the two-level ion , and the operators $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$ where $|e\rangle$ and$|g\rangle$ are the atomic excited and ground states respectively.
We follow the same scheme as in [32] ,and we keep the first and second order term of the Hamiltonian as:

$$\begin{aligned} \bar{H_1} = & -[2(g_1 + g_2^*)a^\dagger + 2(g_1^* + g_2)a + 2(g_3 + g_4^*)a^{\dagger^2} \\ & + 2(g_3^* + g_4)a^2](\sigma_- + \sigma_+) \end{aligned} \tag{35}$$

where

$$\begin{aligned} g_j(t) &= i\Omega_j \eta_j^2 \exp(i\phi_j)\exp(-\frac{\eta_j^2}{2}), j = 1,2 \\ g_j(t) &= -\Omega_l \eta_l^2 \exp(i\phi_l)\exp(-\frac{\eta_l^2}{2}), l = 3,4 \end{aligned} \tag{36}$$

Any atom prepared in the state $(1/2^{1/2})(\mid e\rangle + \mid g\rangle)$ which can be generated from the ground state under this Hamiltonian by applying a $\pi/2$ carrier pulse will stay in this state and will be left unchanged ([37]-[40]) .Thus the dynamics are reduced to those of the motional degrees of freedom only.Under this Hamiltonian the motional dynamics evolve towards the SDFSs $\mid \alpha, z, m\rangle$ by intially preparing the Fock state $\mid m\rangle$, then applying the linear part first, and after that the quadratic part. The Fock state $\mid m\rangle$ can be prepared with very high efficiency according to recent experiments ([20]-[23]). In this case $\alpha = 2i(g_1 + g_2^*)$ and $z = -2(g_3 + g_4^*)$. The prepartion of superposition of these states can be done according to the scheme described in by applying a further displacement operator adjusted in a way to effect the $\mid e\rangle$ state alone, producing $(1/2^{1/2})(\exp i\phi \mid \beta, z, m\rangle \mid e\rangle + k \mid \alpha, z, m\rangle \mid g\rangle)$, then after a carrier $\pi/2$ pulse is applied to give finally the superposition of states. On detecting the ion any one of its internal states $(\mid e\rangle$ or $\mid g\rangle)$ we get the desired superposition $(\mid \alpha, z, m\rangle + k \mid \beta, z, m\rangle)$.

6 Conclusion

We have discussed some properties and generation scheme of superposition of Squeezed Displaced Fock States with different cohernt parameters (SDFSs). We have calculated the photon number distribution, characteristic function and quasiprobability distribution functions. moments have been presented through charactristic function. The second-order correlation function g^2 have been calculated numerically.The squeezeing properties of this state which is presented as analytical and numerical results are increasing by changing the coherent parameters. We have found the basic features of two of superposition of SDFSs, such as the appearance of increased number of separted peaks in the quadrature distribution by increasing the coherent parameters.

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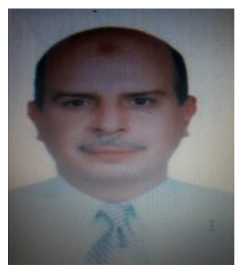
Abdel-Shafy F. Obada, received D. Sc. and Ph. D. from Manchester University, 1994 and 1964. His current research interests include quantum resources, optical and atomic implementations of quantum information tasks and generations. Membership of Scientific Societies: Institute of Physics (London), International Association of Mathematical Physics, International Center for Theoretical Physics (ICTP) Trieste, Italy, and Mathematical and Physical Society of Egypt. He is the Chairman of Egyptian Mathematical Society, member of American Mathematical Society, Egyptian Academy of Sciences, New York Academy of Sciences and African Academy of Sciences. He has awarded several prizes, such as, State prize for scientific publications and citation 2008, State Prize for Scientific Innovation 2004, State Recognition Prize in Basic Sciences for 2005. He has published over 200 papers in international refereed journals, some books and book chapters.



Mohammed M. A. Ahmed Professor of applied Mathematical and Head of Department of Mathematics, Faculty of Science, Al-Azhar University, Cairo (Egypt). He got his Ph.D. degree in 1987 for some studies in Nonlinear Optics (Quantum optics). His research interest is Quantum Optics.

Norhan H. Gerish is a M. Sc. student at Department of Mathematics, Faculty of Science, Suez Canal University, Ismailia (Egypt). She works as Demonstrator of applied Mathematics. Her research interest is Quantum optics .